Keel line of sailing dinghy - Formulation within Gene Hull Dinghy 3.1 - 04 2026

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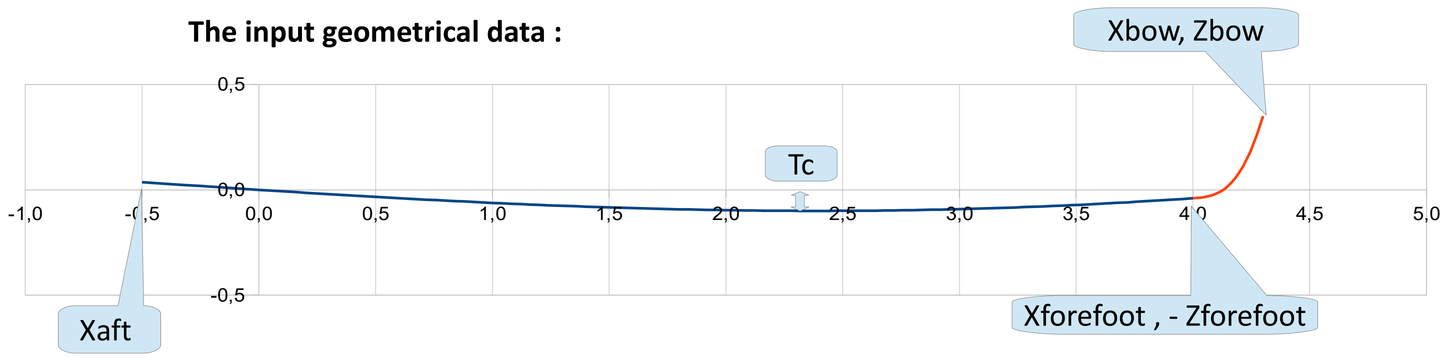
Introduction :

The keel line is defined in the vertical symetrical plan XoZ

Coordinates system :

X = 0 at station C0 (= rear point of the waterline), X positive towards front
Z = 0 at the waterline surface, Z positive towards up

The input geometrical data :



Tc (the hull body draft) and Zforefoot (the forefoot depth) are data input as positive values, and so -Tc and –Zforefoot are the Z coordinates of the corresponding point.

The global keel line is defined by 2 functions Z(X) for respectively :

- the keel line itself , for X = Xaft to Xforefoot , here in blue
- the bow line , for X = Xforefoot to Xbow, here in red

Example (from the .ods file attached) :

Input data (m)		
Xforefoot	4,000	
Zforefoot	0,040	Condition : Zforefoot < Tc
Tc (m)	0,100	Hull draft Tc
Xaft (m)	-0,500	Aft overhang (negative value
n	2	Degree of polynome z"(x)

Input data for the bow line (m)		
Xbow	4,300	Xbow > Xforefoot
Zbow	0,350	Zbow > 0

The rationale

For a sailing dinghy aiming to start early the planing mode and to sustain it at the best, one of the issues recurently evoked is about the optimal shape of the hull bottom center line, here named the keel line the traditional wording, although there is no longer a keel at such for modern dinghies.

The idea is to gradually reduce the keel line curvature from front to aft, up to aiming a curvature equal zero at the transom aft end. So, the more the planing mode develops and the trim increases, the lower the curvature of the hull bottom still in contact with the water. And finally at transom, we want zero curvature for the water exit. This last point, zero curvature at transom, is retained mainly by the elimination of other solutions :

- keeping some convex curvature in the aft zone up to the transom cannot be optimal for the water exit at a planing speed : you deviate the flow from a more horizontal exit while provoking a sucking, i.e. a negative dynamic lift and a bow-up moment. Some argues that the bow-up moment can help prevent the bow from diving into the wave ahead : yes, but it is at the cost of less dynamic lift and less boat speed, it should be better to avoid diving occurrence by another design means.
- adopting a concav shape at transom can give an underisable bow-down moment : as for a sailing dinghy, at contrary of a motorboat, you have already plenty of bow-down moment from the sails center of effort.

