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Report No. 925-P

12th International Symposium on Yacht Design and Construction, HISWA, November 1992.

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by

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Summary

The Delft Systematic Yacht Hull Series has been extended to a total of 39 hull form variations, covering a wide range of length-displacement ratio's and other form parameters.
The total set of modelexperiment results, including upright and heeled resistance as well as sideforce and stability, has been analysed and polynomial expressions to approximate these quantities are presented.
In view of the current interest in the preformance of sailing yachts in waves the added resistance in irregular waves of 8 widely different hull form variations has been calculated. Analysis of the results shows that the added resistance in waves strongly depends on the product of displacement-length ratio and the gyradius of the pitching motion.

CONTENTS

Nomenclature

A_W	- waterline area
A_X	- maximum cross-section area
AR	- aspect ratio
B_{WL}	- waterline breadth
B_{MAX}	- maximum beam
C_F	- frictional resistance coefficient
C_H	- heeled resistance coefficient
C_M	- maximum cross section coefficient
C_P	- prismatic coefficient
C_{Di}	- induced resistance coefficient
C_L	- lift coefficient
F_H	- side force
Fn	- Froude number
GM	- metacentric height
g	- acceleration due to gravity
$\bar{H}_{1/3}$	- significant wave height

k_{yy} - pitch gyradius

LCB - longitudinal center of buoyancy in % L_{WL}

L_{WL} - waterline length

MN - residuary stability

q - stagnation pressure - $\frac{1}{2}\rho V^2$

R_φ - total resistance with heel and leeway

R_T - total resistance in upright posistion

R_F - frictional resistance

R_R - residuary resistance

R_i - induced resistance

R_H - resistance due to heel

R_{AW} - added resistance in waves

S_k - wetted area keel

S_c - wetted area canoe body

S_r - wetted area rudder

S_ζ - spectral density

T_1 - wave period $T_1 = 2\ \pi\ m_0/m_1$

T_e - period of encounter

T_E - effective draught

T - total draught

T_c - draught of canoe body

V - speed

ζ_a - wave amplitude

φ - heel angle

λ - wave length

ρ - density of water

ω - circular frequency

∇_c - volume of displacement

Δ - weight of displacement

β - leeway angle

μ_w - wave direction

ν - kinematic viscosity

1. Introduction

The research on systematic variations of sailing yacht hull forms at the Delft Ship Hydromechanics Laboratory has been extended and completed with model tests of an additional series of eleven hull form variations: Series III.
The total series now consists of thirty nine models.
The experimental results of the last eleven models have been used to increase the reliability of the upright resistance prediction for light displacement yachts, in particular in the high speed range with Fn > 0.45.
The total experimental result of the completed series has been reanalysed, also with regard to sideforce generation, stability and induced resistance
Three modifications of the keel depth of the parent model 1 have also been included in the analysis.

The resistance-speed characteristics of light-displacement yachts for speeds exceeding Fn = 0.45 is quite different as compared with medium and heavy-displacement yachts as shown in [1].
Consequently velocity calculations based on the results of models 1-22 (Serie I) for light-displacement yachts are not correct for the speed range where the vertical hydrodynamic lift forces on the hull cannot be neglected. For instance the IMS approximation of upright resistance, which depends to a large extend on the Delft Series I and II results, seems to underestimate the upright resistance in the high speed range as shown in Figure 1, where the residuary resistance of model 25, as calculated by the IMS and the Delft formulations are compared with experimental results [2].

The stability of a sailing yacht at heel angles up to 30 degrees is important in view of the sailcarrying capacity. In most cases the hydrostatic stability, assuming an undisturbed free surface, can be used as an approximation in a velocity prediction calculation.
However, in the case of ligth-displacement hull forms, with a high beam to draught ratio B_{WL}/T_C, the distortion of the free surface and the corresponding distribution of the hydrodynamic pressure on the hull is quite different form this assumption. A stability reduction of some 20 to 30 % as compared with a hydrostatic calculation has been observed in certain cases, leading to an erroneous velocity prediction, when this reduction is not included in the calculation.
Therefore, the systematic series results also have been used to reanalyse the forward speed effects on stability for all considered hull form variations.
The upright resistance, the heeled resistance, the sideforce and the stability could be expressed in the simple hull form parameters:

$$L_{WL}/\nabla_C^{1/3},\ B_{WL}/T_C,\ T_C/T,\ L_{WL}/B_{WL},\ A_W/\nabla_C^{2/3},$$

LCB, C_p at constant $V/\sqrt{gL_{WL}}$.

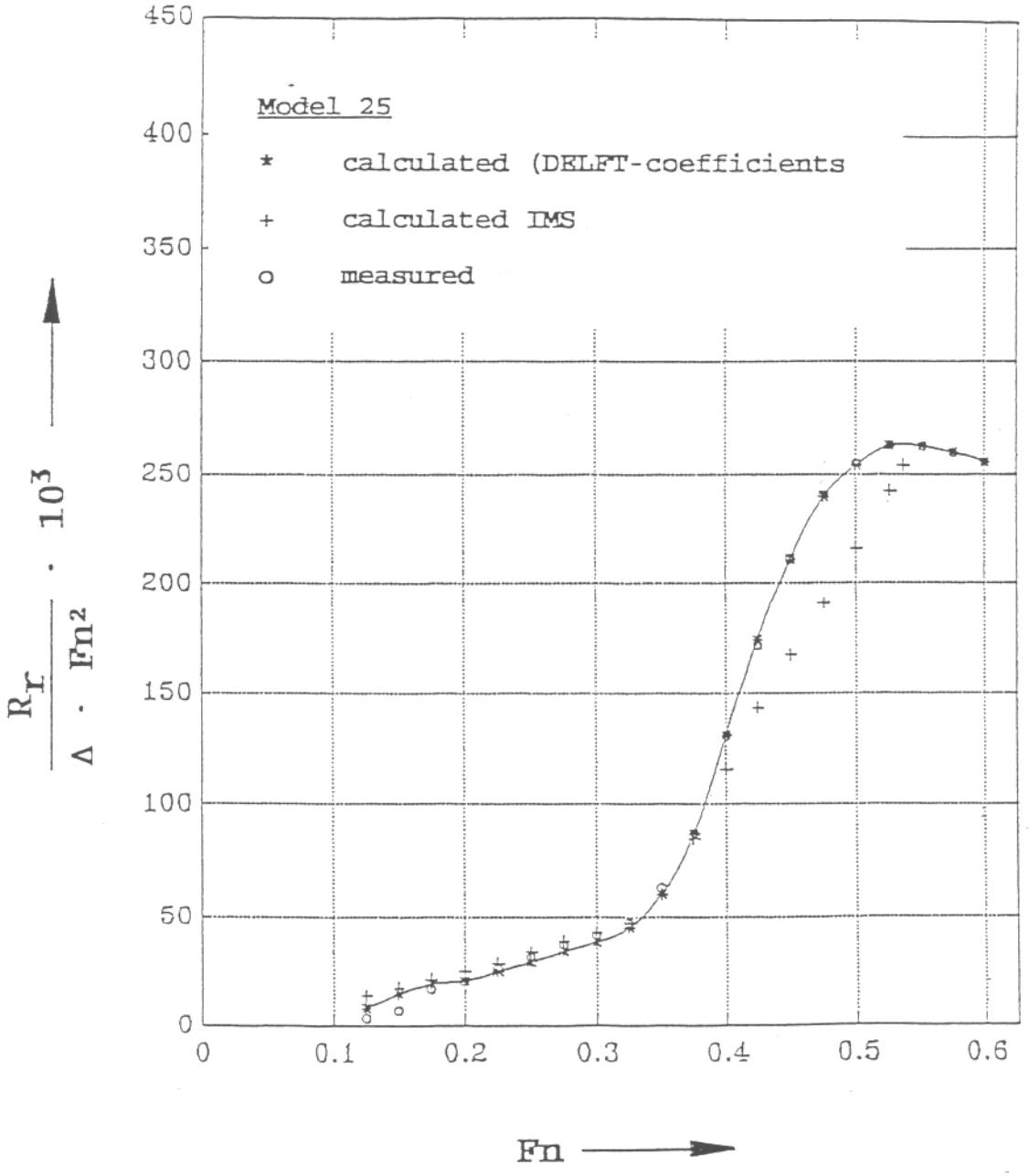


Figure 1: Comparison of IMS and Delft approximations of the residuary resistance with experiments. From [2].

The resulting polynomial expressions may be used for a velocity prediction calculation for a given sailing yacht of known geometry, sailplan and initial stability, assuming that the corresponding sail coefficients are known.
The calculation procedure concerns calm water conditions, assuming that an incident wave system has no influence on the performance of the yacht.

The influence of sea waves with a direction forward of the beam can be estimated when the motions of the yacht due to these waves are known.

The added resistance in waves is related to the damping energy radiated form the oscillating hull. In particular heave- and pitch damping energy is important in this respect, whereas horizontal motions such as sway and yaw can be neglected in this respect [1].
The calculation of the vertical motions and added resistance in waves can be carried out by so called strip-theory methods.
These simplified methods are limited due to neglection of certain 3-dim effects, in particular in resonance conditions. However for practical purposes the simplyfication of the strip theory method is acceptable, at least for analysing purposes. This also applies to the effect of heel angle on the motions in waves.
In general the influence of heel on vertical motions and added resistance is relatively small [1].

The difference of the dynamic response to waves between a light- and medium- or heavy-displacement yacht with comparable length and beam is mainly due to the difference in the natural periods of heave and pitch and the relative damping of these motions.
In general the light displacement yachts have smaller natural pitch and heave periods and larger relative pitch and heave damping.
This causes differences in the added resistance operator which represents the added resistance in regular waves of different length and unit wave height.
In particular there is a shift of the added wave resistance operator to smaller wave-lengths in the case of light-displacement yachts.
When the added resistance response operator for a particular yacht is known, from model experiments or calculation, the added resistance can be determined when the wave spectrum of the considered wave condition is given. The total resistance in waves may be used to carry out a velocity prediction calculation in waves [1].
Directional spreading of wave energy can be included in this procedure, but in view of a lack of data in this respect such a refinement does not seem appropriate.
As a further simplification the wave direction may be set equal to the true wind direction.

With regard to the determination of the added resistance operator for a given yacht it should be remarked that the computing time using strip theory methods is relatively small.
On the other hand it has been shown that the added resistance operator can be very easily expressed by a polynomial expression using only main hull form parameters [3].
In particular for rating purposes such a polynomial expression for the added resistance operator could be useful.
In both the cases standard wave spectra, for instance a Bretschneider formulation using $\bar{H}_{1/3}$ and $\bar{T}_1$, can be used to

compute the added resistance, but in principle any measured wave spectrum can be applied. Added resistance and velocity predictions in seawaves may serve as to show the importance of hull form, mass and the distribution of mass, with some emphasis on the influence of $L_{WL}/\nabla_c^{1/3}$ and the pitch gyradius ratio k_{yy}/L_{WL} [1, 4].

