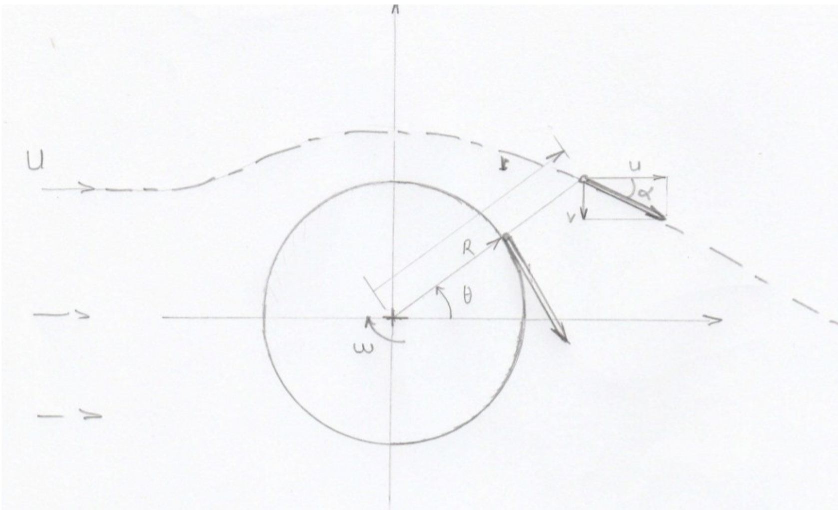
Rotating cylinder in an air stream

Air speed components, module and orientation at any point within the frame of the 2D perfect fluid theory



Notations : U : speed upstream,
v : tangential speed of the rotor at its surface ($v = \omega R$)
R : rotor radius
u,v : speed components at point (r,θ)

Basic formulations, issued from the 2D perfect fluid theory :

Ref : http://air-et-terre.info/aerodyn_theorique/cylindre_2D.pdf

$u(r, \theta) = U. \{1 - \cos(2\theta) .(R/r)^2\} + \Gamma.\sin(\theta)/(2\pi.r)$

$v(r, \theta) = - U. \sin(2\theta) .(R/r)^2 - \Gamma.\cos(\theta)/(2\pi.r)$

, with circulation $\Gamma = 2 \pi v R$

>>> $u(r, \theta) = U. \{1 - \cos(2\theta).(R/r)^2\} + v (R/r) \sin(\theta)$

$v(r, \theta) = - U.\sin(2\theta).(R/r)^2 - v (R/r) \cos(\theta)$

Computation of the speed module S such as : S^2 = u^2 + v^2

Here I detailed all the computation steps so that every one can check more easily

$S^2 = U^2 \{ 1+ \cos(2\theta)^2 (R/r)^4 - 2 \cos(2\theta) (R/r)^2 \} + v^2 (R/r)^2 \sin(\theta)^2 + 2 U \{1 - \cos(2\theta).(R/r)^2\} . v (R/r) \sin(\theta) + v^2 (R/r)^2 \cos(\theta)^2 + 2 (- U.\sin(2\theta).(R/r)^2) . (- v (R/r) \cos(\theta))$

$S^2 = U^2 + U^2 \cos(2\theta)^2 (R/r)^4 - 2 U^2 \cos(2\theta) (R/r)^2 + v^2 (R/r)^2 \sin(\theta)^2 + 2 U v (R/r) \sin(\theta) - 2 U v (R/r)^3 \cos(2\theta) \sin(\theta) + U^2 \sin(2\theta)^2 (R/r)^4 + v^2 (R/r)^2 \cos(\theta)^2 + 2 U v (R/r)^3 \sin(2\theta) \cos(\theta)$

$S^2 = U^2 + U^2 (R/r)^4 - 2 U v (R/r)^3 (\cos(2\theta) \sin(\theta) - \sin(2\theta) \cos(\theta)) -$
$(2 U^2 \cos(2\theta) - v^2) (R/r)^2 + 2 U v (R/r) \sin(\theta)$

as $(\cos(2\theta) \sin(\theta) - \sin(2\theta) \cos(\theta)) = -\sin(\theta)$ >>>

$S = [U^2 + U^2 (R/r)^4 + 2 U v (R/r)^3 \sin(\theta) - (2 U^2 \cos(2\theta) - v^2) (R/r)^2 + 2 U v (R/r) \sin(\theta)]^{0,5}$

Some particular cases :

Case 1 : Speed at the rotor cylinder, when r = R :

$S^2 = U^2 + U^2 + 2 U v \sin(\theta) - (2 U^2 \cos(2\theta) - v^2) + 2 U v \sin(\theta)$

$S^2 = 2 U^2 (1 - \cos(2\theta)) + 4 U v \sin(\theta) + v^2$

as : $1 - \cos(2\theta) = 2 \sin(\theta)^2$

$S^2 = 4 U^2 \sin(\theta)^2 + 4 U v \sin(\theta) + v^2 = (2 U \sin(\theta) + v)^2$

>>> **$S = | 2 U \sin(\theta) + v |$** confirmation of the well-known formula

Case 2 : speeds at θ = 90° , v/U = 1, 2, 3 , 4 and r = R, 1,5R, 2R

$S^2 = U^2 + U^2 (R/r)^4 + 2 U v (R/r)^3 - (2 U^2 . (-1) - v^2) (R/r)^2 +$
$2 U v (R/r)$

