A numerical study of breaking waves

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1 Summary

The focus of the present work is on the numerical simulation of steady flows with spilling breaking waves. In particular, the breaker is modeled through a hydrostatic pressure and a shear stress exerted on the free–surface. Many elements of the exposed model are derived by Cointe and Tulin's theory of steady breaker. The model has been implemented in a RANSE code in a simple but effective way through a modification in the free–surface boundary conditions. At present, the resulting code is valid for two–dimensional flows, and has been accordingly tested against the experimental data obtained by Duncan for the flow generated by a hydrofoil towed under the free–surface at different velocities and depths.

2 Introduction

Breaking waves are often encountered when dealing with ship flows. The appearance of a breaker on the bow wave is generally associated to a strong dumping of the following wave train and to the inception of a turbulent shear flow just under the free–surface. These effects have been experimentally studied by several authors (Dong et al., 1997; Roth et al., 1999), but despite the important changes that a breaking wave induces on the flow around the hull, the presence of a breaker is often overlooked in numerical codes.
This is due to some major difficulties: the choice of a criterion for the onset of breaking, the detection of the points where breaking take place, the calculation of its strength from the wave geometry and/or from the flow variables, the effects on the underlying flow, the description of the turbulent wake that follows the breaker below the free–surface, and so on.
The study of a two–dimensional flow such that described in (Duncan, 1981, 1983), and sketched in figure 2, greatly simplifies the problem, from both the experimental and the numerical point of view. A two–dimensional hydrofoil moving at speed U under the free–surface is considered. Depending on the speed and the depth of the foil, the resulting wave–train can feature an almost steady breaker on the forward face of the leading wave. This experimental set–up has been studied by Cointe and Tulin (1994), who provide useful hints for the calculation of the breaker geometry and the modelization of its influence on the wave–train. The breaker is seen as an eddy riding on the forward face of the breaking wave, exerting 'suitable' pressure and friction on the wave–breaker dividing streamline. To our purposes this theory has an attractive aspect, i.e. it yields some boundary conditions to model the breaker which are simple but effective and readily applicable.
The implementation of Cointe and Tulin's ideas in an existing RANSE code (described in Di Mascio et al. (1998, 2001)) required some further work. The main distinguishing features of the proposed model with respect to Cointe and Tulin's (1994) is the way in which the geometry of the breaking region is related to the wave height. Moreover, the breaking region is not modeled as a sharp triangle,

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but rather as a smoothed geometrical shape in order to mitigate the abrupt transition in the free–surface dynamic boundary conditions and, hence, to enhance convergence to steady state. Finally, we implement the dissipative effects of the breaker through a boundary conditions on the velocity field along the wave–breaker dividing streamline. In particular, we set the normal derivative of the tangential velocity, on the basis of local equilibrium considerations and classical solutions of the mixing layer.
In order to validate the algorithm, we performed a simulation of Duncan's experiment and the results, together with some implementation details, are described in the following.

3 A breaking model for steady viscous flow simulations

According to Cointe and Tulin's theory, the breaker is imagined as an almost stagnant eddy lying on the forward face of the leading wave. Consequently, it exerts on the underlying flow both a pressure, due to its weight, and a shear stress, that keeps the breaker in its position withstanding the gravity force.
From a numerical point of view, this means that we can simulate the presence of the breaker through a slight modification of the boundary conditions and, in particular, by adding a suitable hydrostatic pressure and a shear stress on the breaker–wave dividing streamline.

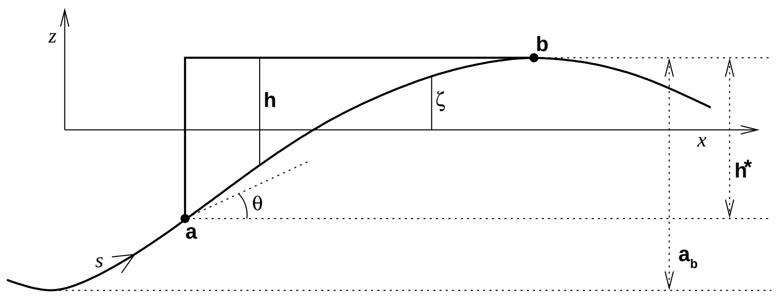


Figure 1: Geometry of the breaking wave.