2. Velocity prediction in calm water

In 1977 the results of model experiments with 9 systematic variations of sailing yacht hull forms were published [5].
The measurements included the determination of the upright resistance, the heeled and induced resistance, the sideforce and the stability.
An extension of this research with another series of 12 hull forms was presented in 1981 [6].
All of the 22 hull form variations were based on the sailing yacht Standfast 43 designed by Frans Maas. (Series I).
In view of the trend towards light-displacements a further extension of the series with 6 models (Series II) was completed providing the same kind of information as for Series I and published in 1988 [7] and 1991 [8].
These hull form variations were based on a van de Stadt & Partners designed parent form.
Finally a third series (Series III) of eleven models has been tested, but only in the upright condition, without leeway.
The speed range for Series I is limited to Fn = 0.45, but for the Series II and III speeds corresponding to Fn = 0.75 have been included. With the parent model of Series I three modifacations of the keel span have been tested.

2.1. Main dimensions and form coefficients

The main dimensions of the models 1 - 39, extrapolated to a waterline length L_{WL} = 10 meter are given in Table 2, whereas in Table 3 the form coefficients and the longitudinal position of the centre of buoyancy are summarized.

In Table 1 the ranges of some ratio's of main dimensions and form coefficients are given.

Table 1

Ranges of hull form paramteres		
L_{WL}/B_{WL}	:	2.76 - 5.00
B_{WL}/T_c	:	2.46 - 19.32
$L_{WL}/\nabla_c{}^{1/3}$	:	4.34 - 8.50
LCB	:	0.0 - -6.0%
C_p	:	0.52 - 0.60

Table 2

Main dimensions

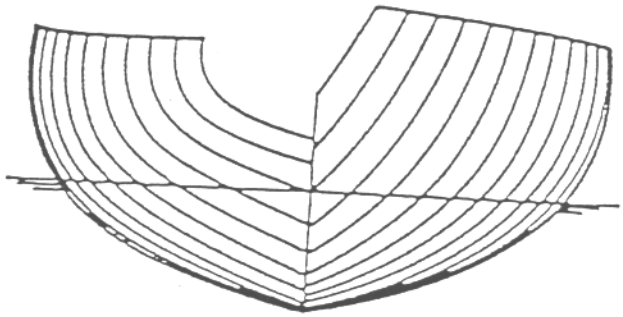
model no.	L_{WL} m	B_{max} m	B_{WL} m	T_C m	T m	∇_C m^3	S_C m^2	A_X m^2	A_W m^2
1	10.04	3.67	3.17	0.794	2.16	9.18	25.4	1.62	21.8
2	10.04	3.21	2.76	0.907	2.28	9.18	23.9	1.62	19.1
3	10.06	4.25	3.64	0.681	2.05	9.16	27.6	1.63	25.2
4	10.06	3.32	2.85	0.722	2.09	7.55	23.0	1.34	19.8
5	10.05	4.24	3.64	0.920	2.29	12.10	29.1	2.15	25.3
6	10.00	3.66	3.17	1.064	2.43	12.24	27.5	2.16	21.9
7	10.06	3.68	3.17	0.640	2.01	7.35	24.1	1.31	21.8
8	10.15	3.54	3.05	0.794	2.16	9.18	25.4	1.57	22.1
9	10.07	3.81	3.28	0.794	2.16	9.18	25.0	1.68	21.5
10	10.00	3.68	3.17	0.794	2.16	9.19	25.6	1.62	22.0
11	10.00	3.68	3.17	0.794	2.16	9.19	25.3	1.62	21.6
12	10.00	3.30	2.85	0.724	2.09	7.52	23.0	1.33	19.8
13	10.00	3.30	2.85	0.724	2.09	7.52	22.8	1.33	19.4
14	10.00	3.30	2.85	0.772	2.14	7.52	22.4	1.42	18.7
15	10.00	3.67	3.16	0.858	2.23	9.29	24.9	1.76	20.8
16	10.00	3.68	3.17	1.128	2.65	12.23	27.3	2.32	20.9
17	10.00	3.68	3.17	0.747	2.12	9.17	26.3	1.53	23.0
18	10.00	3.68	3.17	0.747	2.12	9.17	26.0	1.53	22.6
19	10.00	3.68	3.17	0.845	2.21	9.17	24.8	1.73	21.0
20	10.00	3.68	3.17	0.845	2.21	9.17	24.6	1.73	20.6
21	10.00	3.30	2.85	0.684	2.05	7.54	23.6	1.26	20.5
22	10.00	4.24	3.66	0.865	2.23	12.26	30.2	2.05	26.3
23	10.00	3.20	2.86	0.704	1.80	7.97	23.3	1.46	19.3
24	10.00	3.30	2.86	0.261	1.36	3.00	19.9	0.55	19.0
25	10.00	2.80	2.50	0.464	1.56	4.62	19.0	0.84	16.7
26	10.00	2.90	2.50	0.194	1.29	1.97	17.3	0.36	16.7
27	10.00	2.50	2.22	0.904	2.00	7.92	21.7	1.44	14.9
28	10.00	2.55	2.22	0.329	1.42	2.92	16.2	0.54	14.6
29	10.00	2.93	2.50	0.230	1.33	2.37	17.5	0.43	15.5
30	10.00	2.93	2.50	0.350	1.45	3.64	18.3	0.66	16.7
31	10.00	2.96	2.50	0.160	1.25	1.63	17.1	0.30	14.5
32	10.00	2.95	2.50	0.230	1.33	2.37	17.8	0.43	15.5
33	10.00	2.94	2.50	0.230	1.33	2.37	17.2	0.44	15.3
34	10.00	2.95	2.50	0.240	1.34	2.37	17.0	0.46	16.2
35	10.00	2.95	2.50	0.220	1.32	2.37	18.0	0.41	17.3
36	10.00	3.04	2.50	0.250	1.34	2.37	17.2	0.43	16.3
37	10.00	3.22	2.50	0.270	1.36	2.37	17.0	0.43	16.3
38	10.00	3.92	3.33	0.170	1.27	2.37	22.6	0.43	22.2
39	10.00	2.35	2.00	0.290	1.38	2.37	14.7	0.43	13.4

Table 3

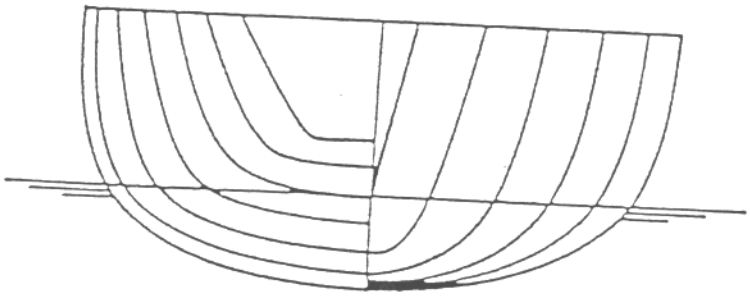
Form parameters

Model no.	L_{WL}/B_{WL}	L_{WL}/B_{max}	B_{WL}/T_c	C_p	$L_{WL}/\nabla_c^{1/3}$	L_{CB}
1	3.17	2.73	3.99	0.568	4.78	-2.3
2	3.64	3.12	3.04	0.569	4.78	-2.3
3	2.76	2.35	5.35	0.565	4.78	-2.3
4	3.53	3.01	3.95	0.564	5.10	-2.3
5	2.76	2.36	3.96	0.574	4.36	-2.4
6	3.15	2.73	2.98	0.568	4.34	-2.4
7	3.17	2.72	4.95	0.562	5.14	-2.3
8	3.32	2.82	3.84	0.585	4.78	-2.4
9	3.07	2.62	4.13	0.546	4.78	-2.2
10	3.15	2.72	3.99	0.565	4.77	0.0
11	3.15	2.72	3.99	0.565	4.77	-5.0
12	3.51	3.03	3.94	0.565	5.10	0.0
13	3.51	3.03	3.94	0.565	5.10	-5.0
14	3.51	3.03	3.69	0.530	5.11	-2.3
15	3.16	2.72	3.68	0.530	4.76	-2.3
16	3.15	2.72	2.81	0.530	4.34	-2.3
17	3.15	2.72	4.24	0.600	4.78	0.0
18	3.15	2.72	4.24	0.600	4.78	-5.0
19	3.15	2.72	3.75	0.530	4.78	0.0
20	3.15	2.72	3.75	0.530	4.78	-5.0
21	3.51	3.03	4.17	0.600	5.10	-2.3
22	2.73	2.36	4.23	0.600	4.34	-2.3
23	3.50	3.13	4.06	0.548	5.00	-1.9
24	3.50	3.03	10.96	0.548	6.93	-2.1
25	4.00	3.57	5.39	0.548	6.01	-1.9
26	4.00	3.45	12.89	0.545	7.97	-2.1
27	4.50	4.00	2.46	0.548	5.02	-1.9
28	4.50	3.92	6.75	0.546	6.99	-1.9
29	4.00	3.41	10.87	0.549	7.50	-4.4
30	4.00	3.41	7.07	0.549	6.50	-4.4
31	4.00	3.38	15.82	0.549	8.50	-4.4
32	4.00	3.39	10.86	0.551	7.50	-2.1
33	4.00	3.40	10.87	0.545	7.50	-6.6
34	4.00	3.39	10.37	0.520	7.50	-4.4
35	4.00	3.39	11.47	0.579	7.50	-4.4
36	4.00	3.29	10.16	0.550	7.50	-4.3
37	4.00	3.11	9.45	0.551	7.50	-4.5
38	3.00	2.55	19.32	0.549	7.50	-4.4
39	5.00	4.26	6.96	0.549	7.50	-4.4

The parent body plans for models 1 - 22 and 29 - 39 are depicted in Figure 2.
The waterline length of all models of Series I (nrs. 1 - 22) is 1.60 meter; for the Series II and III (nrs. 23 - 28 and nrs. 29 - 39) the waterline length is 2.0 meter.



PARENT MODEL (NO. 1 - 22)

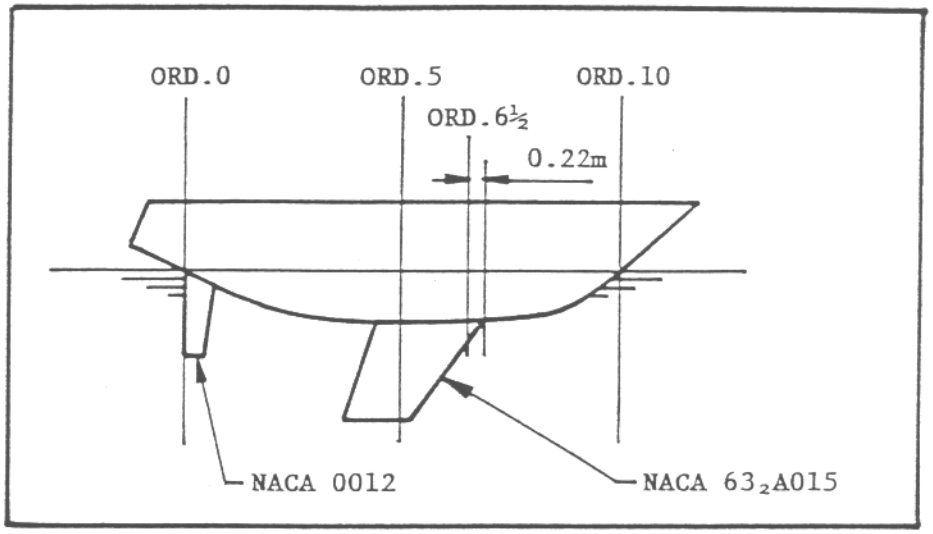


PARENT MODEL (NO. 23 - 39)

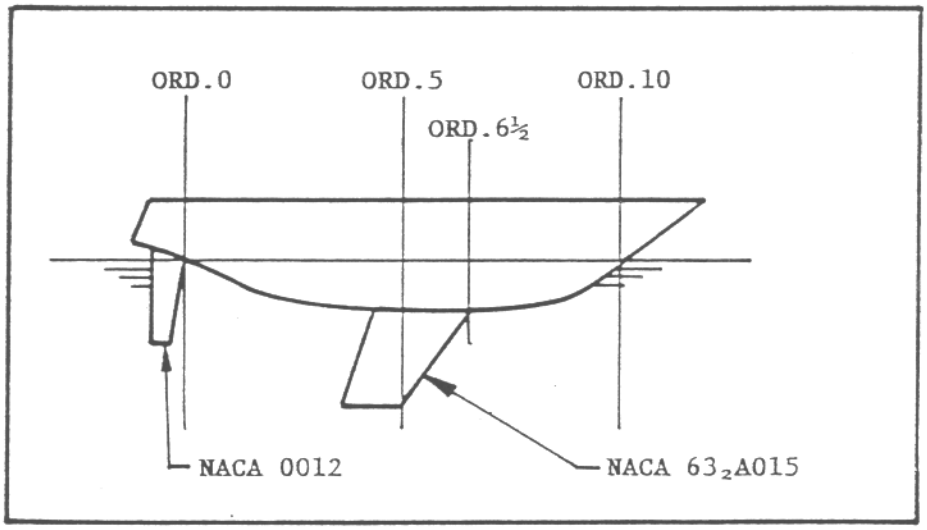
Figure 2: Parent models for the Delft Systematic Yacht Hull Series.

