UNCONSTRAINED SHIPS OF MINIMUM TOTAL DRAG

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SUMMARY

A displacement vessel of a given loaded weight has a theoretical optimum length, usually somewhat longer than the conventional, which minimises its total calm-water drag. Some simple examples are given to illustrate this property. Genetic algorithm techniques are then used to find optimum dimensions for monohull and multihull vessels over a wide range of speeds and displacements, with a fixed assumption about the waterline, cross-section, and buttock shapes. Michell's integral is used for the wave resistance, the 1957 ITTC line for the skin friction, and a simple empirical formula for the form drag.

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1. INTRODUCTION

Consider first a class of monohull ships moving steadily ahead on a flat calm infinitely-deep sea. Fix the displacement, draft, speed, and hull shape. The length is then essentially the only variable allowed. At any given length, adjust the beam by uniform scaling of all offsets, so as to achieve the prescribed displacement; longer ships are thinner. Now vary the length until the total (viscous plus wave) drag $R_t=R_v+R_w$ is minimised.

The above simplified ship optimisation problem, with length as the only variable, usually possesses a non-trivial solution, i.e. a finite optimum length, for the following reason. Viscous drag R_v is predominantly skin friction, which is proportional to surface area, and as a body of a given volume gets longer and thinner, its surface area increases. Hence viscous drag increases with length at fixed displacement.

On the other hand, for conventional ships at conventional speeds, wave resistance R_w generally decreases as the ship length increases. Since we are holding the speed U fixed, as we increase the length L, we are decreasing the length-based Froude number F=U/sqrt(gL). At fixed displacement, and at most relatively low Froude numbers, wave resistance is a (rapidly) increasing function of Froude number, and therefore decreases with increasing length. There are wobbles in the graph of wave resistance versus Froude number, so this is not an absolute conclusion, but it does hold in most cases, and of course this advantage of long ships is very much part of the naval architectural art.

Since there are opposite trends with length in the two constituents of the total drag, there must be a minimum for their sum, at some non-trivial intermediate length. That length is generally somewhat larger than for conventional ships.

In fact, sometimes there is more than one local minimum in the graph of total drag versus length, and this phenomenon is discussed in more detail in the following section. There is often a delicate interplay between local and global optima, which makes for an optimisation process that is quite difficult to analyse. In order to deal with this problem, we use here a powerful general purpose technique, described later, called "genetic algorithms".

In the present paper, we perform an exhaustive treatment of this optimisation problem for a family of monohull and multihull vessels, covering a very large range of speeds and displacements. We hold the hull shape, displacement and speed fixed, and allow the draft (and for multihulls, various other parameters) as well as the length to vary until the minimum total drag is achieved. We treat viscous drag as the sum of skin friction (estimated by the 1957 ITTC line) and a generally small but sometimes crucial form drag contribution which is estimated by an empirical formula. We use Michell's integral for the wave resistance, which is only accurate for thin ships. However, this is a more than usually good assumption for the class of extremely fine hulls that arise from this optimisation process.

When we treat multihull vessels, each separate hull is taken from the same shape family as the monohulls. For catamarans consisting of two identical side-by-side hulls, there is thus just one further parameter that participates in the optimisation, namely the lateral hull separation distance. There are some speed ranges where there is an optimal finite choice for this separation, and others where the best separation is infinite - that is, the optimum "catamaran" is actually two unconnected hulls. We find that, from the point of view of total drag (with no length restriction), a catamaran can never compete with a length-optimised monohull of the same total displacement. This is essentially because of the increased wetted surface area created by splitting the hull in two, which increases further the already dominant viscous drag component.

As the number of hulls increases, many more parameters are involved in the optimisation of multihull vessels, even within the constraint that all hulls have the same shape. For example, for laterally-symmetric trimarans there are a total of seven parameters; namely two drafts, two lengths, one ratio between side-hull and total displacement, one longitudinal and one lateral separation distance. The side-hull displacement ratio parameter is somewhat special, in that the trimaran reduces to a monohull when this ratio is near zero and to a catamaran when it is near unity. If that parameter is included in the optimisation process, a monohull results automatically whenever a length-optimised monohull is superior to a trimaran (which is always!), and similarly a catamaran would result if a catamaran was superior to a trimaran. Hence in order to confine attention to true trimarans, we use only a 6-parameter optimisation, then repeating this optimisation for a range of values of the displacement ratio. The results show that (strictly from the total-resistance point of view of the present paper) trimarans are also uncompetitive with length-optimised monohulls or catamarans.

