Power required to rotate a cylinder with end plates in a moving air

Notations : for more uniformed formulations and ease further comparison,
U is the upstream flow (= the apparent wind when a sailing ship is considered)
v is the tangential speed of rotation at the surface of the rotor (>> speed ratio = v/U)
D and H are the diameter and the height of the rotor.
Sw the "wetted" surface of the rotor is then : π D H
>>> Norwood uses rotor radius a , rotor height 1,5 a and the rotational speed w, so the correspondance are :
D = 2 a , H = 1,5 a and v = a*w
ρ is the air mass volumic of air (kg/m3)
Cf is the friction coefficient, computed using Re = U*D/ν and Cf = 0,075 / (log10(Re) - 2)^2
ν = air cinematic viscosity (m2:s)

Formulations for the cylinder

« Norwood » formulation (rewritten with the above notations) :

Pm = Cf * (1/2 * ρ * Sw) * (2 * U^2 + v^2) * v

« **Norwood bis »** (when considering v for the integration in the 4th quadrant) :

Pm = Cf * (1/2 * ρ * Sw) * (3/2 * U^2 + 2/π * U * v + v^2) * v

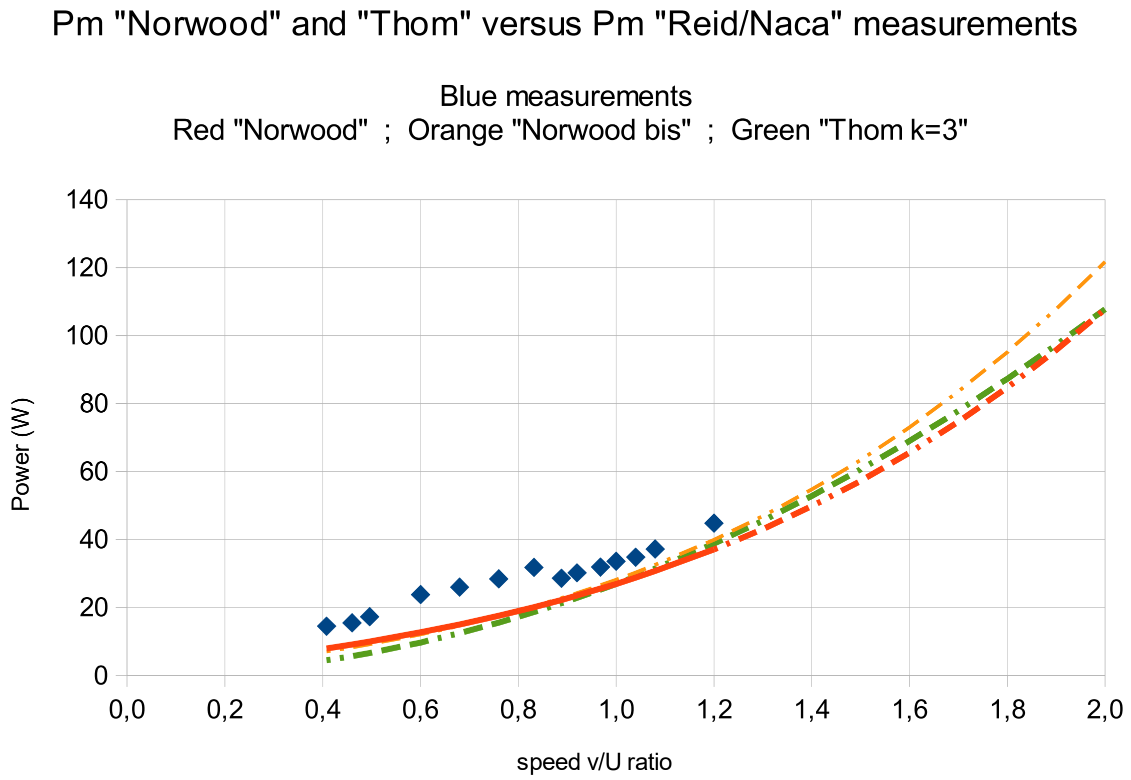
« Thom » formulation , derived from its torque formulation :