All models were tested with the same keel ane rudder and consequently with the uniform extrapolation to L_{WL} = 10 meters there is a difference in keel span for the Series I on the one hand and the Series II and III on the other hand. For Series I the keel span is 1.37 meter and for Series II and III this is 1.10 meter for the corresponding waterline length L_{WL} = 10 meters.
The keel and rudder location is given in Figure 3.
For the additional keel span variations of model 1 the following cases have been considered for the models 1a, 1b and 1c respectively: 1.25 meter, 1.45 meter and 0.69 meter.

MODELS 1 - 22



MODELS 23 - 39



Geometry of keel and rudder

	volume m^3	wetter area m^2	rootchord m	tipchord m	span m	sweep back angle degrees
Keel	0.00262	0.1539	0.414	0.262	0.219	45
Rudder	0.00023	0.0550	0.124	0.096	0.266	5.4

Figure 3: Position of keel and rudder.

2.2. determination of the hydrodynamic resistance

The total hydrodynamic resistance of a sailing yacht in calm water may be split up in three components:

$$R_\varphi = R_T + R_i + R_H \qquad (1)$$

Where: R_T - upright resistance (no leeway)

R_i - induced resistance due to the generation of side force

R_H - resistance due to heel (no side force)

2.2.1. Upright resistance

The upright resistance is split up in frictional resistance R_F and residuary resistance R_R.
The frictional resistance is calculated using the 1957 ITTC extrapolator:

$$C_F = \frac{0.075}{(\log R_n - 2)^2}, \qquad R_n = \frac{VL}{\nu} \qquad (2)$$

where the Reynolds number R_n for the hull is based on $L = 0.7\ L_{WL}$. For keel and rudder the mean chord lengths have been used.
It has been considered to use the so called Prohaska form factors in the extrapolation procedure, but the difference in the final result is not significant.
For the analysis of the model experiment results the kinematic viscosity ν, corresponding to the measured tank water temperature has been used in all cases.
For resistance prediction purposes:

$$\nu = 1.14 * 10^{-6} \text{ and } 1.19 * 10^{-6}\ m^2sec^{-1}$$

for fresh water and seawater respectively at 15 degrees Celsius may be used.

The wetted surface of the canoe body, without keel and rudder can be approximated by:

$$S_C = [1.97 + 0.171 \frac{B_{WL}}{T_C}] * [\frac{0.65}{C_M}]^{1/3} * [\nabla_C * L_{WL}]^{1/2} \qquad (3)$$

with: $$C_M = \frac{\nabla_C}{L_{WL} * B_{WL} * T_C * C_p}.$$

The frictional resistance follows from:

$$R_F = \tfrac{1}{2}\rho V^2 (S_C\ C_{FC} + S_k\ C_{Fk} + S_r\ C_{Fr}) \qquad (4)$$

where the indices c, k and r refer to respectively the canoe body, the keel and the rudder.

Using a least squares method the residuary resistance of all tested models is expressed in a polynomial expression, using hull form parameters as variables.
For the speed range Fn = 0.125(0.025)0.450 the parameters C_p, $L_{WL}/\nabla_C^{1/3}$, LCB and B_{WL}/T_C have been used:

$$\frac{R_R}{\Delta_C} * 10^3 = a_0 + a_1\ C_p + a_2(LCB) + a_3(B_{WL}/T_C) +$$

$$+ a_4(L_{WL}/\nabla_C^{1/3}) + a_5\ C_p^2 + a_6\ C_p * (L_{WL}/\nabla_C^{1/3}) +$$

$$+ a_7(LCB)^2 + a_8(L_{WL}/\nabla_C^{1/3})^2 + a_9(L_{WL}/\nabla_C^{1/3})^3 \qquad (5)$$

For the speed range Fn = 0.475(0.025)0.750 the polynomial fit is as follows:

$$\frac{R_R}{\Delta_C} * 10^3 = C_0 + C_1(L_{WL}/B_{WL}) + C_2(A_W/\nabla_C^{2/3}) + C_3(LCB) +$$

$$+ C_4(L_{WL}/B_{WL})^2 + C_5(L_{WL}/B_{WL}) * (A_W/\nabla_C^{2/3})^3 \qquad (6)$$

The coefficients a and c are given in the Tables 4 and 5.
It should be noted that Δ_C is the weight of displacement of the canoe body, without keel and rudder. ∇_C is the corresponding volume of displacement.
The waterline area A_W may be approximated with sufficient accuracy by:

$$\frac{A_W}{L_{WL} * B_{WL}} = 1.313C_p + 0.0371(L_{WL}/\nabla_C^{1/3}) - 0.0857C_p * (LWL/\nabla_C^{1/3}) \qquad (7)$$

Table 4

Fn	a0 a5	a1 a6	a2 a7	a3 a8	a4 a9
0.125	-6.735654 -38.86081	+38.36831 +0.956591	-0.008193 -0.002171	+0.055234 +0.272895	-1.997242 -0.017516
0.150	-0.382870 -39.55032	+38.17290 +1.219563	+0.007243 +0.000052	+0.026644 +0.824568	-5.295332 -0.047842
0.175	-1.503526 -31.91370	+24.40803 +2.216098	+0.012200 +0.000074	+0.067221 +0.244345	-2.448582 -0.015887
0.200	+11.29218 -11.41819	-14.51947 +5.654065	+0.047182 +0.007021	+0.085176 -0.094934	-2.673016 +0.006325
0.225	+22.17867 +7.167049	-49.16784 +8.600272	+0.085998 +0.012981	+0.150725 -0.327085	-2.878684 +0.018271
0.250	+25.90867 +24.12137	-74.75668 +10.48516	+0.153521 +0.025348	+0.188568 -0.854940	-0.889467 +0.048449
0.275	+40.97559 +53.01570	-114.2855 +13.02177	+0.207226 +0.035934	+0.250827 -0.715457	-3.072662 +0.039874
0.300	+45.83759 +132.2568	-184.7646 +10.86054	+0.357031 +0.066809	+0.338343 -1.719215	+3.871658 +0.095977
0.325	+89.20382 +331.1197	-393.0127 +8.598136	+0.617466 +0.104073	+0.460472 -2.815203	+11.54327 +0.155960
0.350	+212.6788 +667.6445	-801.7908 +12.39815	+1.087307 +0.166473	+0.538938 -3.026131	+10.80273 +0.165055
0.375	+336.2354 +831.1445	-1085.134 +26.18321	+1.644191 +0.238795	+0.532702 -2.450470	-1.224173 +0.139154
0.400	+566.5476 +1154.091	-1609.632 +51.46175	+2.016090 +0.288046	+0.265722 -0.178354	-29.24412 +0.018446
0.425	+743.4107 +937.4014	-1708.263 +115.6006	+2.435809 +0.365071	+0.013553 +1.838967	-81.16189 -0.062023
0.450	+1200.620 +1489.269	-2751.715 +196.3406	+3.208577 +0.528225	+0.254920 +1.379102	-132.0424 +0.013577

Table 5

Fn	c0	c1	c2	c3	c4	c5
.475	180.1004	-31.50257	-7.451141	2.195042	2.689623	0.006480
.500	243.9994	-44.52551	-11.15456	2.179046	3.857403	0.009676
.525	282.9873	-51.51953	-12.97310	2.274505	4.343662	0.011066
.550	313.4109	-56.58257	-14.41978	2.326117	4.690432	0.012147
.575	337.0038	-59.19029	-16.06975	2.419156	4.766793	0.014147
.600	356.4572	-62.85395	-16.85112	2.437056	5.078768	0.014980
.625	324.7357	-51.31252	-15.34595	2.334146	3.855368	0.013695
.650	301.1268	-39.79631	-15.02299	2.059657	2.545676	0.013588
.675	292.0571	-31.85303	-15.58548	1.847926	1.569917	0.014014
.700	284.4641	-25.14558	-16.15423	1.703981	0.817912	0.014575
.725	256.6367	-19.31922	-13.08450	2.152824	0.348305	0.011343
.750	304.1803	-30.11512	-15.85429	2.863173	1.524379	0.014031

2.2.2. Induced resistance

The induced resistance coefficient for a lifting surface with an effective aspect ratio AR_E is given by:

$$C_{Di} = \frac{C_L^2}{\pi\ AR_E} \qquad (8)$$

Similarly, for the hull, keel and rudder combination the induced resistance, resulting from the generated sideforce F_H can be written as:

$$R_i = \frac{1}{\pi\ AR_E} * \frac{F_H^2}{qS_C} \qquad (9)$$

where AR_E is the effective aspect ratio of the hull, keel and rudder combination, and $q = \frac{1}{2}\rho V^2$.

Using the results of the resistance measurements with heel angle and leeway, the induced resistance could be expressed by:

$$R_i = (C_0 + C_2 \varphi^2) \frac{F_H^2}{qS_C} \qquad (10)$$

where C_0 and C_2 depend on the geometry of the hull, keel and rudder combination.
The expression (10) works well for Series I (nrs. 1 - 22) but for the Series II and III (nrs. 23 - 39 an additional term with the Froude number Fn was necessary to cope with a significant free surface influence on the induced resistance.
Thus:

$$R_i = (C_0 + C_2 \varphi^2 + C_3 Fn) \frac{F_H^2}{qS_C} \qquad (11)$$

For Series I a fair agreement between (10) and (11) exists for Fn = 0.325.
With (9) and (10) we find:

$$AR_E = \frac{1}{\pi (C_0 + C_2 \varphi^2)} \qquad (12)$$

We now define an effective draught T_E with:

$$AR_E = \frac{T_E^2}{S_C} \text{ , than:}$$

$$T_E^2 = \frac{S_C}{\pi (C_0 + C_2 \varphi^2)} \qquad (13)$$

and:

$$R_i = \frac{F_H^2}{\pi \, T_E^2 \, q} \qquad (14)$$

With the measured F_H values for models 1 - 28 and model 1a, model 1b and model 1c the effective draughts T_E have been determined for heel angles 0, 10, 20 and 30 degrees.
The relative effective draught T_E/T appears to be strongly dependent on T_C/T, B_{WL}/T_C and φ.