When there are length (or other) restrictions, and hence (for shorter-than-optimal ships) a greater contribution of wave resistance to the total drag, multihulls can have less total drag than monohulls of the same length, because of the potential for favourable hull-hull cancellation of wave resistance. For example, in work to be reported elsewhere, we have examined a 3500 tonne vessel of length 160m designed to operate at 40 knots speed. In that case, the best catamaran has 10% less total drag than the best monohull of the same length, and there are indications that further improvements are possible with optimised trimarans.

The main purpose of the present study is to provide a benchmark, from which extended studies can follow. One class of such extensions obviously involves allowing the shape of the hull to vary. For the present study, we have used a very fine type of hull, appropriate for high-speed and sporting-type vessels, and there is a need to repeat the study with more commercial shapes of hull.

However, perhaps of greater importance is inclusion of further constraints. When the only quantities held fixed are speed and displacement, it is not surprising that the resulting ship proportions are somewhat (but not outrageously) unconventional. Further constraints, such as constraints on maximum length or minimum beam, arise inevitably from commercial, structural, safety, seakeeping, or sporting requirements. When these constraints are imposed, the ship proportions will return to the more conventional range, but at a price in terms of increased total drag. It is of value to know just how much of a price is being paid.

1.1 An illustrative example

In the present section, we first give an example illustrating the character of the results obtained in the present study. Further results are presented in more generality and in nondimensional form later. For this example, we confine attention to a "ship" of one-tonne displacement, representative of a (large) rowing shell, and use dimensional units.

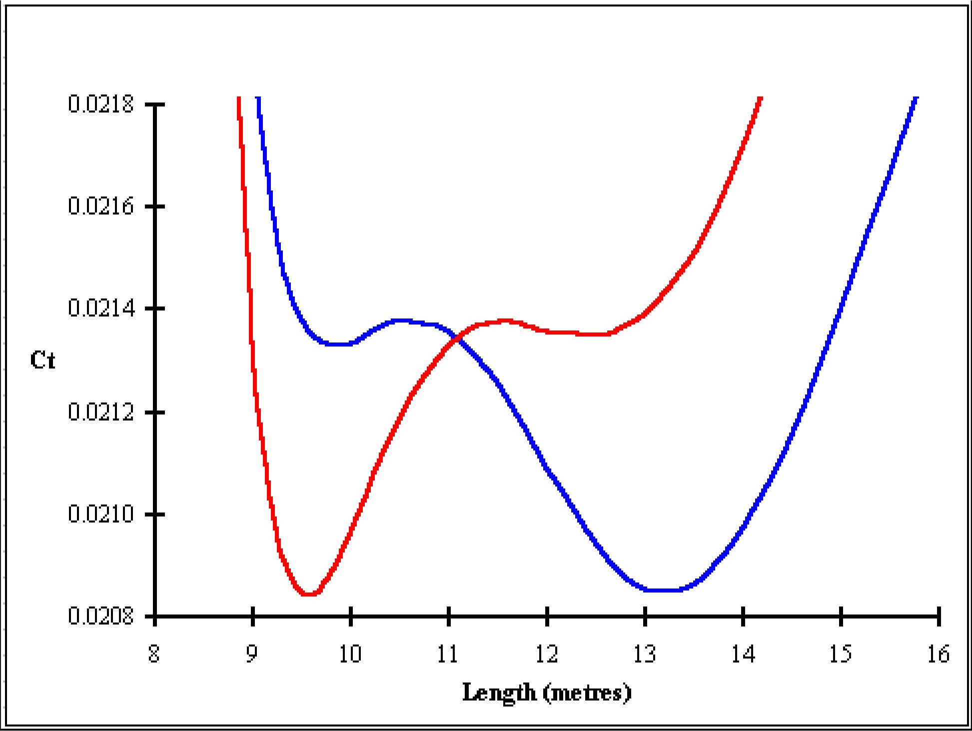

Fig. 1.1: Comparison of total resistance for two one tonne monohulls.