If we agree with this approach, then the curvature evolution is directly deduced : a curvature gradually reduced from forefoot to aft means that the curvature is maximum at the forefoot, logic. And as we want also zero curvature at transom level, that means that the curvature should evolve regularly from a maximum at forefoot to zero at transom. These two conditions + the geometrical ones above can be laid down as a basis for deducing an analytical formulation where, for simplification, the curvature 1/R of the keel line is approximated by the second derivative Z''(X), which is true by more than 99% due to the low slope of the line (Z'(X) ~ 0). (see in annex the mathematical demonstration).

It should also be noted that another result of this approach is that the position XTc of the maximum body draft Tc is always forward of the waterline midpoint, it is given in the output data. This aligns with the intuition of the early naval architects of planing dinghies.

Keel line formulation, from Xaft (aft transom) to Xforefoot

To lighten the formulation writing :

Xf = Xforefoot
Zf = - Zforefoot (remind : Zforefoot is a positive input data)
Z''f = Z''forefoot
Xa = Xaft

Z(X) = **Z''f** / 2 * (Xf – X)^2 – **k** * (Xf – X)^(n+2) / (n+1) / (n+2) – **c** * (Xf – X) + Zf for X = Xa to Xf

where

k = Z''f / (Xf – Xa)^n

c = Z''f / 2*Xf – **Z''f** / (Xf – Xa)^n / (n+1) / (n+2)*Xf^(n+1) + Zf/Xf

Z''f is obtained through an iterative process detailed here below

Input data used for this formulation and for Z''f calculation below :

Geometrical : Xf, Zf, Tc, Xa
Adimensional parameter : n

Iterative calculation for Z''f :

Start with initial values (index 0) :

- **ratio0 = 0,5** : « ratio » defined such as XTc = ratio*Xf is the position of the hull body maxi draft Tc. Remind : Tc is an input data, but its position which is an output of the process, and so not known at the beginning and is part of the iteration.
- **Z''f0 = 1**

First iteration (index 1 versus index 0) :

ratio1 = 1/2 - (Xf – **ratio0***Xf)^(n+1) / (Xf – Xa)^n / (n+1) / Xf + Xf^n / (Xf – Xa)^n / (n+1) / (n+2) – Zf/Xf^2/**Z''f0**

Z''f1 = (- Tc - Zf) / **{**1/2 * (Xf – **ratio1***Xf)^2 - (Xf – **ratio1***Xf)^(n+2) / (Xf – Xa)^n / (n+1) / (n+2) - (Xf – **ratio1***Xf) * [Xf/2 – Xf^(n+1) / (Xf – Xa)^n / (n+1) / (n+2) + Zf/Xf/**Z''f0**] **}**

Then second iteration, etc …, up to 8 iterations are sufficient for a high convergence (see the .ods file) , giving final values of : **ratio , Z''f , c , k** and then the Z(X) formula above can be calculated for X = Xa to Xf

One can derive from Z(X) the angles of the keel line, and in particular the angle of attack (estimated at X = 0,9 Lw) and the exit angle (at aft transom Xa), through tangent(angle) = Z'(X) :

Z'(X) = - Z''f * (Xf – X) + k / (n+1) * (Xf – X)^(n+1) + c >>> Angle = Arctan[Z'(X)]

Bow line formulation, from Xforefoot to Xbow

To lighten the formulation writing :

Xb = Xbow ; Zb = Zbow

Bow line formula :

Z(X) = A X^3 + B X^2 + C X + D

where,

A = (Q5 * 2 – Q6 * Q8) / (Q7 * 2 – Q9 * Q8)

B = (Q5 – A * Q7) / Q8

C = Z'f – 3 * A * Xf^2 – 2 * B * Xf
(with Z'f = c , result from the iterative process, see previous page)

D = Zb – A * Xb^3 – B * Xb^2 – C * Xb

Q5 = Zb – Zf – Z'f * (Xb – Xf)

Q6 = Z''f (Z''f result of the previous iterative process, see previous page)

