PRELIMINARY EVALUATION OF AXIAL COMPRESSION FORCE ON THE MAST

First of all let's make a bit of an introduction perhaps useful to clarify the issue ………..
The existing methods for mast and rigging calculation are based on the determination of the maximum compression load acting on the mast being a function of the boat Righting Moment.
The ISO guide lines state as follows: *….*
."For large sailing multihulls, the righting moment is so huge that it would not be realistic to assess the load on rig by equating heeling and righting moment. Therefore for multihulls, the design heeling moment is taken when the fist reef is taken, as calculated in ISO 12217-3, unless the boat is stated by its manufacturer to be designed to "fly a hull"……………
……well…..we will consider it as ….just flying the windward hull !….…as this situation is most likely to occur on a regular basis on racing multi-hulls and many cruising multi-hulls as well.

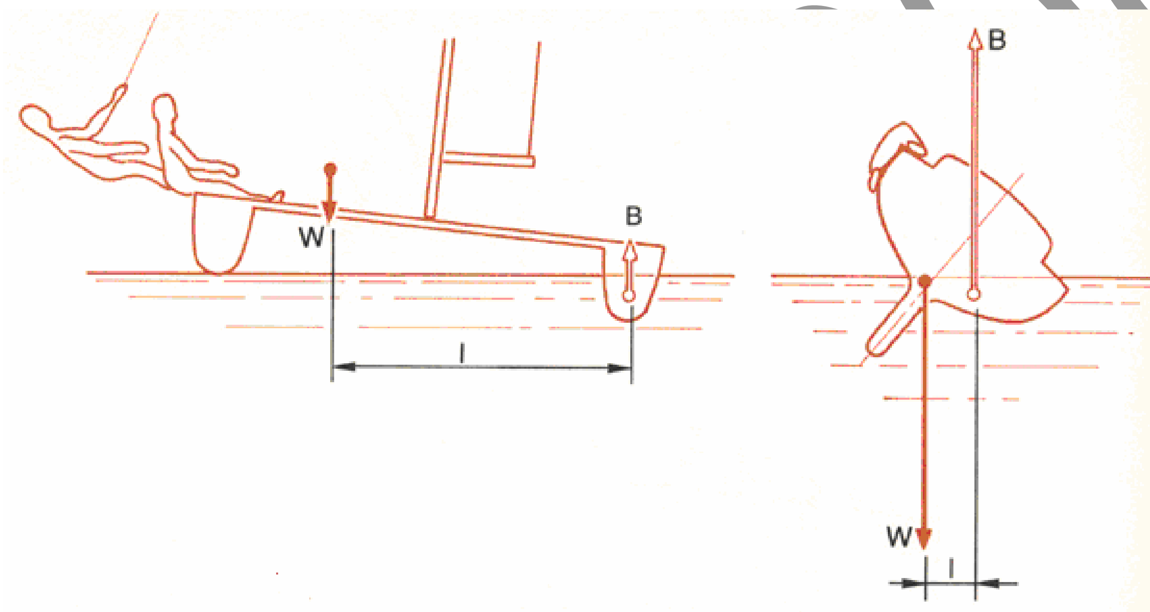

Figure extracted from Pierre Gutelle," The Design of Sailing Yachts*",(International Marine Publishing, first edition, 1979), showing the difference in Rigthing Moment, where, even on small angles of heel, multihulls will face a maximum load much more frequently than a monohull will do.*

Hence, the main problem encounter with a multihull is that it has a very high Righting Moment if compared with a monohull even at smaller angles of heel and it obviously reaches the maximum as soon as the windward hull rises fully out of the water, as previously stated.

This aspect must be considered in the evaluation of a mast compression load.

As a result, the dimensions of the mast section has to be considerable larger in respect of the vessel size as well as the wall thickness which has to be a lot thicker on a multihull rather than for a monohull. On the other hand, a multihull, by virtue of its width, has a much broader staying base than a monohull so that high loads in rigging are mitigated somewhat by the angles the shrouds and stays make to the mast. This has a modifying effect on the compression in the mast, allowing the mast to be made somewhat lighter and, therefore, cheaper to build than one would otherwise expect, considering the high Righting Moments involved.

An aluminum stayed mast is a reasonable lightweight and cheap option.

Just to give the idea of a computational procedure involved in the calculation of the compression force along a standard mast, taking into account a **Knock-down by Spinnaker** (which is slightly simpler then main+genoa) could be itemized as follows:

