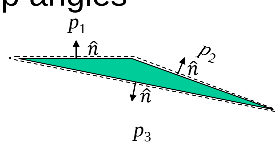
Supersonic Airfoils

- Early on in supersonic aircraft design, recognized that thin airfoils with sharp edges were advantageous
  - avoid detached shocks
  - reduced drag
- Examine use of analysis based on oblique shocks and expansions to investigate aerodynamic forces on an airfoil in supersonic flight conditions
  - shock-expansion theory

Images from NASA

Shock-Expansion Theory

- Goal is to determine/estimate lift and drag coefficients for thin supersonic airfoils
- Limit focus to airfoil shapes composed of straight line segments connected by sharp angles



- Recall from mom. conservation for CS around body (or from statics)

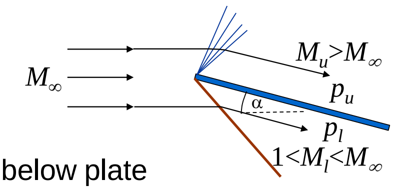
$$\sum \vec{F}_{body} = \sum \vec{F}_{surface}$$

  - assuming inviscid flow, just need to know pressures along surface

Georgia Tech College of Engineering
School of Aerospace Engineering

Example: Thin Flat Plate

- Begin with “thin” flat plate at angle of attack (α) to oncoming flow
  - – thin means can ignore thickness
- What will flow look like?
  - – below?
    - attached oblique shock below plate to turn flow downward
  - – above?
    - expansion fan above plate to turn flow downward
- Can use what we’ve learned to calculate pressure after turns (upper and lower sides), but first let’s examine lift and drag coefficients



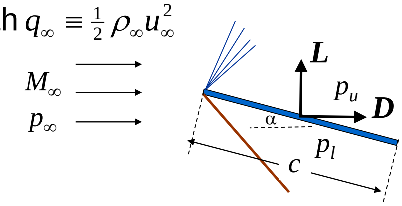
Georgia Tech College of Engineering
School of Aerospace Engineering

Thin Flat Plate: Lift and Drag Coeffs.

- Recall lift and drag coefficients for 2-d airfoil

$$c_\ell \equiv \frac{L/S}{q_\infty c} \qquad c_d \equiv \frac{D/S}{q_\infty c} \qquad \text{with } q_\infty \equiv \tfrac{1}{2}\rho_\infty u_\infty^2$$

- Also, lift and drag are defined relative to the oncoming flow direction
  - – so to get lift and drag per span, we can use the projected “area”



$$\frac{L}{S} = (p_l - p_u)\times c(\cos\alpha) \qquad \frac{D}{S} = (p_l - p_u)\times c(\sin\alpha)$$

- Finally, $u_\infty = a_\infty M_\infty \qquad p_\infty = \rho_\infty R T_\infty$

$$q_\infty = \tfrac{1}{2}\rho_\infty u_\infty^2 = \tfrac{1}{2}\rho_\infty(\gamma R T_\infty)M_\infty^2 = \tfrac{1}{2}\gamma p_\infty M_\infty^2$$

Georgia Tech College of Engineering
School of Aerospace Engineering

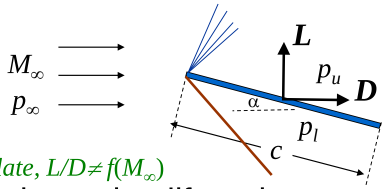
Thin Flat Plate: Lift and Drag Coeffs.

- Combining, we find

Lift, drag coeff. functions only of flight Mach number and angle of attack

$$c_\ell = \frac{2}{\gamma M_\infty^2}\left(\frac{p_l}{p_\infty} - \frac{p_u}{p_\infty}\right)\cos\alpha$$

$$c_d = \frac{2}{\gamma M_\infty^2}\left(\frac{p_l}{p_\infty} - \frac{p_u}{p_\infty}\right)\sin\alpha$$



For flat plate, $L/D \neq f(M_\infty)$

- So what we have to find to determine lift and drag coefficients for our airfoil is
  - p_l/p_∞, the pressure ratio across the oblique shock
  - p_u/p_∞, the pressure ratio across the PM expansion

