Defining it formally, the *centre of gravity* of a body is that point through which, for statical considerations, the whole weight of the body may be assumed to act. The first moment of weight about the centre of gravity is zero.

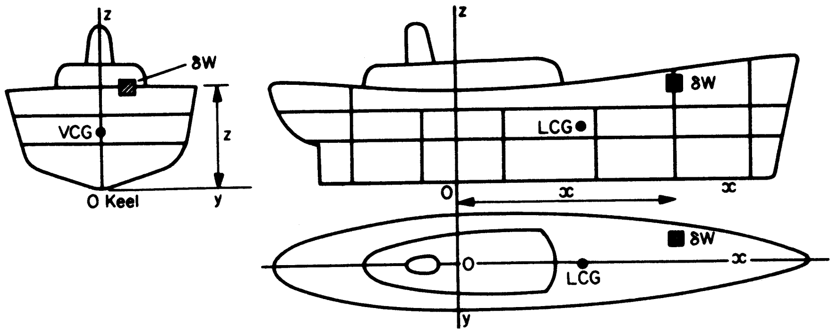

Fig. 2.21 Centre of gravity projections

METACENTRE

Consider any body floating upright and freely at waterline WL, whose centre of buoyancy is at B. Let the body now be rotated through a small angle in the plane of the paper without altering the volume of displacement (it is more convenient to draw if the body is assumed fixed and the waterline rotated to W_1L_1). The centre of buoyancy for this new immersed shape is at B_1. Lines through B and B_1

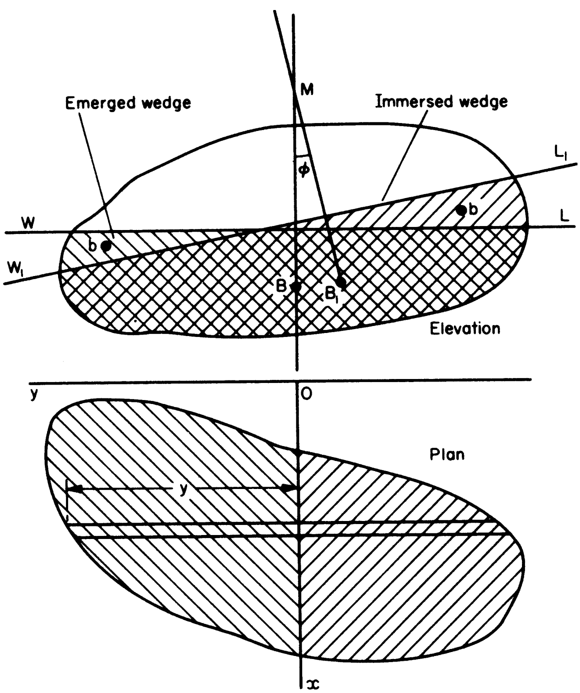

Fig. 2.22

normal to their respective waterlines intersect at M which is known as the *metacentre* since it appears as if the body rotates about it for small angles of rotation. The *metacentre* is the point of intersection of the normal to a slightly inclined waterplane of a body, rotated without change of displacement, through the centre of buoyancy pertaining to that waterplane and the vertical plane through the centre of buoyancy pertaining to the upright condition. The term metacentre is reserved for small inclinations from an upright condition. The point of intersection of normals through the centres of buoyancy pertaining to successive waterplanes of a body rotated infinitesimally at any angle of inclination without change of displacement, is called the *pro-metacentre*.

If the body is rotated without change of displacement, the volume of the immersed wedge must be equal to the volume of the emerged wedge. Furthermore, the transfer of this volume from the emerged to the immersed side must be responsible for the movement of the centre of buoyancy of the whole body from B to B_1; from this we conclude:

(*a*) that the volumes of the two wedges must be equal
(*b*) that the first moments of the two wedges about their line of intersection must, for equilibrium, be equal and
(*c*) that the transfer of first moment of the wedges must equal the change in first moment of the whole body.

Writing down these observations in mathematical symbols,

$$\begin{aligned}
\text{Volume of immersed wedge} &= \int y \times \tfrac{1}{2} y\phi \, \mathrm{d}x \\
&= \text{Volume of emerged wedge} \\
\text{1st moment of immersed wedge} &= \int \left(\tfrac{1}{2} y^2 \phi\right) \times \tfrac{2}{3} y \, \mathrm{d}x \\
&= \text{1st moment of emerged wedge} \\
\text{Transfer of 1st moment of wedges} &= 2 \times \int \tfrac{1}{3} y^3 \phi \, \mathrm{d}x = \tfrac{2}{3}\phi \int y^3 \, \mathrm{d}x
\end{aligned}$$

$$\text{Transfer of 1st moment of whole body} = \nabla \times \overline{\mathrm{BB}'} = \nabla \cdot \overline{\mathrm{BM}} \cdot \phi$$

$$\therefore \; \nabla \cdot \overline{\mathrm{BM}} \cdot \phi = \tfrac{2}{3}\phi \int y^3 \, \mathrm{d}x$$

But we have already seen that $I = \frac{2}{3}\int y^3 \, \mathrm{d}x$ about the axis of inclination for both half waterplanes

$$\therefore \; \overline{\mathrm{BM}} = \frac{I}{\nabla}$$

This is an important geometric property of a floating body. If the floating body is a ship there are two $\overline{\mathrm{BM}}$s of particular interest, the transverse $\overline{\mathrm{BM}}$ for rotation about a fore-and-aft axis and the longitudinal $\overline{\mathrm{BM}}$ for rotation about a transverse axis, the two axes passing through the centre of flotation of the waterplane.