A satisfactory fit to the expermental data is given by:

$$\frac{T_E}{T} = A_1 \left(\frac{T_C}{T}\right) + A_2 \left(\frac{T_C}{T}\right)^2 + A_3 \left(\frac{B_{WL}}{T_C}\right) \qquad (15)$$

with:

$$A_1 = 4.080 + 0.0370\ \varphi - 4.9830\ \varphi^3$$

$$A_2 = -4.179 - 0.8090\ \varphi + 9.9670\ \varphi^3$$

$$A_3 = 0.055 - 0.0339\ \varphi - 0.0522\ \varphi^3$$

φ in radians.

2.2.3. Resistance due to heel

For each of the models 1 -28 the resistance due to heel, R_H, has been determined.
It was found that a reasonable approxination of R_H is given by:

$$\frac{R_H}{qS_C} = C_H\ Fn^2\ \varphi \qquad (16)$$

φ in radians.

The C_H was expressed in the keel and hull parameters T_C/T and B_{WL}/T_C.

$$C_H * 10^3 = 6.747 \left(\frac{T_C}{T}\right) + 2.517 \left(\frac{B_{WL}}{T_C}\right) + 3.710 \left(\frac{B_{WL}}{T_C}\right) * \left(\frac{T_C}{T}\right) \qquad (17)$$

The resistance due to heel and side force, the heeled resistance is given by:

$$R_i + R_H = \frac{F_H^2}{\pi\ T_E^2\ q} + (C_H\ Fn^2\ \varphi)\, qS_C \qquad (18)$$

with T_E and C_H as shown in (15) and (17).

For φ > 30 degrees an extra resistance increase can be included to allow for the influence of deck immersion.

By analogy with the IMS formulation the following expression is used for velocity predictions:

$$R_\varphi = R_{\varphi 0} \, [1 + 0.0004(\varphi - 30)^2] \qquad [19]$$

φ in degrees.

This results in a resistance increase of 1% and 4% for respectively φ = 35 degrees and φ = 40 degrees.

2.3. Side force as a function of heel and leeway

For the models 1 - 22 (Series I) and model 1c (half keel span) the realtion between leeway and side force is approximated by:

$$\beta = \frac{F_H \cos \varphi}{qS_C} (B_0 + B_2 \varphi^2) \qquad (20)$$

β and φ in radians

Due to larger B_{WL}/T_C an additional term depending of the heel angle and the Froude number is necessary for the models 23-28 (Series II) to satisfy the experimental evidence which indicates free surface effects.
Thus:

$$\beta = \frac{F_H \cos \varphi}{qS_C} (B_0 + B_2 \varphi^2) + B_3 \varphi^2 \, Fn \qquad (21)$$

If the combination of hull, keel and rudder is considered as a side force (lift) generating element, the "lift" slope will be given by the first two terms of (21):

$$\frac{F_H \cos \varphi}{\beta \, qS_C} = \frac{1}{B_0 + B_2 \varphi^2} \qquad (22)$$

The slope depends on the effective aspect ratio of the underwater part of the hull, keel and rudder, which in this case is related to side force generation.

It was found that the "lift" slope can be expressed with sufficient accuracy by: T_C/T and T^2/S_C:

$$\frac{F_H \cos\varphi}{\beta\ q\ S^C} = b_1\left(\frac{T^2}{S_C}\right) + b_2\left(\frac{T^2}{S_C}\right)^2 + b_3\left(\frac{T_C}{T}\right) + b_4\left(\frac{T_C}{T}\right) * \left(\frac{T^2}{S_C}\right) \qquad (23)$$

with:

	$\varphi = 0°$	$\varphi = 10°$	$\varphi = 20°$	$\varphi = 30°$
b_1	+ 2.025	+ 1.989	+ 1.980	+ 1.762
b_2	+ 9.551	+ 6.729	+ 0.633	- 4.957
b_3	+ 0.631	+ 0.494	+ 0.194	- 0.087
b_4	- 6.575	- 4.745	- 0.792	+ 2.766

The coefficient B_3 in (21) has been determined with the experimental results of models 23 - 28 (Series II):

$$B_3 = 0.0092 \left(\frac{B_{WL}}{T_C}\right) * \left(\frac{T}{T_C}\right) \qquad (24)$$

The contribution of the B_3 is relatively small, except in the case of very large B_{WL}/T_C and T/T_C, such as models 24 and 26. Than there is a certain heel angle at which no side force is generated, which follows from:

$$\beta = B_3\ \varphi^2\ Fn.$$

2.4. Stability

The data reduction of the experimental stability data has been carried out as follows, see Figure 4.

$$GN \sin\varphi = GM \sin\varphi + MN \sin\varphi \qquad (25)$$

where GM is the calculated hydrostatic value at V = 0.
The residuary stability lever can be expressed in: φ, Fn and B_{WL}/T_C:

$$\frac{MN \sin\varphi}{L_{WL}} = (D_2 * \varphi * Fn + D_3\ \varphi^2) \qquad (26)$$

with:

$$D_2 = -0.0406 + 0.0109 \left(\frac{B_{WL}}{T_C}\right) - 0.00105 \left(\frac{B_{WL}}{T_C}\right)^2$$

$$D_3 = +0.0636 - 0.0196 \left(\frac{B_{WL}}{T_C}\right)$$

φ in radians

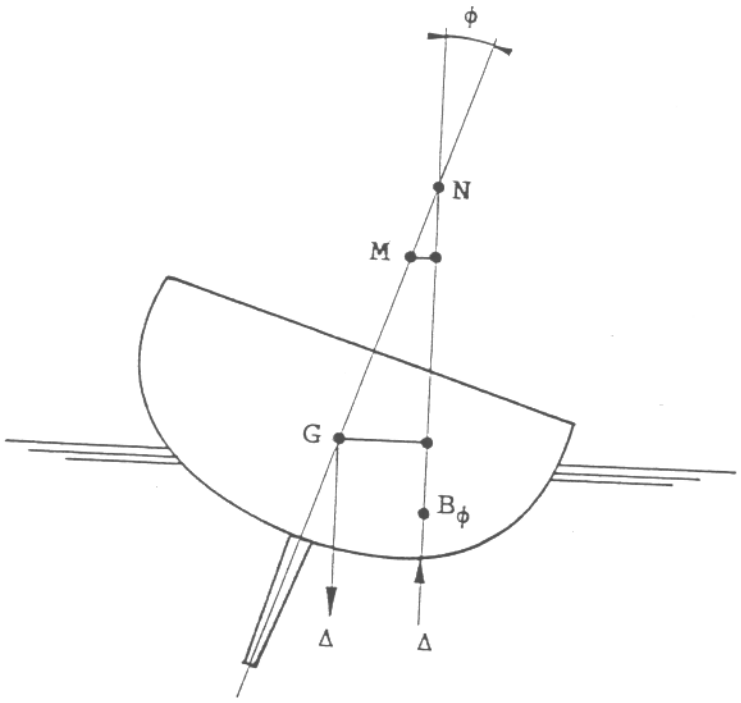
Finally the distance of the centre of lateral resistance to the waterline is given by:

$$D_4 * T \qquad (27)$$

with:

$$D_4 = 0.414 - 0.165 \left(\frac{T_C}{T}\right)$$

Apparently for $T_C/T \longrightarrow 0$ D_4 approaches the value for an elliptic distribution of the sideforce from the tip of the keel to the waterline.



GN sin φ + (GM + MN) sin φ

Figure 4: Definition of residual stability lever MN sin φ.

To show the goodness of fit of the various polynomials as given for resistance, side force and stability some results are given in the Figures 5 - 8.
In Figure 5 the measured and predicted upright resistance for the models 16 and 37 (a heavy- and light-displacement hull) are compared. The typical difference in character of the resistance curve for speeds excluding Fn = 0.45 is clearly shown.
In Figure 6 the heeled resistance, predicted with equation (18) is compared with the experimental results for models 16 and 28, and in Figure 7 the generated side force as a function of leeway and heel angle predicted according to equation (21) is compared with the measurements.
Finally a simular comparison has been made for the stability lever at 10, 20 and 30 degrees as a function of the Froude number using equation (25) and (26).
The examples include some rather extreme hull forms, but the prediction in all considered cases is satisfactory.
The importance of the length-displacement ratio $L_{WL}/\nabla_C^{1/3}$ and the beam to draught ratio B_{WL}/T_C is clearly shown in the Figures 5 - 8.
In particular the attention is drawn to the loss of stability at forward speed for the wide beam models 31 and 33 as depicted in Figure 8.

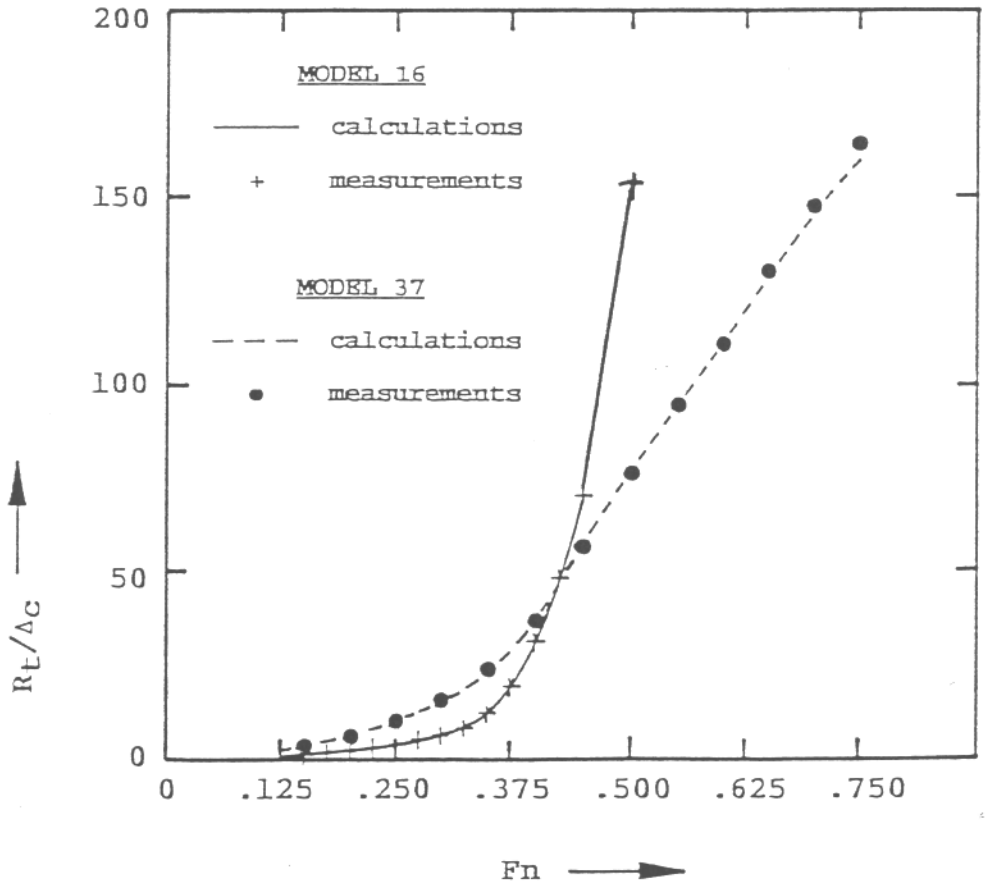


Figure 5: Measured and predicted upright resistance.