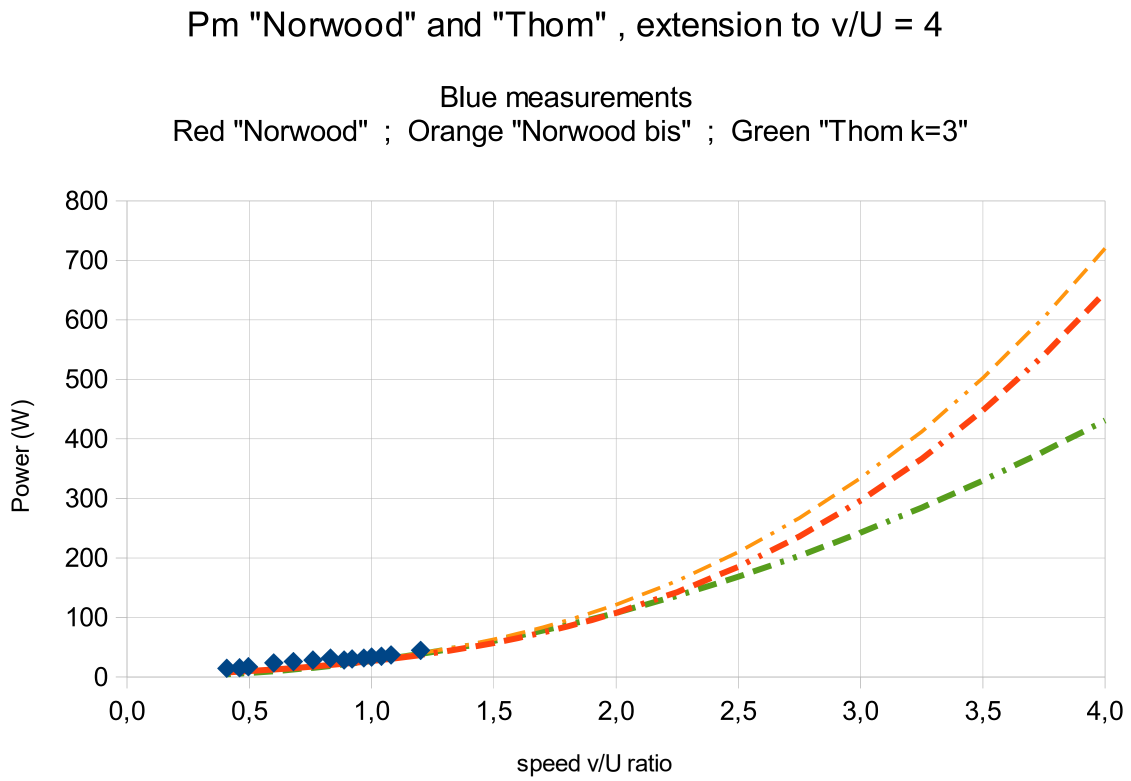
Pm = Cf * (1/2 * ρ * Sw) * (k * U * v) * v , with k ~ 3 (?)

>>> that presentation of the formulations highlights on the origin of the power computation (a friction force x a tangential speed of rotation) and especially on the quadratic function of (U,v) dimensioned as a squared speed and resulting from the integration on the cylinder perimeter.

Comparison with Reid Naca 1924 measurements :

Data of the experiments :
D = 0,1143 m ; H = 1,5232 m
ν = 1,429 E-5 m2/s (estimated)
ρ = 1,23 kg/m3 (estimated)
Cf computed with Re = U D / ν and Cf = 0,075 / (log10(Re) - 2)^2 >> Cf = 0,0079
Measurements done with U = 15 m/s and for 14 values of v/U from 0,4 to 1,2


Pm "Norwood" and "Thom" versus Pm "Reid/Naca" measurements
Blue measurements
Red "Norwood" ; Orange "Norwood bis" ; Green "Thom k=3"
140
120
100
80
60
40
20
0
Power (W)
0,0
0,2
0,4
0,6
0,8
1,0
1,2
1,4
1,6
1,8
2,0
speed v/U ratio