Referring to figure 1, the model is turned on when $a_b > 0.69\,\mathsf{Fr}^2$ (Tulin and Cointe, 1986), where a_b is the crest–trough distance of the first wave and Fr the Froude number.
Once the model is activated, we first detect the location of the crest (i.e., the top of the breaker) and the trough of the leading wave. Then, the breaker height is computed from

$$h^* = 0.64\, a_b \tag{1}$$

in accordance with experimental findings by Duncan (1981). This expression was preferred to the one found in (Cointe and Tulin, 1994)

$$h^* = \frac{\mathsf{Fr}^2}{2} - \zeta_b \tag{2}$$

because the latter holds only once the flow has attained its steady state, but it is not useful within the iterative algorithm to reach the steady state itself.
On all points of the free–surface between x_{toe} and x_{top}, the pressure is enforced to be

$$p(x) = \frac{\zeta(x) + \rho_b\, h(x)}{\mathsf{Fr}^2} \tag{3}$$

where $h(x)$ is the top–flat function, smoothed in order to improve convergence, and ρ_b is the density of the breaker, which we assume, as in the Cointe and Tulin's model, equal to 0.6. For the velocity we assign the normal derivative of the tangential velocity:

$$\frac{\partial q^t}{\partial n} = \delta \, \frac{1-\chi}{1+\gamma\chi} \qquad \text{with} \qquad \chi = \frac{x - x_a}{x_b - x_a} \tag{4}$$

which resembles the solution of the Tollmien mixing layer in the nearby of the toe.

4 Discussion

We have applied the model to the simulation of the wavy flow past a submerged profile, in the same conditions as the experiments reported in (Duncan, 1981, 1983) and illustrated in fig. 2. A `NACA 0012` profile, whose chord is `20.3 cm`, is towed in a tank at a speed of 0.8 `m/s`, with an angle of attack of 5°. As in the towing tank experiments, the depth is varied by changing the water level, whereas the profile is kept fixed with respect to the bottom. For the sake of brevity, we consider here only the immersion d=`18.5 cm`, which is the first depth considered by Duncan for which breaking spontaneously occurs, and still is not so strong to spoil completely the following wave train.

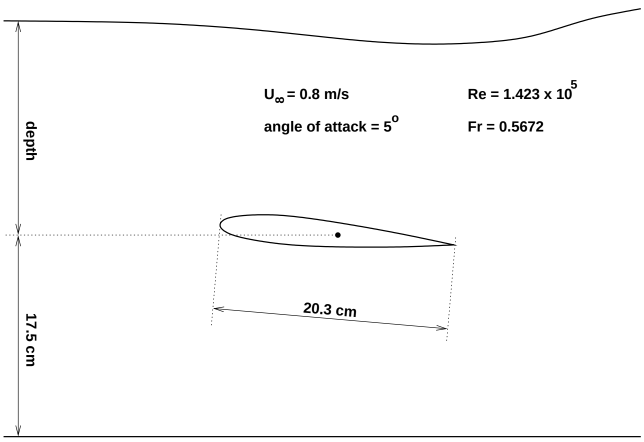
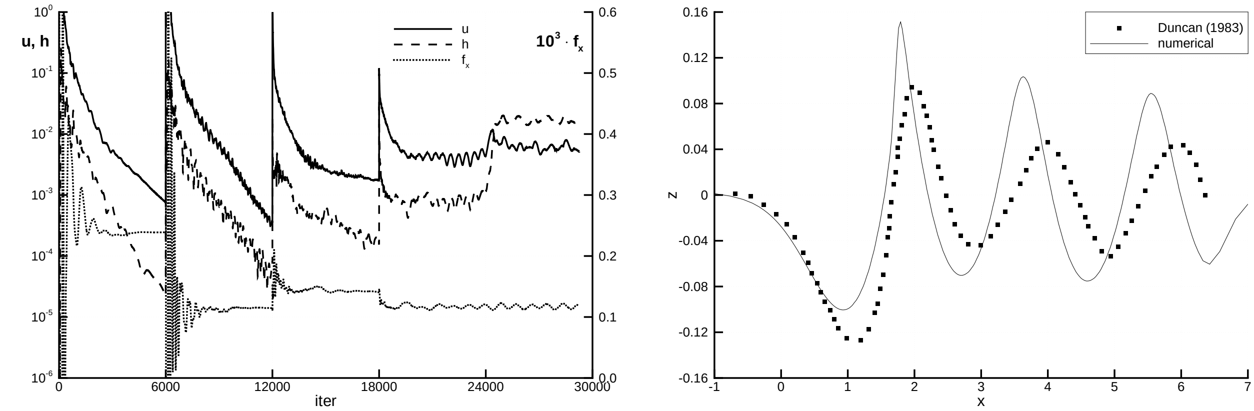