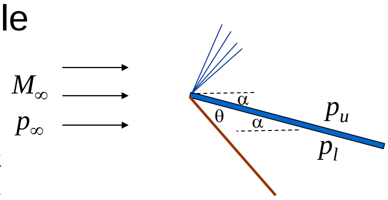
Wave pressure ratios functions only of M_∞ and turn angle

Georgia Tech College of Engineering
School of Aerospace Engineering

Thin Flat Plate: Pressure Calculations

- Lower side – oblique shock
  - 1st determine turn angle
    - from geometry = α
  - from V.B22

$$\frac{p_u}{p_\infty} = \frac{2\gamma}{\gamma+1} M_\infty^2 \sin^2\theta - \frac{\gamma-1}{\gamma+1}$$



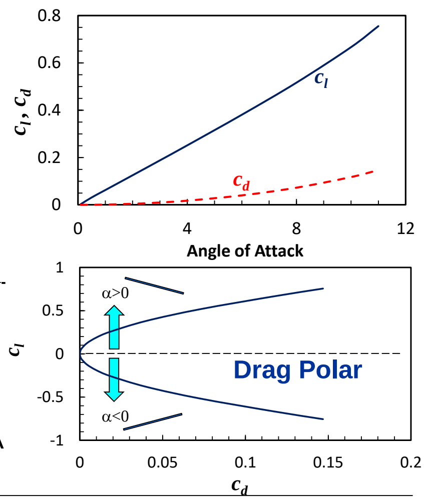
    - and θ from V.B26 or 27 using α as turn angle
- Upper side – PM expansion
  - get p from isentropic relations
  - and get post-expansion Mach number from PM relation (V.C3 or 4)

$$\frac{p_l}{p_\infty} = \left(\frac{1+\frac{\gamma-1}{2}M_l^2}{1+\frac{\gamma-1}{2}M_\infty^2}\right)^{\frac{\gamma}{\gamma-1}}$$

Georgia Tech | College of Engineering | School of Aerospace Engineering

Thin Flat Plate: Results

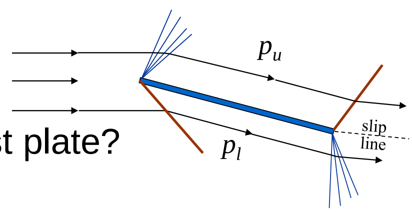
- Results of this analysis shown for air (γ=1.4) and M_∞=1.5
- Lift and drag increase with angle of attack
  - larger $p_l - p_u$
- Very high L/D for small angles
  - only included **wave and form drag**
  - neglected viscous (skin friction) drag
- c_l vs c_d called Drag Polar
  - symmetric about 0 AOA for symmetric airfoil



Georgia Tech | College of Engineering | School of Aerospace Engineering

Thin Flat Plate: Trailing Edge

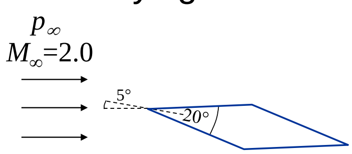
- What happens at trailing edge of our airfoil?
- Boundary conditions past plate?
  - pressure BC?
    - must match across streamlines
    - $p_u < p_l$ so…
    - but what is final pressure?
  - use velocity BC
    - directions must match (parallel streamlines)
    - will this produce horizontal flow?...up?...down?



Georgia Tech | College of Engineering | School of Aerospace Engineering

Example: Diamond Airfoil

- **Given:** Symmetric diamond airfoil flying supersonic at 50 kft
  - M_∞= 2.0
  - 20° leading edge
  - 5° angle of attack

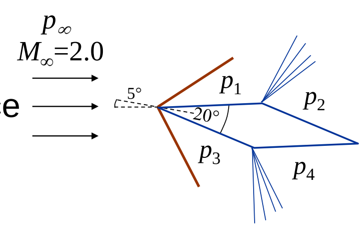


- **Find:** lift and drag coefficients will need to find pressures along each surface
- **Assume:** air is tpg/cpg with γ=1.4, isentropic flow except in shocks