Pm "Norwood" and "Thom" , extension to v/U = 4
Blue measurements
Red "Norwood" ; Orange "Norwood bis" ; Green "Thom k=3"
800
700
600
500
400
300
200
100
0
Power (W)
0,0
0,5
1,0
1,5
2,0
2,5
3,0
3,5
4,0
speed v/U ratio

Pm formulations / Naca Reid measurements

D (m)	H (m)	nu (m2/s)	Rho (kg/m3)	U (m/s)	>>> Re	Cf	½*Rho*Sw
0,1143	1,5232	1,43E-005	1,23	15	1,20E+005	0,00791	0,33638

Pm Reid (Watts)	N (rpm)	v/U	v (m/s)	Pm Norwood 2*U^2 + v^2	P (Watts)	Pm Norwood bis Quadratic	P (Watts)	PmThom k=3 P (Watts)
14,5	1000	0,408	6,12	487,45	7,94	433,40	7,06	4,48
15,5	1115	0,460	6,90	497,61	9,14	451,00	8,28	5,70
17,3	1240	0,496	7,44	505,35	10,00	463,90	9,18	6,63
23,8	1500	0,600	9,00	531,00	12,72	504,44	12,08	9,70
26,0	1700	0,680	10,20	554,04	15,04	538,94	14,63	12,46
28,4	1900	0,760	11,40	579,96	17,59	576,32	17,48	15,56
31,8	2080	0,832	12,48	605,75	20,12	612,43	20,34	18,65
28,6	2220	0,888	13,32	627,42	22,24	642,12	22,76	21,25
30,2	2300	0,920	13,80	640,44	23,52	659,72	24,23	22,80
31,9	2420	0,968	14,52	660,83	25,53	686,99	26,54	25,25
33,6	2500	1,000	15,00	675,00	26,94	705,74	28,17	26,94
34,8	2600	1,040	15,60	693,36	28,78	729,83	30,30	29,14
37,2	2700	1,080	16,20	712,44	30,71	754,64	32,53	31,43
44,8	3000	1,200	18,00	774,00	37,07	833,39	39,92	38,80
		1,3	19,50	830,25	43,08	903,96	46,91	45,53
		1,4	21,00	891,00	49,79	979,04	54,71	52,81
		1,5	22,50	956,25	57,25	1058,61	63,38	60,62
		1,6	24,00	1026,00	65,52	1142,68	72,98	68,97
		1,7	25,50	1100,25	74,66	1231,26	83,55	77,86
		1,8	27,00	1179,00	84,71	1324,33	95,15	87,29
		1,9	28,50	1262,25	95,73	1421,90	107,83	97,26
		2	30,00	1350,00	107,77	1523,98	121,66	107,77
		2,25	33,75	1589,06	142,71	1798,85	161,55	136,40
		2,5	37,50	1856,25	185,23	2101,85	209,74	168,39
		2,75	41,25	2151,56	236,17	2432,97	267,06	203,75
		3	45,00	2475,00	296,37	2792,22	334,35	242,48
		3,25	48,75	2826,56	366,67	3179,59	412,46	284,58
		3,5	52,50	3206,25	447,92	3595,09	502,24	330,04
		3,75	56,25	3614,06	540,95	4038,71	604,51	378,88
		4	60,00	4050,00	646,62	4510,46	720,13	431,08

Formulation for an end plate diameter De > D :

For simplification and due to uncertainties linked with this approach, a formulation in line with the « Norwood » one for the cylinder is proposed for the surface of the end plate at the cylinder side :

Pm = Cf * (1/2 * ρ * Sw) * (a * U^2 + b * v^2) * (c * v)