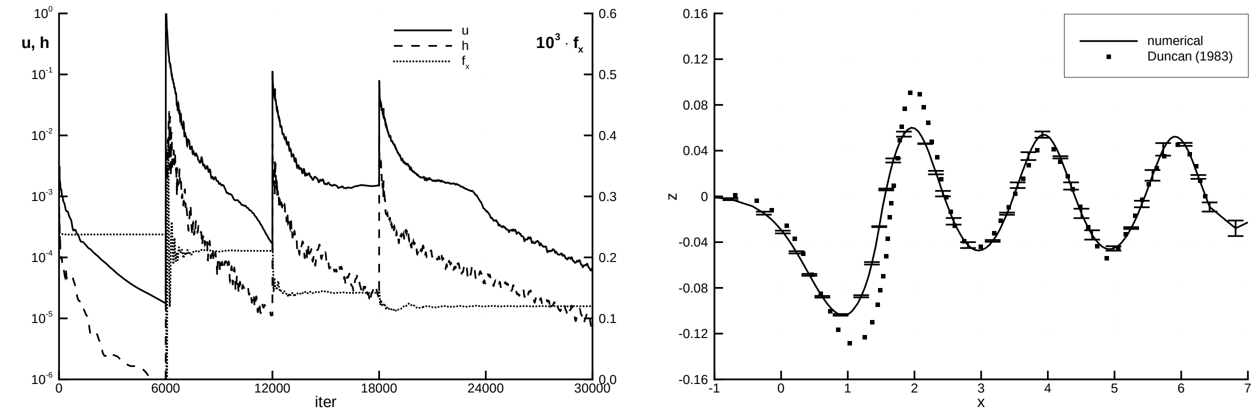
Figure 2: Experimental setup

We use a multi-grid approach with the finest grid consisting of about 60,000 cells, and each coarser grid obtained by halving the finer one.
The L_2-norm of residuals and non-dimensional resistance histories for the simulation without breaking model (fig. 3a) are good for the coarser grids but on the finest one the residuals oscillate around a small but constant value. In fact, the physical solution features a breaker, whereas the first wave of the wave–train in the simulation is too step and its shape is simply due to the fact that the code cannot cope with multivalued surface height (fig. 3b).
Using the breaking model dramatically improves the solution in terms of both convergence (fig. 4a) and wave pattern (fig. 4b, with error bars for the numerical data calculated as suggested in (ITTC Quality Manual, 2001; Roache, 1997)). Although the first wave height is not well captured, the following train shows a very good accordance with experiments in both height and phase.
The major difference with respect to the experimental results is located on the first trough, which is not as deep as in the measurements by Duncan. This same problem is shared by other Navier–Stokes



(a) (b)

Figure 3: No breaking model; (a) residuals and resistance histories, (b) computed wave pattern vs. experiments.



(a) (b)

Figure 4: Simulation with breaking model; (a) residuals and resistance histories, (b) computed wave pattern vs. experiments.

simulations, especially at relatively low Reynolds' numbers (Mori and Shin, 1988; Hino, 1997; Rhee and Stern, 2001).

In fig. 5 the velocity profile is shown near the free–surface under the second trough after the breaker. Duncan's results are shown both with symbols corresponding to actual measurements and with the fitting curve computed in (Duncan, 1983). Numerical results mirror the expected trend, that is the profile for a Stokes' wave, up to the wake of the breaker. The velocity values out of the wake are slightly overpredicted, within 1% from the experiments, whereas the wake thickness is underestimated, as can be evinced from the experiments which are, in this respect, rather spread.

Finally, we have verified that the height of the breaker computed through eq. (1) is in accordance with that yielded by eq. (2). The two values, $h^*_{(1)} \simeq 0.105$ and $h^*_{(2)} \simeq 0.101$, are in very good agreement, thus validating the use of eq. (1) within the iterative algorithm.

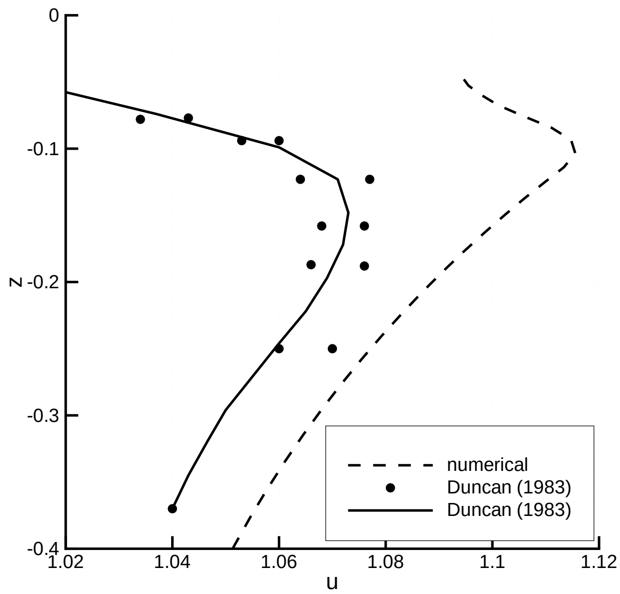


Figure 5: Velocity profile under the second trough after the breaker.

5 Conclusions

A numerical model for steady spilling breakers simulation with RANSE codes has been proposed. The basic ideas for the model are derived from Cointe and Tulin's theory. Nevertheless, important changes were found to be necessary in order to get a stable iterative algorithm when coupling the model with a steady state computation of wavy flow with breaking waves.
Even if the underlying theory is somewhat over–simplified with respect to the physics of the problem (see Lin and Rockwell (1995) for a more accurate description of a two–dimensional flow in presence of breaking) the model can be implemented straightforwardly and yields excellent results for the test case considered.
The proposed model will be generalized and extended to three–dimensional flows past ship hulls in a future work.

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