Q7 = (Xb^3 – Xf^3) – 3 * Xf^2 * (Xb – Xf)

Q8 = (Xb^2 – Xf^2) – 2 * Xf * (Xb – Xf)

Q9 = 6 * Xf

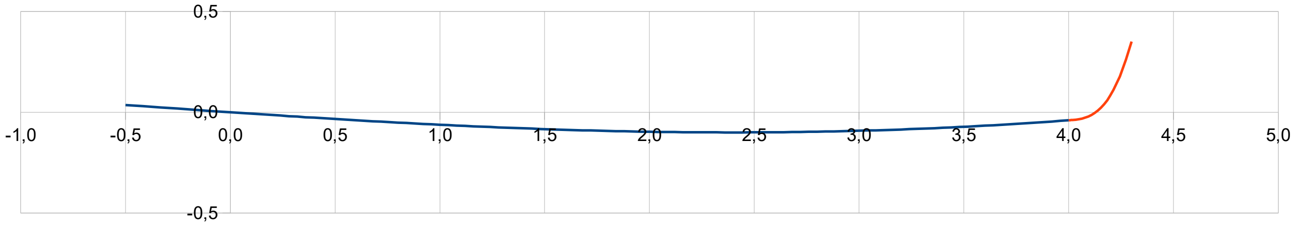
Besides Z'f and Z''f, all other geometrical data used are : Xf, Zf, Xb, Zb

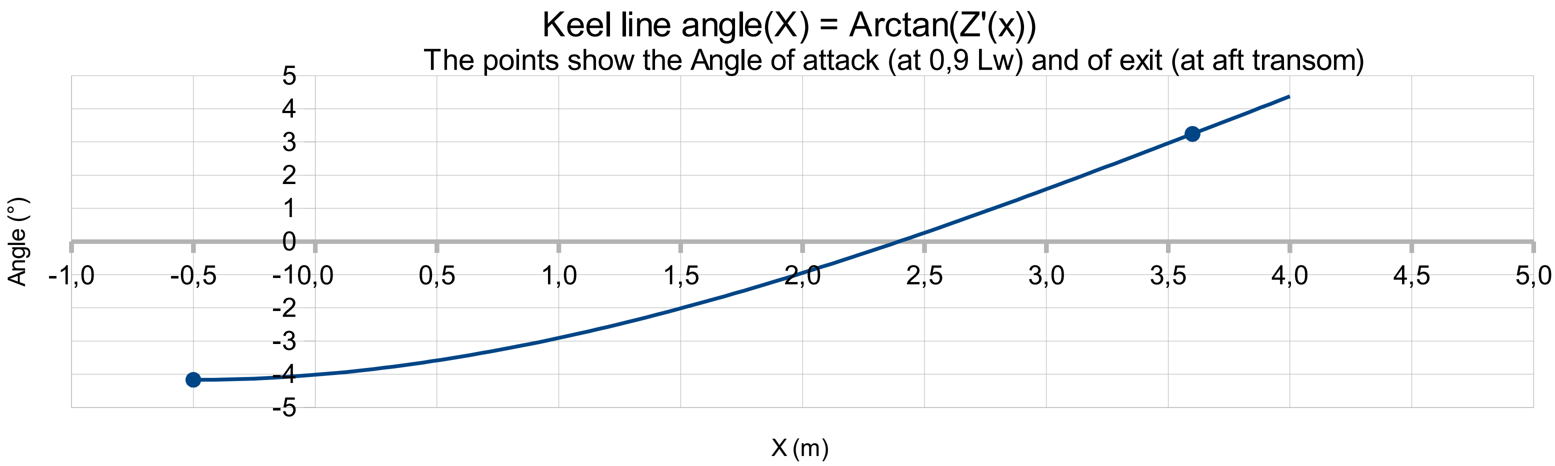
Typical example (done with the .ods file attached) :