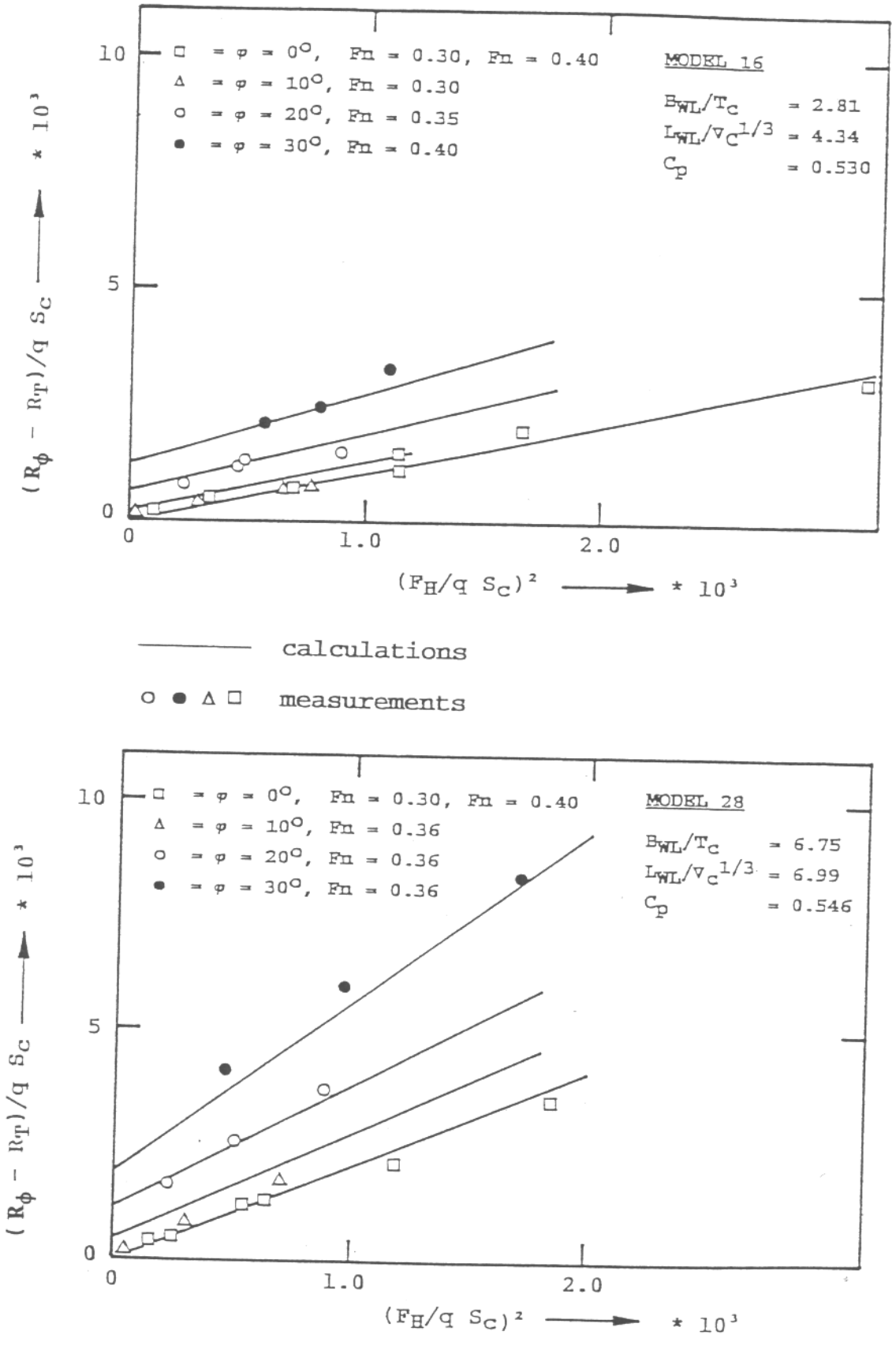


Figure 6: Measured and predicted heeled resistance.

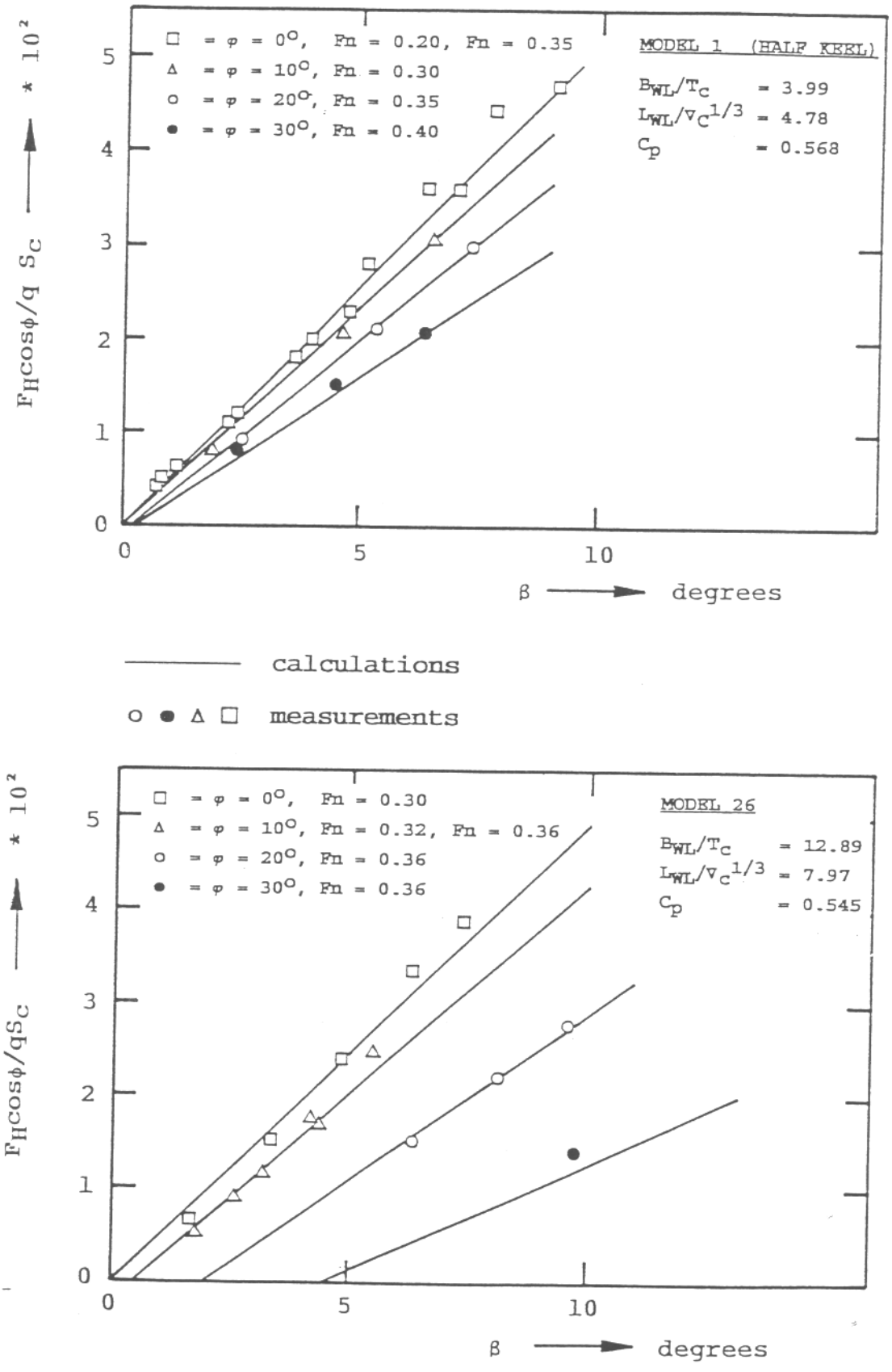


Figure 7: Measured and predicted sideforce.

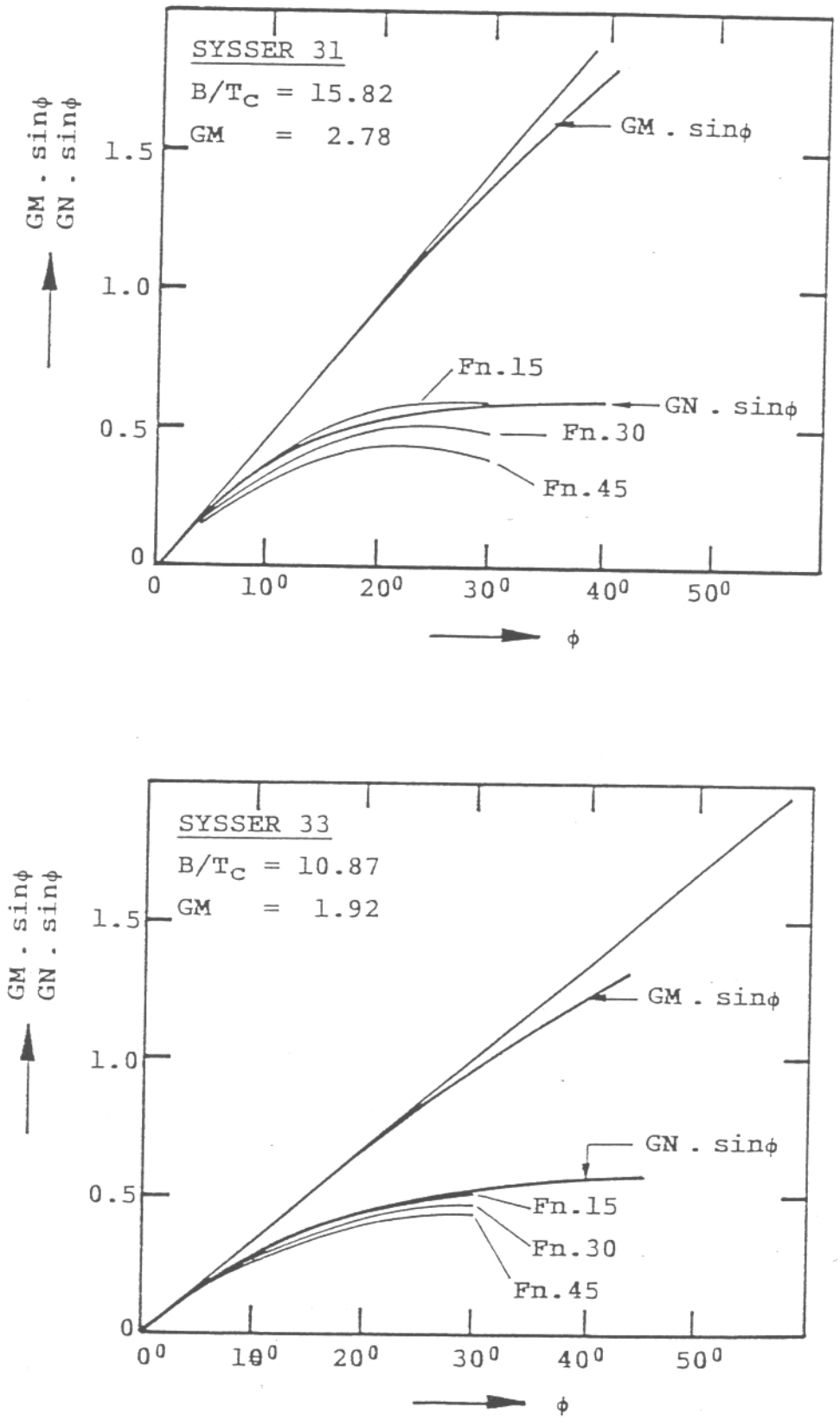


Figure 8: Stability lever GN sin φ as a function of Fn. The measured values for Fn = .15, .30 and .45 coincide with the drawn lines.