First of all, make some realistic assumptions, for the case considered, such as:

a. Max RM occurring @ **20°** (**JUST FOR MULTIHULLS)** though max stability can be approximated by multiplying **0.50** by the beam between centerlines of outer hulls (assumed in the region of **8,5m**), or **0,75** by the Bmax for narrow cruisers or up to **0,90** by the Bmax for wide racers, times, or better saying, ..multiplied the displacement.
b. LOA and Bmax assumed being = **10m** approx.
c. Max Width between chain plates assumed to be about **9.0m (i.e. 0,9*Bmax)**
d. Halyards compression due to main **1/5** of max BL and due to staysail is **2/5** of max BL. (a tot of 60% of nominal Breaking Load to be added on to the mast tot vertical axial compression load).
e. Add on to point (d) also the allowance for 180° pulley friction (i.e. multiply by **1,85** correction factor the halyards compression load)
f. Consider the approximate mast weight being of about **14 kgm** (weight per unit length)
g. Add on an allowance to mast weight to incorporate fittings weight i.e. **1,25** (a multiplying factor of the mast weight)
h. Consider the $RM_{20°} \cong 18 * RM_{1°}$ (quite realistic for a trimaran of around **10m,** a LOA value assumed being quite reasonable for a modern wide cruiser/racers for a given Δ_{mass} of about **2500kg**)
i. Reasonable initial GM_T for trimaran hulls of about **10m** could be considered in the region of about **13-14m.**, slightly higher then the Open 60 monohull yachts wide model size.
j. Halyards (galvanized steel flexible wire) reasonable dimension is $\cong$ **6mm** for a 10m vsl approx., having a nominal Breaking Load of **2300 kg** $\approx$.
k. Distance from masthead (halyard pulley) to CLR assumed to be **16.5m** approx.
l. Mast height. **15m** approx
m. Angle between shrouds and top of mast $\approx 16°$; angle between forestay and mast $\approx 19°$; angle between mast and shrouds on the longitudinal plane $\approx 10°$.
n. No pre-tension on rig.

N.B. ASSUMING NO SPREADERS ARRANGEMENTS - (not an optimum solution as such)

1	$RM_{1°}$	$= \frac{W_{(displ.mass)} * GM_T}{57.3} * \phi = \frac{2600 * 13.5 * 1°}{57.3}$	=	**613 kgm**
2.	$RM_{20°}$	$= 18 * 613$	=	**11000 kgm** $\approx$
3.	**MAST wt / unit length**	Approx rigged mast weight /m incl. fittings: P=**14*1.25**	=	**17.5 kg/m**
4.	**Halyards compression acting axially along mast**	Compression due to staysail+main as a percentage of max BL i.e. $60\%\ of\ 2300$ (max Breaking Load)	=	**1380 kg**
5.	**Pulley Friction**	allowance for 180° pulley friction $P_2 = 1380 * 1.85$	=	**2550 kg**
6.	**load at SPI halyard attachment normal to mast**	$P_{T1} = \frac{RM_{20°}}{(height\ masthead\ to\ CLR)} = \frac{11000}{16.50}$ (**normal** to the mast)	=	**667 kg**
7.	**total mast weight**	applied normal to the mast at the masthead $17.5 * 15$	=	**263 kg**
8.	**Mast weight load component (i)**	Applied @ the masthead (and **normal** to the mast) $P_{T2} = 263 * \cos 20°$	=	**247 kg** $\approx$
9.	**load P_T normal to mast**	$P_T = P_{T1} + P_{T2} = 667 + 247$ (due to mast weight)	=	**914 kg** $\approx$

10	**shroud tension T**	$T = \dfrac{P_T}{\sin 16°} = \dfrac{\mathbf{914}}{\mathbf{0.28}}$ =	**3316 kg**
11	**axial load on mast**	$P_1 = T * \cos \mathbf{16}° = \mathbf{3316 * 0.96}$ =	**3316 kg**
12	**TOT axial vertical load on mast**	$P = P_1 + P_2 = \mathbf{3188 + 2550}$ = (quite reasonable result for this vsl LOA and the type of rig envisaged !)	**5738 kg**
13	**Minimum required Mom of Inertia I_{XX} - (ii)**	$= P * K_1 * K_3 * L^2 * 10^{-7} = \mathbf{5738 * 2.4 * 1.35 * 1500^2 * 10^{-7}}$ =	**4183 cm⁴**
14	**Minimum required Mom of Inertia I_{YY} - (iii)**	$= P * K_2 * K_3 * L^2 * 10^{-7} = \mathbf{5738 * 2 * 1.35 * 1500^2 * 10^{-7} * 1.9}$ =	**6623 cm⁴**
15	**From SPARCRAFT std catalog** we choose the PERFORMANCE Range Mast - F4600 - giving I_{XX} / I_{YY} **DUE TO AN EXTREMELY HIGH VALUE OF I_{YY} IT IS HIGHLY ADVISABLE TO CONSIDER A TAPERED MAST.** The **dimensions** being **along the XX axis** / **along the YY axis** and **weight per unit length**		**4400 cm⁴** **12600 cm⁴** **223 mm** **400 mm** **18 kg/m**

(i) the mast is considered as having a single panel neglecting the UDL when calculating P the "critical load" applied normal to masthead, as <u>no spreaders</u> are fitted onto the mast

(ii) (from: Principle of Yacht Design by Larsson & Eliasson)
K_1=panel factor =2.4 (for <u>no spreaders rig)</u>
K_3=factor for deck stepped mast=1.35
L=actual panel length (in our case mast head +no spreaders rig means tot mast height to be taken for calculation)

(iii) (from: Principle of Yacht Design by Larsson & Eliasson)
K_2=staying factor=2 (for <u>no spreaders rig)</u>
K_3=factor for deck stepped mast=1.35
L=actual panel length (in our case mast head +no spreaders rig means tot mast height to be taken for calculation)
1.9=factor of safety allowing for stays, halyards and sheeting.