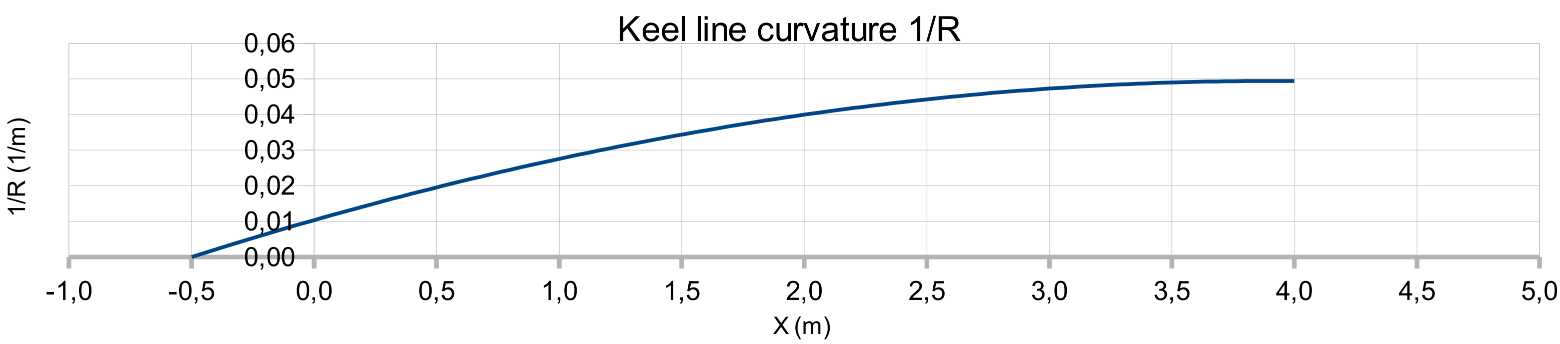
>>>>

Input data (m)			Output data	
Xforefoot	4,000			
Zforefoot	0,040	Condition : Zforefoot < Tc	>> Zforefoot /Tc (%)	
Tc (m)	0,100	Hull draft Tc	40	
Xaft (m)	-0,500	Aft overhang (negative value)	ratio = XTc / Xforefoot	0,599
n	2	Degree of polynome z"(x)	>> XTc (m)	2,396
			Lw (m)	4,130
			>> XTc (% Lw)	58,0

Input data for the bow line (m)			Keel line angles :		
Xbow	4,300	Xbow > Xforefoot	> Aattack (°)	3,2	at 0,9 Lw
Zbow	0,350	Zbow > 0	> Aexit (°)	-4,2	at aft transon