3. Velocity prediction in waves

The added resistance in waves has an important influence on the performance of a sailing yacht. The oscillatory motion of the yacht generates damping waves, which superimpose on the incedent seawaves. The damping waves, which are mainly due to pitching and heaving motions, radiate the damping energy. The resulting added resistance is forced by equalizing the work done by the added resistance force and the radiated wave damping energy.

To estimate the relative importance of the added resistance in waves the simple strip theory may be used, as discussed in [7].
The mean added resistance in waves, R_{AW}, follows from:

$$R_{AW} = \frac{1}{\lambda} \int_0^{L_{WL}} \int_0^{T_e} b' V_z^2 \, dx_b dt \qquad (28)$$

where:

λ - wave length

t - time

b' - cross sectional damping coefficient, corrected for the forward speed

V_z - relative vertical velocity of the considered cross-section with respect to the water.

T_e - period of wave encounter

x_b - length ordinate of the hull.

The vertical relative motion V_z is determined by vectorial summation of heave, pitch and incident wave velocity.
The strip theory calculation of motions and added resistance agrees quite well with model experiments, as shown in [7] and [9].

Using the superposition principle the added resistance of a yacht in an irregular seaway can be determined when the added resistance response operator, as well as the wave spectrum are known.
The wave spectrum may be available from actual wave buoy-measurements or approximated using visual estimates of the significant wave height $\bar{H}_{1/3}$ and the average period $\bar{T}_1$ in a standard formulation for the wave energy distribution, as given for instance by Bretschneider [7].
With the computational capacity of to day's personal computers, the added resistance response operator of a yacht can be easily determined, using strip theory methods, when the

linesplan and the longitudinal distribution of mass are given.
The mean added resistance R_{AW} in a wave spectrum S_ζ follows from:

$$\bar{R}_{AW} = 2 \int_0^\infty \frac{R_{AW}}{\zeta_a^2} * S_\zeta(\omega_e) d\omega_e \qquad (29)$$

where ω_e is the frequency of encounter.

The added resistance operator R_{AW}/ζ_a^2, or a corresponding dimensionless presentation, such as : $R_{AW}/\rho g \zeta_a^2 L_{WL}$, depends on the hull geometry, the longitudinal gyradius k_{yy}, the wave period or frequency and the wave direction μ_w.

For all models of the Delft Series the added resistance operator has been calculated by Reumer for range of Froude numbers, wave frequencies and wave directions [3].
With a least squares procedure the resulting added resistance operators could be expressed in a polynomial expression:

$$\frac{R_{AW}}{\zeta_a^2} = a_1(L_{WL}/\nabla_C^{1/3}) + a_2(L_{WL}/\nabla_C^{1/3})^2 +$$
$$+ a_3(L_{WL}/\nabla_C^{1/3})^3 + a_4(L_{WL}/B_{WL}) + a_5(L_{WL}/B_{WL})^2 +$$
$$+ a_6(B_{WL}/T_C) + a_7 C_p + a_8 C_p^2 + a_9 C_p^3. \qquad (30)$$

The coefficients a_i are a function of the wave direction, wave frequency and the Froude number. The calculations have been carried out for L_{WL} = 10 meters and a gyradious k_{yy}/L_{WL} = 0.25.
In Figure 9 the result of (30) is compared with a direct computation, using strip theory, for the models 1 and 25 for μ_w = 165° (15° off the bow) and Fn = 0.25.

Computed added resistance operators have been used to analyse the influence of the pitch gyradius and the displacement-length ratio on the mean added resistance in an irregular seaway defined by:

$$S_\zeta = A\omega^{-5} \exp(-B\omega^{-4}) \qquad (31)$$

with: $A = 173\ H_{1/3}/\overline{T_1}^4$

$B = 691/\overline{T_1}^4$

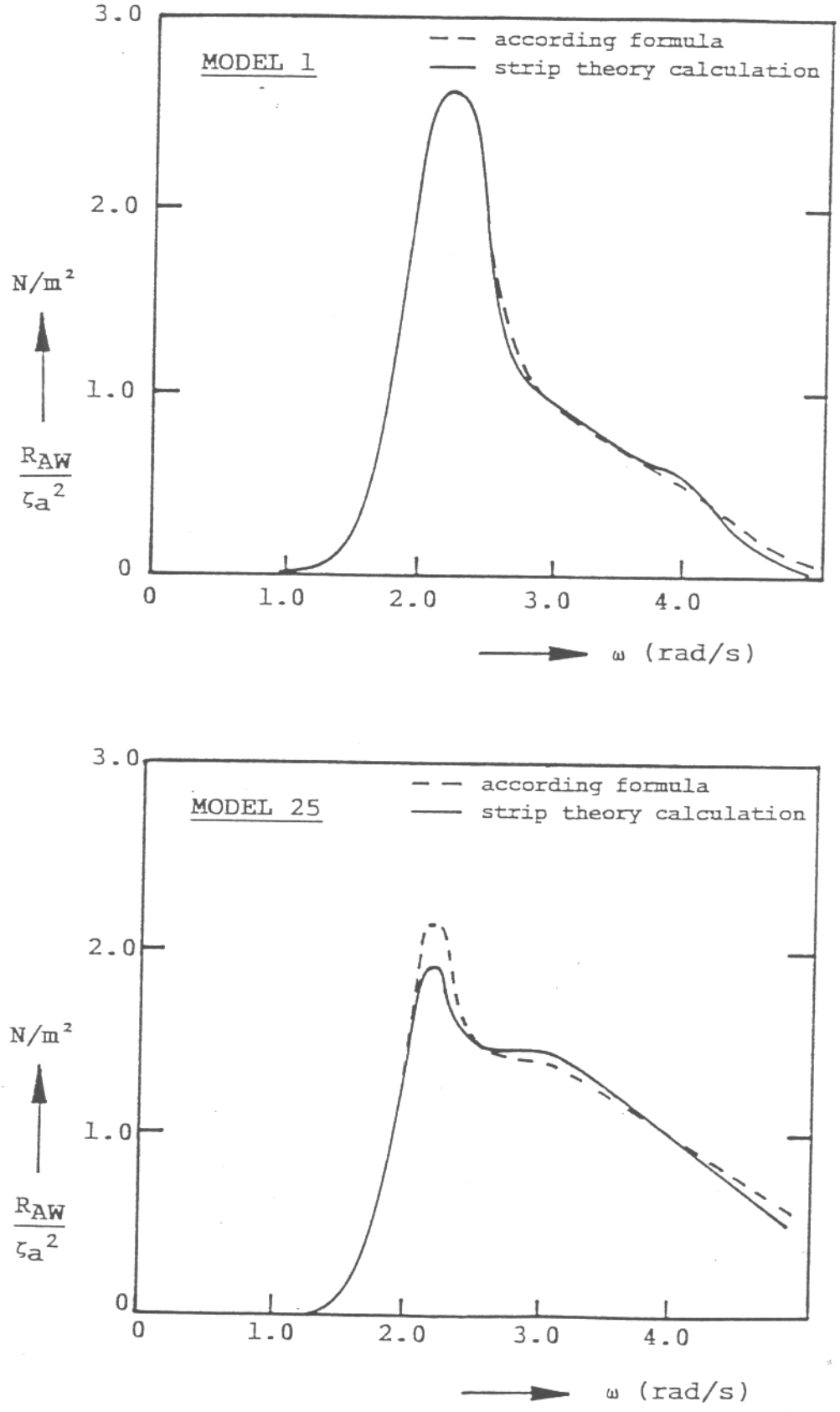


Figure 9: Added resistance operators for models 1 and 25.

The calculated $\bar{R}_{AW}$ for $\bar{T}_1$ = 2, 4 and 6 seconds, k_{yy}/L_{WL} = 0.23, 0.27 and 0.31, and L_{WL} = 10 meters is depicted in Figure 10 on a base of:

$$\frac{\nabla_c^{1/3}}{L_{WL}} \cdot \frac{k_{yy}}{L_{WL}}$$, as a function of wave direction.

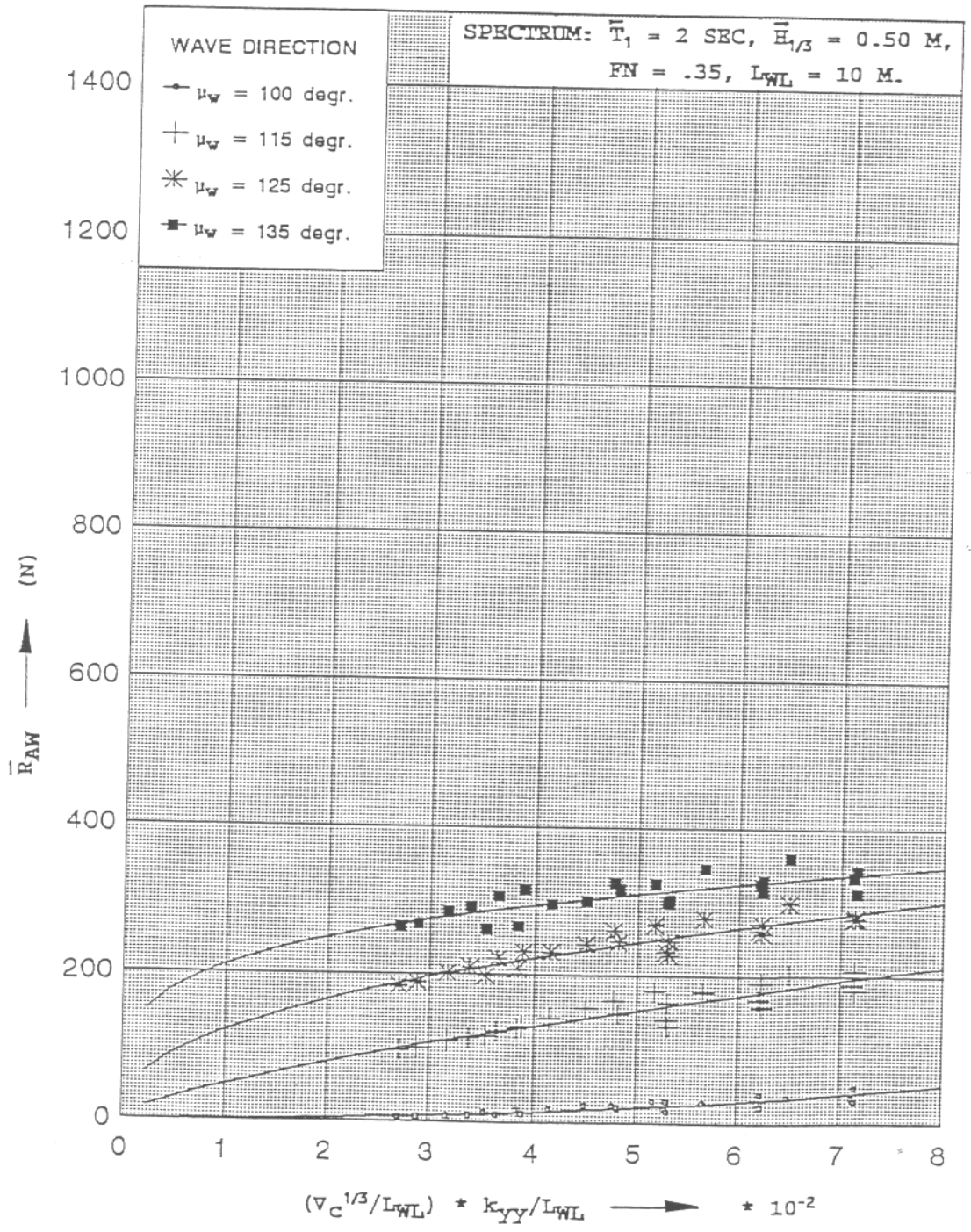


Figure 10a: Added resistance.