$S^2 = U^2 + U^2 (R/r)^4 + 2 U v (R/r)^3 + (2 U^2 + v^2) (R/r)^2 + 2 U v (R/r)$

r = R >>> $S_R = 2 U + v = U. (2 + v/U)$ = *« Norwood » used for the cylinder*
r = 1,5 R >> $S_{1,5R}^2 = U^2 + 0{,}19753 U^2 + 0{,}5926 U v + 0{,}4444 (2 U^2 + v^2) + 1{,}3333 U v$
$S_{1,5R} = U. [2{,}08642 + 1{,}92593 (v/U) + 0{,}4444 (v/U)^2]^{0,5}$
instead of « Norwood » : $S_{1,5R} = U (2 + 1{,}5 (v/U))$ used for the end plates
r = 2 R >>> $S_{2R}^2 = U^2 + 0{,}0625 U^2 + 0{,}25 U v + 0{,}25 (2 U^2 + v^2) + Uv$
$S_{2R} = U. [1{,}5625 + 1{,}25 (v/U) + 0{,}25 (v/U)^2]^{0,5}$
instead of « Norwood » : $S_{2R} = U (2 + 2 (v/U))$ used for the end plates

θ = 90°	v/U = 1	2	3	4
>>> $Speed_R$	3 U	4 U	5 U	6 U
>>> $Speed_{1,5R}$	2,11 U	2,78 U	3,45 U	4,11 U
« Norwood » $S_{1,5R}$	*3,5 U*	*5 U*	*6,5 U*	*8 U*
>>> $Speed_{2R}$	1,75 U	2,25 U	2,75 U	3,25 U
« Norwood » S_{2R}	*4 U*	*6 U*	*8 U*	*10 U*

Case 3 : speeds at θ = 0° , v/U = 1, 2, 3 , 4 and r = R, 1,5R, 2R

$S^2 = \mathbf{U^2 + U^2 (R/r)^4 – (2 U^2 – v^2) (R/r)^2}$

r = R >>> $S_R = v = U\,(v/U)$
r = 1,5 R >> $S_{1,5R}^2 = U^2 + 0{,}19753\,U^2 - 0{,}4444\,(2U^2 - v^2)$
$S_{1,5R} = U.\,[\,0{,}30873 + 0{,}4444\,(v/U)^2\,]^{0,5}$
r = 2 R >>> $S_{2R}^2 = U^2 + 0{,}0625\,U^2 - 0{,}25\,(2U^2 - v^2)$
$S_{2R} = U.\,[0{,}5625 + 0{,}25\,(v/U)^2\,]^{0,5}$

θ = 0°	v/U = 1	2	3	4
>>> $Speed_R$	U	2 U	3 U	4 U
>>> $Speed_{1,5R}$	0,87 U	1,44 U	2,08 U	2,72 U
>>> $Speed_{2R}$	0,90 U	1,25 U	1,68 U	2,14 U

Computation of the Speed orientation α such as tg(α) = v / u

$\mathbf{tg(\alpha) = [- U.\sin(2\theta).(R/r)^2 - v\,(R/r)\cos(\theta)] / [\,U.\{1 - \cos(2\theta).(R/r)^2\} + v\,(R/r)\sin(\theta)\,]}$

Some particular cases :

Case 1 : to check that for r = R, the speed is tangential to the rotor cylinder

$tg(\alpha) = [- U.\sin(2\theta) - v\cos(\theta)] / [\,U.\{1 – \cos(2\theta)\} + v\sin(\theta)\,]$

as : $1 - \cos(2\theta) = 2\sin(\theta)^2$ and $\sin(2\theta) = 2\sin(\theta)\cos(\theta)$ >>> :

$tg(\alpha) = [- 2U.\sin(\theta)\cos(\theta) - v\cos(\theta)] / [\,2U\sin(\theta)^2 + v\sin(\theta)]$

$tg(\alpha) = - [\,\cos(\theta)\,(2U\sin(\theta) + v\,] / [\,\sin(\theta)\,(2U\sin(\theta)+ v\,)\,]$

>>> $tg(\alpha) = - 1 / tg(\theta)$, meaning that speed is indeed tangential to the cylinder

Case 2 : speed orientation at θ = 90°

$tg(\alpha) = [- U.\sin(2\theta).(R/r)^2 - v\,(R/r)\cos(\theta)] / [\,U.\{1 - \cos(2\theta).(R/r)^2\} + v\,(R/r)\sin(\theta)\,]$

$tg(\alpha) = [- 0 - 0\,] / [\,U.\{1 + (R/r)^2\} + v\,(R/r)\,] = 0$ whatever r and v/U, all the speed orientations are in the Ox direction

Case 3 : speed orientation at θ = 0° , v/U = 1, 2, 3 , 4 and r = R, 1,5R, 2R

$tg(\alpha) = [- v\,(R/r)\,] / [\,U.\{1 – (R/r)^2\}\,] = - (v/U)\,(R/r) / (1 – (R/r)^2)$
$\alpha_R = - 90°$
$\alpha_{1,5R} = \text{Atan}\,(- 1{,}2\,(v/U))$
$\alpha_{2R} = \text{Atan}\,(-0{,}6667\,(v/U))$

θ = 0°	v/U = 1	2	3	4
>>> α_R	- 90°	- 90°	- 90°	- 90°

>>> $\boldsymbol{\alpha}_{1,5R}$	-50,2°	-70,9°	-74,5°	-78,2°
>>> $\boldsymbol{\alpha}_{2R}$	-33,7°	-53,1°	-63,4°	-69,4°