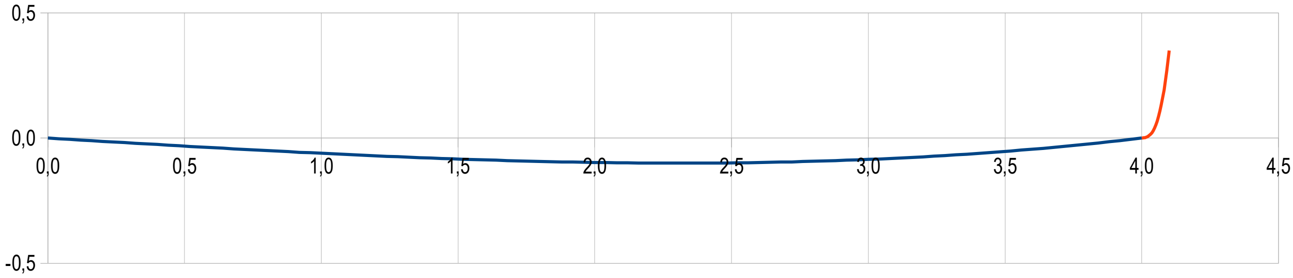


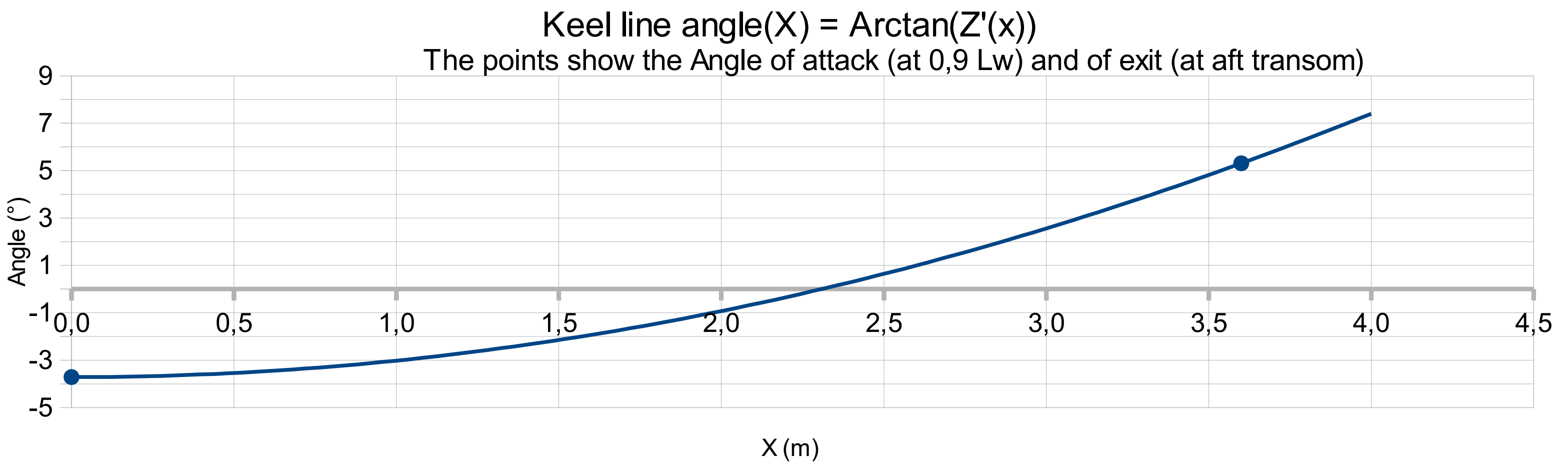
Another example (in red the changed data from above example)

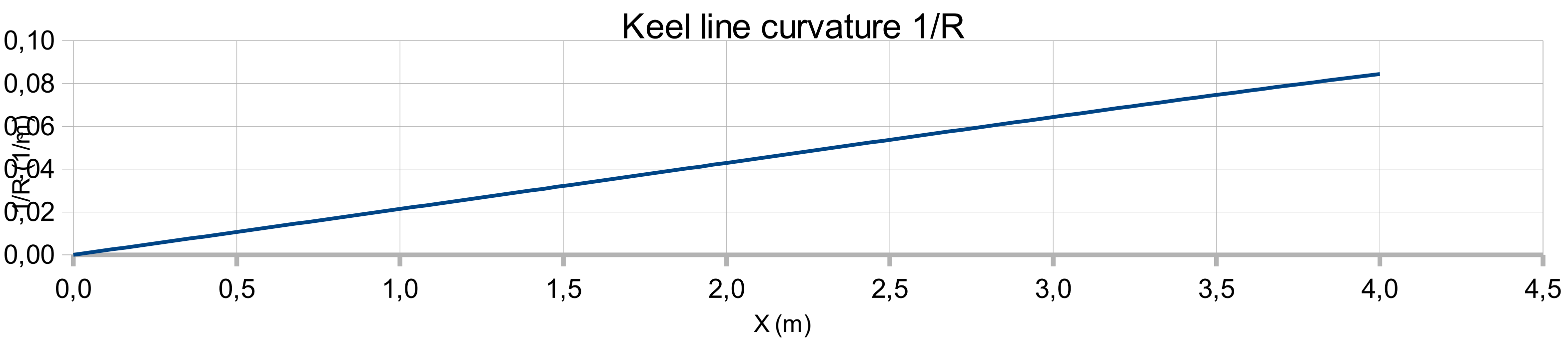
>>>>

Input data (m)			Output data	
Xforefoot	4,000			
Zforefoot	0,000	Condition : Zforefoot < Tc	>> Zforefoot /Tc (%)	
Tc (m)	0,100	Hull draft Tc	0	
Xaft (m)	0,000	Aft overhang (negative value)	ratio = XTc / Xforefoot	0,577
n	1	Degree of polynome z"(x)	>> XTc (m)	2,309
			Lw (m)	4,000
			>> XTc (% Lw)	57,7