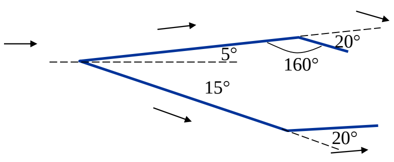
Georgia Tech | College of Engineering | School of Aerospace Engineering

Diamond Airfoil: Flow Features

- **Analysis:**
- To find lift and drag coefficients will need to find pressures along each surface

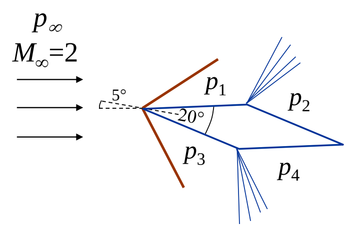


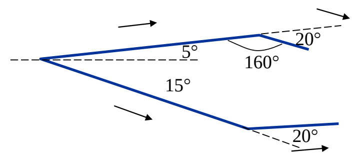
- What will flow look like?
  - upper surfaces
    - 5° upward turn
    - then 20° down
  - lower:
    - 15° downward turn
    - then 20° up



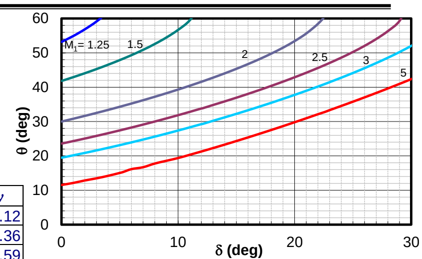
Diamond Airfoil: Surface Pressures

- **Analysis:**
- First find pressures
  - surface 1, after shock
  - surface 2, after expansion
  - surface 3, after shock
  - surface 4, after expansion





AE2010



M	ν	M	ν	M	ν	M	ν	M	ν	M	ν
1.40	8.99	1.70	17.81	1.90	23.59	2.10	29.10	2.30	34.28	2.50	39.12
1.41	9.28	1.71	18.10	1.91	23.87	2.11	29.36	2.31	34.53	2.51	39.36
1.42	9.57	1.72	18.40	1.92	24.15	2.12	29.63	2.32	34.78	2.52	39.59
1.43	9.86	1.73	18.69	1.93	24.43	2.13	29.90	2.33	35.03	2.53	39.82
1.44	10.15	1.74	18.98	1.94	24.71	2.14	30.16	2.34	35.28	2.54	40.05
1.45	10.44	1.75	19.27	1.95	24.99	2.15	30.43	2.35	35.53	2.55	40.28
1.46	10.73	1.76	19.56	1.96	25.27	2.16	30.69	2.36	35.77	2.56	40.51
1.47	11.02	1.77	19.86	1.97	25.55	2.17	30.95	2.37	36.02	2.57	40.74
1.48	11.32	1.78	20.15	1.98	25.83	2.18	31.21	2.38	36.26	2.58	40.96
1.49	11.61	1.79	20.44	1.99	26.10	2.19	31.47	2.39	36.50	2.59	41.19
1.50	11.91	1.80	20.73	2.00	26.38	2.20	31.73	2.40	36.75	2.60	41.41
1.51	12.20	1.81	21.01	2.01	26.66	2.21	31.99	2.41	36.99	2.61	41.64
1.52	12.49	1.82	21.30	2.02	26.93	2.22	32.25	2.42	37.23	2.62	41.86
1.53	12.79	1.83	21.59	2.03	27.20	2.23	32.51	2.43	37.47	2.63	42.09
1.54	13.09	1.84	21.88	2.04	27.48	2.24	32.76	2.44	37.71	2.64	42.31
1.55	13.38	1.85	22.16	2.05	27.75	2.25	33.02	2.45	37.95	2.65	42.53
1.56	13.68	1.86	22.45	2.06	28.02	2.26	33.27	2.46	38.18	2.66	42.75
1.57	13.97	1.87	22.73	2.07	28.29	2.27	33.53	2.47	38.42	2.67	42.97
1.58	14.27	1.88	23.02	2.08	28.56	2.28	33.78	2.48	38.66	2.68	43.19
1.59	14.56	1.89	23.30	2.09	28.83	2.29	34.03	2.49	38.89	2.69	43.40