, introducing coefficients a, b and c :

a = 1 + [2 D / (De + D)]^2 >>> converge towards 2 when De = D and to 1 when De infini

b = 2 D / (De + D) >>> converge towards 1 when De = D and to 0 when De infini

c = (De +D) / 2 D >>> converge towards 1 when De = D and infini when De infini

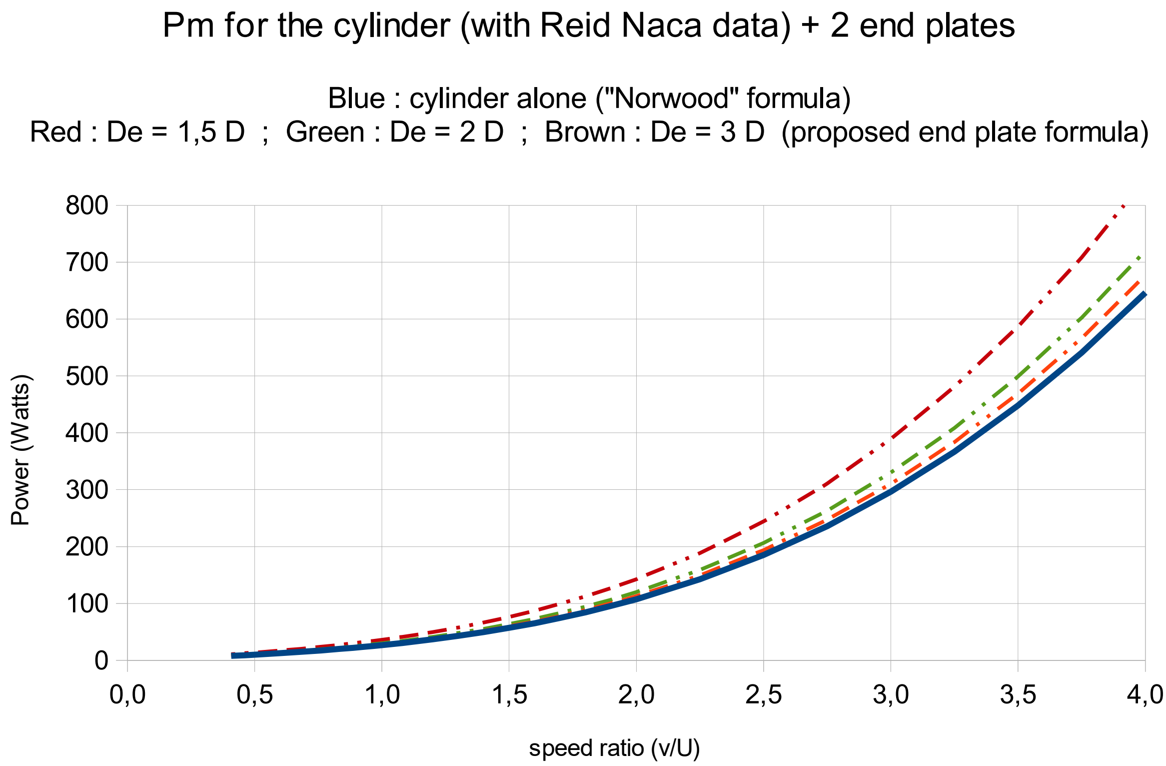
Cofficients a and b are derived from the comparison of the air flow speed at teta 90° at the cylinder (radius R) and at radius r > R (according to 2D perfect fluid theory) :

$U_r = U [1 + (R/r)^2 + (v/U) (R/r)]$ compare with $U_R = U [2 + (v/U)]$ when r = R = D / 2

, and using for r the mean value of the end plate radius >> r = (D + De) / 4

Sw = the « wetted » surface of the end plate between diameters D (cylinder) and De (Diameter end plate) = π / 4 *(De^2 – D^2)

Example with the Naca Reid cylinder + 2 end plates diameters De = 1,5 D , 2 D, 3D



As a preliminary conclusion :

This approach keep both simple and easy to compare with experimental results if any.

For the cylinder, the « Norwood » formula :
Pm = Cf * (1/2 * ρ * Sw) * (a * U^2 + b * v^2) * (c * v)
with :
a = 2 b=1 and c=1

For an end plate, a formula derived from « Norwood » :
Pm = Cf * (1/2 * ρ * Sw) * (a * U^2 + b * v^2) * (c * v)
with :
a = 1 + [2 D / (De + D)]^2
b = 2 D / (De + D)
c = (De +D) / 2 D