Input data for the bow line (m)			Keel line angles :		
Xbow	4,100	Xbow > Xforefoot	> Aattack (°)	5,3	at 0,9 Lw
Zbow	0,350	Zbow > 0	> Aexit (°)	-3,7	at aft transom

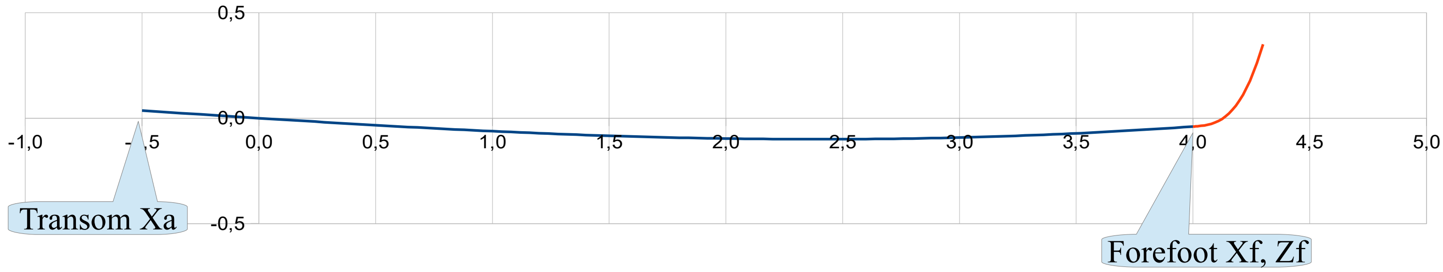






Annex 1 - The formulation of the keel line shape from the function of its curvature

The goal : a curvature evolving from a maximum at bow to zero at transom, in order to both generate a positive attack angle in the fore part to initiate the dynamic lift, and then to have a regular decreasing of the curvature up to zero at the tansom to accompany the development of the full planing mode.



Let's Z(X) the equation of the keel line, the curvature general formula is : $1/R = Z'' / (1 + Z'^2)^{3/2}$

In our case, due to low slopes of the keel line for such dinghy, i.e. with angles mostly < 5° everywhere, one can make the assumption that the first derivate Z' (which is equal to the tangent of the angle) is close to zero and the denominator $(1 + Z'^2)^{3/2}$ # 1 (e.g. for an angle of 5° it is true at 98,9 %) . So, that allows a drastic simplification : $1/R = Z''$ the second derivate.

Let's set : $Z''(X) = Z''f - k (Xf - X)^n$ in line with the goal described above, where :

- Z''f : is the maximum curvature positioned at the forefoot Xf, not known at this stage
- n : the degree of the polynome, it is an input data by the User (e.g. if n = 1, this is a linear decreasing of the curvature, if n =2 this is a parabolic decreasing of the curvature)

To have zero curvature at Xa (aft transom) , i.e. $Z'' (Xa) = 0$, this leads to **$k = Z''f / (Xf - Xa)^n$**

By double integration, we have :

$$Z'(X) = - Z''f (Xf - X) + k/(n+1) \cdot (Xf - X)^{(n+1)} + c$$

$$Z(X) = Z''f/2 \cdot (Xf - X)^2 - k/(n+1)/(n+2) \cdot (Xf - X)^{(n+2)} - c (Xf - X) - d$$

To determine the coefficients c and d, these formulas should respect the other conditions which are :

At X = 0 >> Z = 0 (the keel line crossing the waterline at aft perpendicular)

>>> $Z(0) = 0 = Z''f/2 \cdot Xf^2 - Z''f / (Xf - Xa)^n / (n+1) / (n+2) \cdot Xf^{(n+2)} - c\, Xf - d$