…………."Furthermore it is important to underline that in the classification societies construction rules as "Bureau Veritas Rule" or "Rules for construction of vessel less then 15 metres (1983)" of Det Norske Veritas there are also rules for mast scantlings determination. These methods are based on simplified models of structural behaviour coupled to certain assumptions involving loadings and allowable stresses, tempered by a knowledge of mast failures in service…………..." (*Sailing Yacht Design: Theory - 1998)*

The most popular method used is without any doubt:

<u>SKENE'S METHOD</u>

The critical load P_T at the basis of the mast is calculated from the $RM_{30°}$

$P_T = \dfrac{1.5 * RM_{30°}}{0.5 * b}$; therefore the Maximum Compressive Load $P = 1.85 * P_T$ where:

$RM_{30°}$	=	righting moment at 30° heel
b	=	lower shroud spreading (or chain plate width)
1.50	=	allowance for heeling angles >30°

1.85 = factor of safety for stays, halyards and sheeting

The minimum required moment of inertia in cm^4 for the mast section is given by the following formula:

$I_{MIN} = \mathbf{1.422} * C * P * L^2 * \mathbf{10}^{-7}$ [cm4] (applied to the lower mast panel):

C = is a coefficient used depending on, mast step, n° of spreaders and type of rig as per the table below (assuming per default the mast material being aluminum)

P = Maximum Compressive Load in (kg)

L = actual mast panel length considered:

a) for Ixx use mast panel length from deck step to first set of spreaders [cm]

b) for Iyy use mast panel length from deck step to inner forestay (or baby stay) attachment

Ixx calc'n	Applying C for Ixx (athwartship)		
Standing rigging arrangement	**Mast step**	**Sing. Spreader**	**2/3 sets of spreaders**
double Lower shrouds	Keel stepped	1.29	1.51
	Deck stepped	1.62	1.94
single Lower shroud + Lower stay	Keel stepped	---	1.15
	Deck stepped	1.65	1.70

Iyy calc'n	Applying C for Iyy (longitudinal)				
Yacht dimensions:	**Standing rigging arrangement**	**Mast step**	**Masthead rigged**		**7/8 rigged**
			Short	**Tall**	
< 9,75 m LWL	double Lower shrouds	Keel stepped	0.76	0.80	0.72
		Deck stepped	0.95	0.98	0.89
	single Lower shroud + Lower stay	Keel stepped	0.64	0.70	0.62
		Deck stepped	0.79	0.83	0.77
>9.75<13.70 m LWL	double Lower shrouds	Keel stepped	0.51	0.55	0.74
		Deck stepped	0.64	0.68	0.87
	single Lower shroud + Lower stay	Keel stepped	0.43	0.49	0.63
		Deck stepped	0.54	0.60	0.79
>13.70 m LWL	double Lower shrouds	Keel stepped	0.43	0.46	0.61
		Deck stepped	0.54	0.58	0.73
	single Lower shroud + Lower stay	Keel stepped	0.58	0.61	0.80
		Deck stepped	0.71	0.78	0.91

For shrouds scantling, the load distribution, derived from experimental measurements, can be assumed as a percentage of the critical load P_T with a FS = 3.

Test for P value obtained using Skene's formula (iv)	$P = \frac{(\mathbf{0.87} * \mathbf{11000}) * \mathbf{1.5} * \mathbf{1.85}}{\mathbf{0.5} * \mathbf{9.0}}$ RM_{30° =87% of max RM (i.e. RM_{20° for multihulls)	=	**5900kg ≈** **(3% ≈ difference between results)**

(iv) 1.5=safety factor used as allowance to RM_{30°

1.85=allowance for stay-backstay and running rigging

Max chain plate width ≅ 9.0m (i.e. 0,9*Bmax)

N.B.

If hydrostatic characteristics are not available there is a formula proposed by ***Dr Gerritsma (1993)*** which can be useful in obtaining a preliminary value of the Righting Moment as follows:

$$M_R(\varphi) = \Delta\left[\left(\mathbf{0.664}T_C + \mathbf{0.111}\frac{BWL^2}{T_C} - G_{DWL}\right)\sin\phi + LWL\left(D_2 * \phi * F_N + D_3 * \phi^2\right)\right]$$

Where:

Δ = displacement of the vessel

T_C = draught at canoe body

B_{WL} = breadth water line

G_{DWL} = center of gravity of the yacht (-ve if above DWL)

φ = heel angle

L_{WL} = length waterline

D_2 = $$-0.0406 + 0.0109 * \left(\frac{B_{WL}}{T_C}\right) - 0.00105 * \left(\frac{B_{WL}}{T_C}\right)^2$$ a stability coefficient

F_N = $$\frac{V_S}{\sqrt{g * L_{wL}}}$$ Froude Number

D_3 = $$0.0636 - 0.0196 * \left(\frac{B_{WL}}{T_C}\right)$$ a stability coefficient

N.B. when considering knock down by main+genoa the procedure is slightly different !