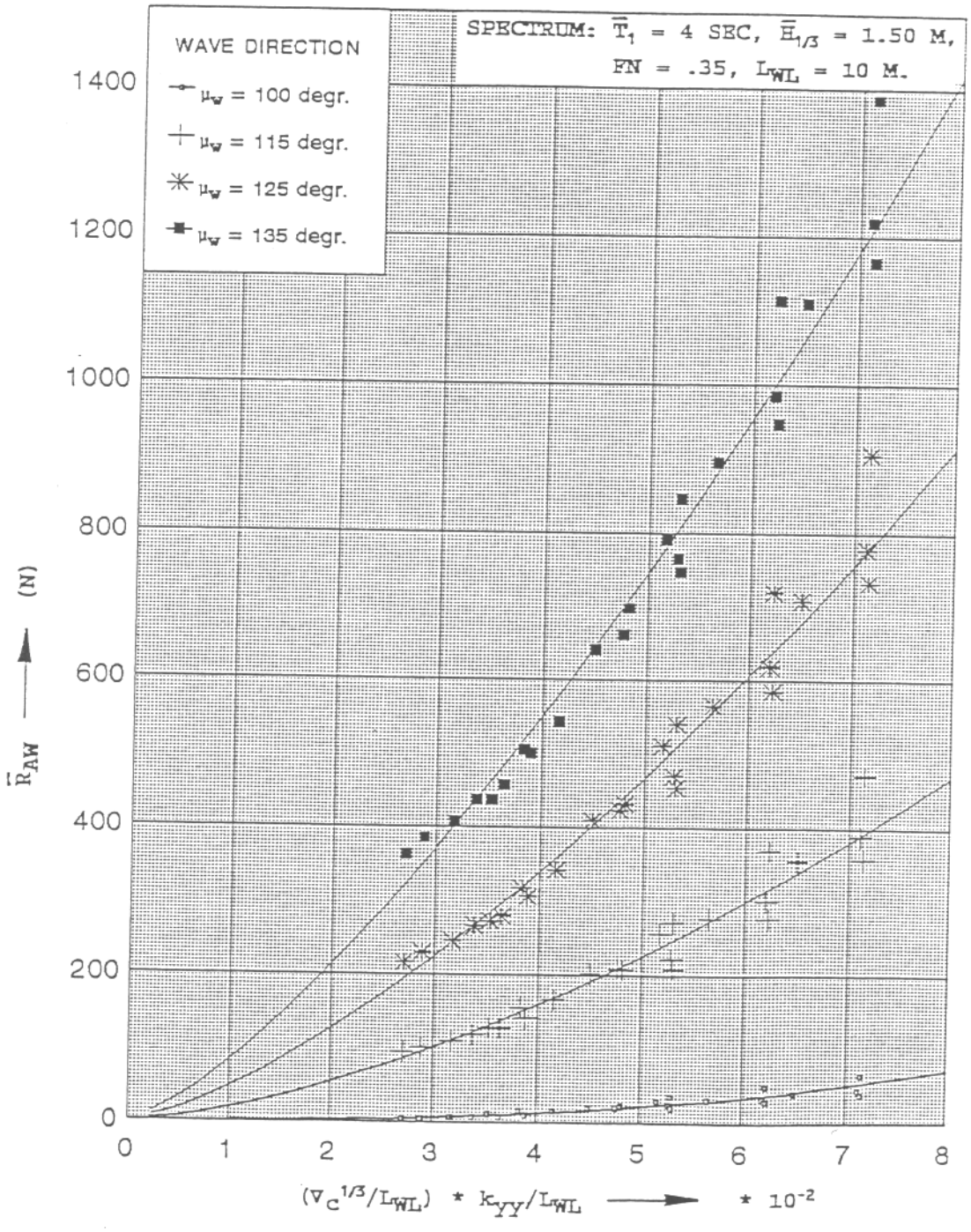


Figure 10b: Added resistance.

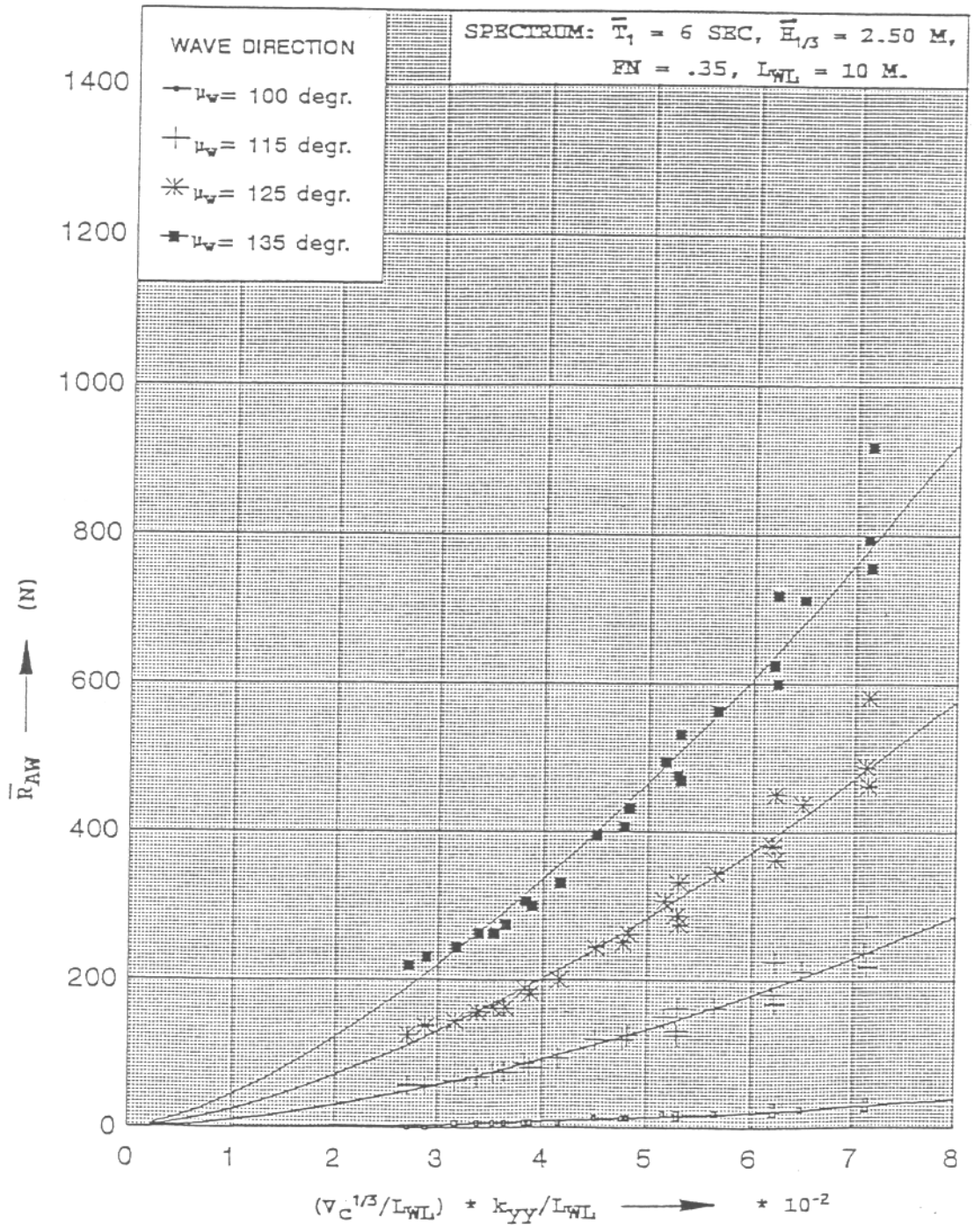


Figure 10c: Added resistance.

The eight models 1, 5, 6, 22, 25, 26, 30 and 31 constitute a very large range of hull form variation. Therefore the data in Figure 10 can be regarded to represent the total series with respect to the added resistance in waves. Also the pitch gyradius range, as chosen, is very wide in particular for the medium- and heavy-displacement hull forms.

For one wave direction μ_w = 135 degrees the added resistance has been plotted on a base of mean wave period (Fn = 0.35, L_{WL} = 10 meters) see Figure 11, which shows the importance of the mean wave period or mean wave length for the added resistance in waves, as well as the influence of the pitch gyradius.
The importance of the added resistance is shown by relating $\bar{R}_{AW}$ to the upright resistance R_T, which is 1261 N for hull nr. 1 and 657 N for nr. 26.