At X = Xf >> Z = Zf (at the forefoot point)

>>> **$d = - Zf$**

>>> **$c = Z''f/2 \cdot Xf - Z''f / (Xf - Xa)^n / (n+1) / (n+2) \cdot Xf^{(n+1)} + Zf/Xf$**

At the hull body max draft, where Z = - Tc : here we don't know its X location, we can introduce a parameter named « ratio » such as, at X = ratio*Xf, we have Z(ratio*Xf) = -Tc. And, as it is a minimum of the curve, we have also Z'(ratio*Xf) = 0. So at this point, we have two equations for 2 unknows : **ratio** and **Z''f**

Z'(ratio*Xf) = 0 :
>>> 0 = - Z''f (Xf – ratio*Xf) + Z''f / (Xf – Xa)^n / (n+1) * (Xf – ratio*Xf)^(n+1) + Z''f/2*Xf – Z''f / (Xf – Xa)^n / (n+1) / (n+2)*Xf^(n+1) + Zf/Xf

We divise by Z''f :
>>> 0 = - (Xf – ratio*Xf) + (Xf – ratio*Xf)^(n+1) / (Xf – Xa)^n / (n+1) + Xf/2 – Xf^(n+1) / (Xf – Xa)^n / (n+1) / (n+2) + Zf/Xf/Z''f

We divise by Xf :
>>> **ratio** = 1 / 2 - (Xf – **ratio***Xf)^(n+1) / (Xf – Xa)^n / (n+1) / Xf + Xf^n / (Xf – Xa)^n / (n+1) / (n+2) – Zf/Xf^2/**Z''f** **(Equation 1)**

Z(ratio*Xf) = - Tc :
>>> -Tc = Z''f/2*(Xf – ratio*Xf)^2 – Z''f / (Xf – Xa)^n /(n+1)/(n+2)*(Xf – ratio*Xf)^(n+2) - (Xf – ratio*Xf) * [Z''f/2*Xf – Z''f / (Xf – Xa)^n / (n+1) / (n+2)*Xf^(n+1) + Zf/Xf] + Zf

>>> - Tc = Z''f * **{**1/2 * (Xf – ratio*Xf)^2 - (Xf – ratio*Xf)^(n+2) / (Xf – Xa)^n / (n+1) / (n+2) - (Xf – ratio*Xf) * [Xf/2 – Xf^(n+1) / (Xf – Xa)^n / (n+1) / (n+2) + Zf/Xf/Z''f] **}** + Zf

>> - Tc - Zf = **Z''f** * **{**1/2 * (Xf – **ratio***Xf)^2 - (Xf – **ratio***Xf)^(n+2) / (Xf – Xa)^n / (n+1) / (n+2) - (Xf – **ratio***Xf) * [Xf/2 – Xf^(n+1) / (Xf – Xa)^n / (n+1) / (n+2) + Zf/Xf/**Z''f**] **}** **(Equation 2)**

The intrication of **ratio** and **Z''f** within these 2 equations is too high to allow a direct formula for each, but we can easily solve the system by iterations :

Intial values to start : ratio0 = 0,5 ; **Z''f0= 1**

Iteration 1
Equation 1 :
ratio1 = 1/2 - (Xf – **ratio0***Xf)^(n+1) / (Xf – Xa)^n / (n+1) / Xf + Xf^n / (Xf – Xa)^n / (n+1) / (n+2) – Zf/Xf^2/**Z''f0**
Equation 2 :
- Tc - Zf = **Z''f1** * **{**1/2 * (Xf – **ratio1***Xf)^2 - (Xf – **ratio1***Xf)^(n+2) / (Xf – Xa)^n / (n+1) / (n+2) - (Xf – **ratio1***Xf) * [Xf/2 – Xf^(n+1) / (Xf – Xa)^n / (n+1) / (n+2) + Zf/Xf/**Z''f0**] **}**

Iteration 2 with ratio1 and Z''f1 , etc ... , after 8 iterations we have a high degree of convergence.