Figure 1.1 shows two typical examples of graphs of total drag versus length (in metres) at a fixed speed, for such a vessel. For the present purpose, it is not essential how the drag is determined or scaled, but we should note that it does include an allowance for form drag, discussed later. The blue curve is at a fixed speed of 5.56 knots and the red curve at only a very slightly higher speed of 5.59 knots. In both cases, there are two prominent minima, i.e. two distinct (and remarkably different) lengths are locally favourable, and define "best" and "second-best" ships. At the lower speed, the longer ship (13.2 metres length) is better than the shorter ship (9.8m), whereas at the higher speed, the shorter ship (9.6m) is superior to the longer ship (12.3m). Thus, as we vary the speed and other parameters, there may occur an interchange between two local optima, so that the optimum length may appear to change discontinuously. These changes can occur over a remarkably narrow range of speeds.

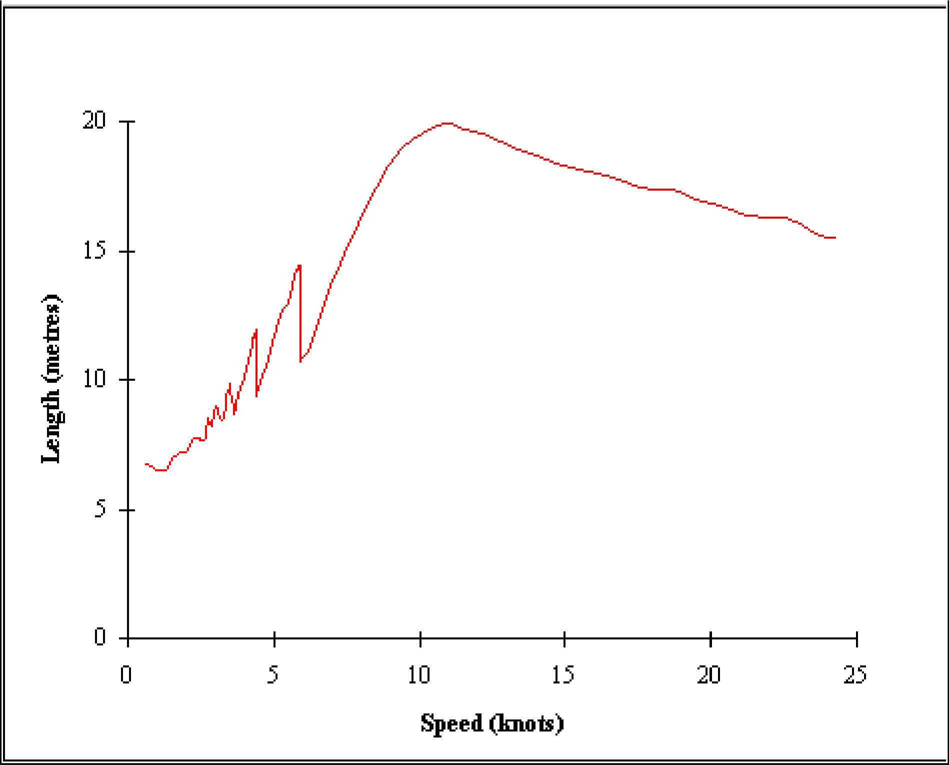

Fig. 1.2: Optimum length for a one tonne monohull

This type of discontinuity in the optimum length is shown in Figure 1.2, again taken from the family of one-tonne monohulls. This figure gives the optimum length in metres as a function of the speed in knots. The discontinuities indicated above occur only at relatively low speeds, notably at about 5.6 knots (where the change between the two curves of Figure 1.1 occurs) and 4.3 knots, with smaller discontinuities at even lower speeds.

At speeds between the discontinuities, the Froude number based on shiplength remains essentially constant, and examination of the variation of wave resistance with Froude number indicates that this constant value corresponds to a local minimum of wave resistance. What is happening as we increase the speed is that, in attempting to design for minimum total drag, we simultaneously increase the shiplength, in order to stay at that local minimum. This continues as long as possible while we increase the speed, and when it is no longer possible, the optimum ship suddenly decreases its length, so that the Froude number suddenly jumps to the next higher local minimum, avoiding the local maximum in between. This process is intuitively like changing gears!