M_1	M_2	p_2/p_1	T_2/T_1	ρ_2/ρ_1	p_{o2}/p_{o1}
1.10	0.912	1.245	1.065	1.169	0.999
1.12	0.897	1.297	1.078	1.203	0.998
1.14	0.882	1.350	1.090	1.238	0.997
1.16	0.868	1.403	1.103	1.272	0.996
1.18	0.855	1.458	1.115	1.307	0.995
1.20	0.842	1.513	1.128	1.342	0.993
1.22	0.830	1.570	1.141	1.376	0.991
1.24	0.818	1.627	1.153	1.411	0.988
1.26	0.807	1.686	1.166	1.446	0.986
1.28	0.796	1.745	1.178	1.481	0.983
1.30	0.786	1.805	1.191	1.516	0.979
1.32	0.776	1.866	1.204	1.551	0.976
1.34	0.766	1.928	1.216	1.585	0.972
1.36	0.757	1.991	1.229	1.620	0.968
1.38	0.748	2.055	1.242	1.655	0.963
1.40	0.740	2.120	1.255	1.690	0.958
1.42	0.731	2.186	1.268	1.724	0.953
1.44	0.723	2.253	1.281	1.759	0.948

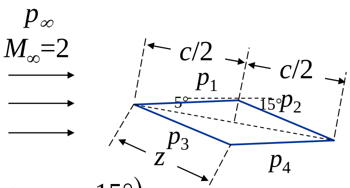
Georgia Tech College of Engineering
School of Aerospace Engineering

Diamond Airfoil: Lift and Drag Coeff.

- **Analysis:** Lift/drag coefficients

$$c_\ell = \frac{L/cS}{\frac{1}{2}\gamma p_\infty M_\infty^2} \qquad c_d = \frac{D/cS}{\frac{1}{2}\gamma p_\infty M_\infty^2}$$

- Use projected "areas" in horizontal/vertical directions



$$\frac{L}{S} = z\left(p_3\cos 15° + p_4\cos 5°\right) - \left(p_1\cos 5° + p_2\cos 15°\right)$$

$$z = \frac{(c/2)}{\cos 10°}$$

  - so

$$c_\ell = \frac{1}{\cos 10°}\frac{1}{\gamma M_\infty^2}\left[\left(\frac{p_3}{p_\infty}\cos 15° + \frac{p_4}{p_\infty}\cos 5°\right) - \left(\frac{p_1}{p_\infty}\cos 5° + \frac{p_2}{p_\infty}\cos 15°\right)\right]$$

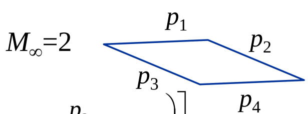
  - and

$$c_d = \frac{1}{\cos 10°}\frac{1}{\gamma M_\infty^2}\left[\left(\frac{p_1}{p_\infty}\sin 5° + \frac{p_3}{p_\infty}\sin 15°\right) - \left(\frac{p_2}{p_\infty}\sin 15° + \frac{p_4}{p_\infty}\sin 5°\right)\right]$$

Georgia Tech College of Engineering
School of Aerospace Engineering

Diamond Airfoil: Lift and Drag Coeff.

- **Analysis:** Numbers



$$c_\ell = \frac{1}{\cos 10°}\frac{1}{\gamma M_\infty^2}\left[\left(\frac{p_3}{p_\infty}\cos 15° + \frac{p_4}{p_\infty}\cos 5°\right) - \left(\frac{p_1}{p_\infty}\cos 5° + \frac{p_2}{p_\infty}\cos 15°\right)\right]$$

$$c_\ell = \frac{1}{\cos 10°}\frac{1}{1.4(2)^2}\times\left[\begin{array}{l}(2.195\cos 15° + 2.195(0.3457)\cos 5°) \\ -\ (1.315\cos 5° + 1.315(0.2990)\cos 15°)\end{array}\right]$$

$$c_\ell = 0.215 \qquad c_d = 0.0933$$

  - so L/D = 2.3
    - << result for flat plate at 5° AOA
    - why?