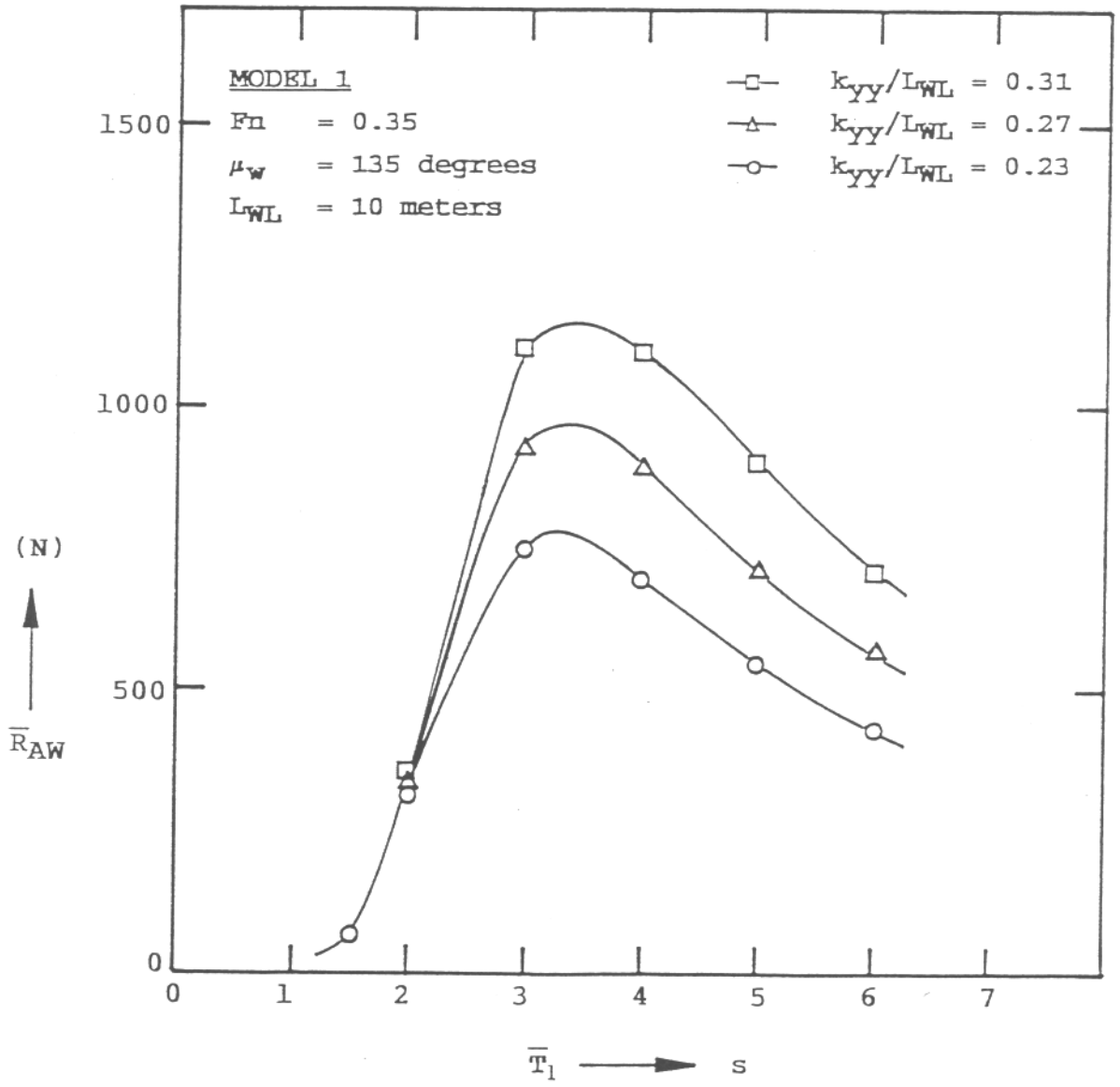


Figure 11a: Added resistance versus mean wave period Model 1.

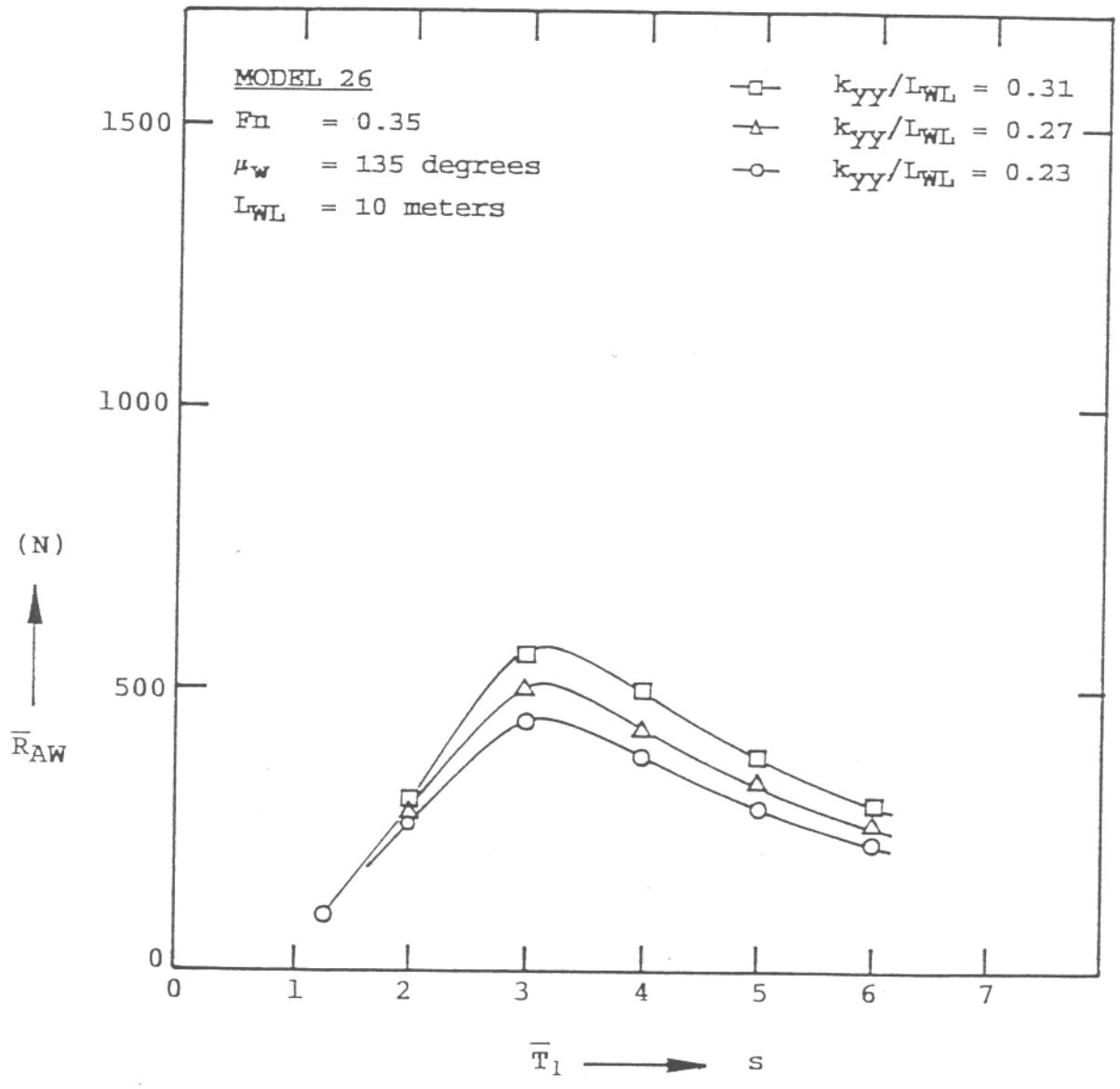


Figure 11b: Added resistance versus mean wave period model 26

The added resistance calculation has been carried out for Fn = 0.15, 0.25, 0.35, 0.45 and 0.60.
The total result is given in dimensionless form to facilitate the use for waterline lengths other than L_{WL} = 10 meters.
To this end the added resistance operator is expressed as $\overline{R}_{AW}/\rho g L_{WL} \cdot \overline{H}^2_{1/3}$ as a function of $\overline{T}_1\sqrt{g/L_{WL}}$, Fn and μ_w.

The following example illustrates the calculation procedure for a yacht with L_{WL} = 15 meters, $L_{WL}/\nabla_c^{1/3}$ = 5.8, k_{yy}/L_{WL} = 0.30 in a wave-spectrum $\bar{H}_{1/3}$ = 1.2 meter, $\bar{T}_1$ = 3 seconds and Fn = 0.35, μ_w = 135 degrees, see Figure 12.
In this case $\nabla_c^{1/3}/L_{WL} \cdot k_{yy}/L_{WL}$ = 0.052 and $\bar{R}_{AW}/\rho g L_{WL} \bar{H}^2_{1/3}$ = 0.0079.
Thus, $\bar{R}_{AW}$ = 0.0079 * 1025 * 9.81 * 15 * 1.2^2 = 1716 N.

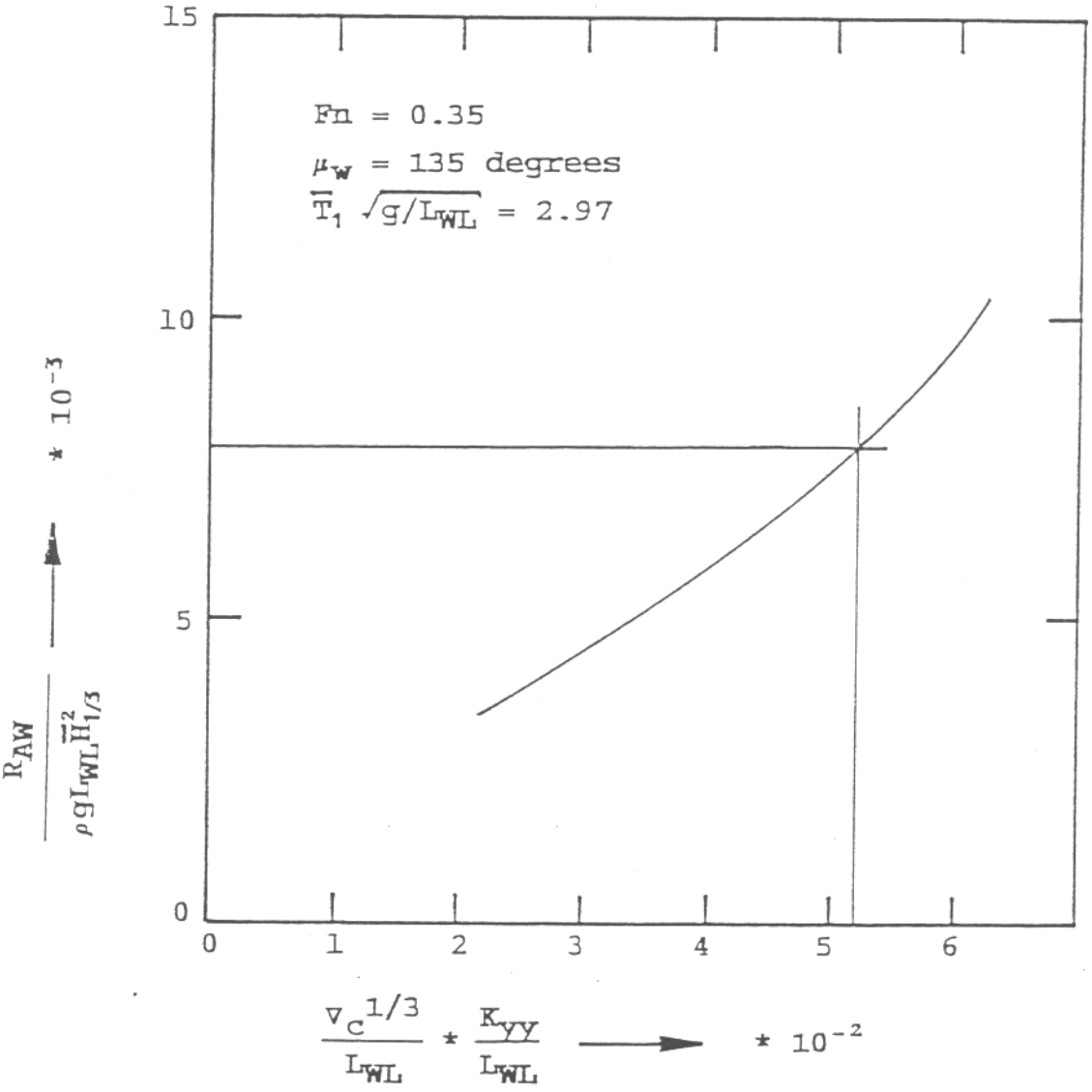


Figure 12: Example R_{AW} calculation.

The total result of the added resistance calculation has been approximated by a least squares procedure to enable the use of the data for a velocity predition program.
It should be remarked that the added resistance calculation, as presented, is an approximation, based on linear strip theory.
It is assumed that the wave direction is equal to true wind direction.

Also the influence of the heel angle is not included. As shown in [7] this may be acceptable in many cases, but some discrepancies compared with experimental results have been observed.
In general strip theory methods are a reasonable tool to estimate ship motions and added resistance in seawaves. More accurate 3-dimensional methods are now available, but in view of other uncertainties such as non-linearities and the description of the irregular sea surface, the increased complexity of such methods seems not justified.

However the present method, as described in the paper is thought to be adequate for design and rating purposes.

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