The length variation in the example of Figure 1.2 is continuous for all speeds above 5.6 knots. However, as is discussed later, if form drag is neglected, there can also be an apparent high-speed discontinuity. It is important to note that, as indicated by Figure 1.1, there is no discontinuity in the actual total drag at these speeds, merely an interchange in the roles of "best" and "second-best" ships. At the speeds where the optimum length changes discontinuously, the residual total drag tends to reach a local maximum, where its rate of change with respect to speed changes discontinuously.

Although these discontinuities are of interest in their own right, they are not necessarily the most important feature of Figure 1.2. They depend on the fact that the wave resistance possesses minima, and these minima are to a certain extent magnified by the theoretical procedure (here Michell's integral) used to compute wave resistance. If more empirical means are used to estimate wave resistance, with the effect of smoothing out the humps and hollows in the wave resistance variation, there will be a consequent reduction in the size of the discontinuities. However, so long as there are at least two minima in the wave resistance curve, a discontinuity is inevitable, no matter what method is used to estimate wave resistance.

Above 5.6 knots, the optimum length of a one-tonne vessel varies smoothly, and it is unlikely that the optimum length is sensitive to the procedure for wave resistance computation. In fact, the range of speeds above that where discontinuous length changes occur is the one of greatest interest in practice; for example, it is the competitive speed range for rowing shells.

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2. BASIC CONSIDERATIONS

2.1 Hull geometry

In this study, we present results for one hullform only - a canoe body defined by parabolic waterlines, elliptical cross-sections, and a parabolic keel line. Although this form is an obvious idealisation, there has recently been an appeal (Insel and Molland 1991) for further work on similar hull shapes.

For this hullform, the block coefficient is C_b = 0.417 and the prismatic coefficient is C_p = 4 C_b/pi = 0.533. Clearly this is a much finer type of hull than that of a typical merchant ship, but is relevant to sporting canoes and hulls of special high-speed vessels. It is particularly appropriate for slender vessels with the high length/beam ratios that we shall find optimal.

2.2 Wave Resistance

We use Michell's integral (Michell 1898; see also Tuck 1989) to estimate the wave resistance R_w of the ship. This requires evaluation of a triple integral, one integral in each of the length-wise and draft-wise co-ordinate directions, and one integral with respect to the angle theta of propagation of the ship-generated waves. The numerical method used here for evaluating these integrals both for monohulls and catamarans is described fully in Tuck (1987). For monohulls, we use up to 81 stations, 81 waterlines, and 640 intervals for the integration with respect to theta. This is an unusually high degree of precision, and is sufficient to eliminate any numerical artefacts in the integration, which is a common source of error in use of Michell's integral. For multihulls we sometimes need an even larger number of intervals of theta, because the interference between the wave patterns produced by the individual hulls tends to produce a more oscillatory variation in the wave spectrum with propagation direction than for monohulls.

Michell's integral depends for its validity on the ship being thin, and is sometimes considered (perhaps unfairly) to be insufficiently accurate for use with ships of conventional proportions. However, the hulls produced by the optimisation process in this study are significantly thinner than conventional ships, and there is good evidence that for such slender vessels Michell's integral is satisfactory. For example, Hanhirova et al 1995 (see also Tuck 1989 and Chapman 1972) report that for length-based Froude numbers above 0.35, accuracies relative to measured residuary resistance of better than 10% are achieved by Michell's integral for hulls with length/beam ratios of the order of 10.0. The optimised hulls in the present study are even more slender.

In any case, the hulls resulting from the optimisation process also have the property that their wave resistance is generally only about 10% of the total, so that the absolute accuracy of the wave resistance measure is not critical. This proportion of wave resistance to total drag is lower than what is usually encountered with conventional ships, since the present optimum is in part achieved by increasing the length beyond the conventional, so as to reduce the influence of wave resistance. Even though the wave resistance is then only a small component of the final total drag, it remains a critically important component nevertheless in controlling the optimisation process; after all, if there was no wave resistance at all, short ships of minimum surface area would be preferred.

There is really no actual Michell integral for multihulls. What we use here is the assumption that each separate hull can be represented by the same singularity distribution (namely sources distributed over its own centreplane) as if that hull were alone. This neglects one type of interaction between the hulls, namely the influence of one hull on another in creating a cross-flow which modifies this singularity distribution, in particular inducing vortices as well as sources. On the other hand, it does not prevent interference between the wave systems generated by the centreplane sources. Little is known of the relative importance of these two types of interactions, but the present assumption seems to yield quite good results for the wave resistance (Tuck 1987, Salvesen et al 1985). It is notable that the assumption that there are no induced vortices due to other hulls can be exactly satisfied by allowing the hull centrelines to possess a suitable small camber (Lin 1974). This camber has no effect on the wave resistance, and may be desirable in eliminating induced drag. It is also important (Tuck 1987, Newman and Tuck 19xx) that the size of this induced-crossflow effect relative to that due to the hull's own thickness is proportional to the draft/length ratio, and hence negligible for conventional (and *a forteriori* the present optimal) slender ships of small draft, even if formally of the same order as self-thickness effects for the thin ships of finite draft for which Michell derived his integral.

2.3 Viscous Resistance

The viscous resistance R_v can be written as

R_v=1/2*rho*U^2*S*C_v

where rho is the water density and S the wetted surface area of the hull. When skin friction dominates, the drag coefficient C_v approximately equals C_f, where C_f is a skin friction coefficient which can be estimated using the ITTC 1957 ship correlation line (Proc. 8th ITTC).

$C_f = 0.075/(\log_{10}R-2)^2$

where R = UL/nu is the Reynolds number; nu approx equals 1.054 x $10^{-6}m^2s^{-1}$ is the kinematic viscosity.

We have used the full length of the waterline for L in the definition of the Reynolds number; however there are other possibilities. Gerritsma et al. (1981) use 0.7L in their study of the resistance of a systematic yacht hull series, reasoning that this defines a kind of average length.

2.4 Form Effects

As a correlation line, the ITTC 1957 line already contains some allowance for three-dimensional effects, and two recent ITTC Committees have recommended that additional corrections not be made in routine resistance predictions of high speed craft (Insel and Molland 1991, p. 16). However, including a form factor specific to the hullform under consideration can often give better estimates of the viscous drag. This factor is difficult to estimate and may vary with speed because of (among other things) changes in trim and sinkage.

In their examination of eight-oared rowing shells, which have a hullform not unlike the canoe body examined here, Scragg and Nelson (1991) found a simple empirical formula for the form factor of these hulls. The viscous resistance coefficient is written as

$C_v = (1+k)C_f$

where

$k = 0.0097(\text{theta}_{entry} + \text{theta}_{exit})$

Here, theta_{entry} and theta_{exit} are the half-angles (in degrees) of the bow and stern, respectively, at the waterplane.

Care must be taken in applying this form factor. In an optimisation problem where shape is not constrained, there could be many undesirable side-effects. For example, since there is a tendency of the optimisation to reduce the above half-angles, bows could tend to be overly cuspy. In addition, since more of the displacement can be placed deeper without penalty, there could be a tendency away from wall-sided hulls and towards some tumblehome below the waterline. This is not an issue here, since our parabolic waterlines and elliptic sections do not allow so much freedom. We shall also find that use of this form factor leads to an improved optimisation process in the high-speed range.

2.5 Some Effects Neglected

Wave-breaking and spray resistance is neglected. Wave-breaking resistance for our fine, sharp-bowed hulls, would be negligible at relatively low speeds. Spray resistance seems to be one of the reasons form factors are difficult to calculate at high speeds.

We assume that there is no effect of dynamic vertical forces, which at low speeds account for sinkage and trim. These are small effects, but notably for multihulls can be substantially different than for monohulls. At high speeds, dynamic forces are upward and yield a lift rather than a sinkage; hence planing, and we neglect that. The present results are for displacement rather than planing conditions, although for completeness we exhibit them even in speed ranges where planing would be expected.

Asymmetric flow around each demihull of a catamaran has been observed. This manifests itself in differences in draft and wetted surface area between both sides of the demihulls. Asymmetrical flow can cause lift and inevitably, induced drag; see Insel and Molland (1991).

Viscous interference between the demihulls of catamarans also seems to be an as yet incompletely understood effect, which can complicate the estimation and application of simple form factors. Insel and Molland (1991) state that "catamarans show substantially higher resistance than twice that of the monohulls, even at ... low speeds where wave interactions are negligible, therefore indicating viscous interactions. Additionally, flow visualisation experiments ... on a catamaran model indicated a change of flow lines and pressure field, hence some form of viscous interaction."

Shallow water effects can be important in some applications, e.g. see Millward (1992) for catamarans, and Scragg and Nelson (1993) for rowing shells. However, we retain the infinite-depth assumption here. We also neglect any lateral flow domain restrictions; see Doctors and Day (1995) and Day and Doctors (1996) for the case of a ship moving in a channel.

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3. PREDICTION OF OPTIMAL PARAMETERS

Once we have a theory that gives reasonable predictions of the total resistance, it seems natural to search for "sensible" parameter configurations minimising that resistance. Many engineering design problems can be cast into the form of optimisation problems. For example the problem addressed in this paper can be formulated as
Minimise the real-valued function $f(x_1,x_2,...,x_n)$, with each real parameter x_i subject to (domain) constraints $a_i<=x_i<=b_i$ for some real constants a_i and b_i

Many techniques exist for solving optimisation problems such as the one described above, however, they vary greatly in efficiency and the quality of the final solution for a given number of function evaluations. No single technique is best for all design problems. Gradient-based methods work well with smooth, unimodal functions, but may yield local optima for multimodal functions. Heuristic algorithms can increase search efficiency, but at the expense of guaranteed optimality - they do not always find the global optimum.

3.1 Genetic Algorithms (GAs)

GAs are adaptive search methods that use heuristics inspired by natural population dynamics and the evolution of life. They differ from other search and optimisation schemes in four main respects (Dhingra and Lee 1994):

- Search proceeds from a population of points, not from a single point.
- They use a coding of the parameters, not the parameters themselves.
- Objective function values guide the search process. They do not use gradients or other problem-specific information.
- State transition rules are probabilistic, not deterministic.

In the present study, we use a non-traditional GA similar to Eshelman's (1991) *CHC*, augmented with, among other features, hill-climbing routines, cataclysmic restarts and incest prevention. The resulting computer program, called "GODZILLA" for *Genetic Optimisation and Design of Zoomorphs*, is described in Lazauskas (1996 in prep.).

3.2 GODZILLA

GODZILLA's general operation can be described quite succintly: create and evaluate new (candidate) designs until some termination criterion is met. Termination can occur when a certain number of designs have been evaluated, or after a prescribed amount of time has elapsed, or when the algorithm seems to be making no further progress.

GODZILLA begins the optimisation process by creating an initial population of (real-valued) design vectors and calculating the total resistance for each design. Initial designs are randomly generated, although the population can also be "seeded" with previously found good designs.

Genetic operators and hill-climbing operators are used to create candidate designs. Genetic operators create new (offspring) vectors from two parent vectors in the population, using heuristics inspired by the recombination of DNA. There are too many varieties to here discuss individual strengths, deficiencies and peculiarities. GODZILLA's primary genetic operator is one gleaned from fuzzy set theory described in Voigt et al (1995). After evaluating the total resistance of the offspring, GODZILLA replaces the worst individual in the population with the offspring if the offspring's total resistance is lower. This replacement strategy guarantees that the best individual in the population is never replaced by an inferior individual.

The method used to select parent vectors from the population can have a substantial influence on the performance of GAs. GODZILLA uses *binary tournament selection*. In this method, two individuals are selected without replacement from the population. The individual with the lower total resistance becomes the first parent. A second binary tournament determines the other parent.

One form of hill-climbing operator used by GODZILLA, *Stochastic Bit-climbing*, creates a candidate vector by adding or subtracting small increments from each of the parameters of the best design vector found so far. This allows the program to explore more closely promising regions of the search space found by the genetic operators. GODZILLA also incorporates another hill-climbing technique called the *Simplex Search Method*. This method, which is not to be confused with the *Simplex Method* of linear programming, is described in Reklaitis et al (1983).

The field of evolutionary computation is expanding very quickly, and almost all communication occurs via the electronic Internet. The USENET group, *comp.ai.genetic*, is a very useful and important resource.

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4. RESULTS

4.1 Method of presentation

Since there is no length restriction, but the displacement D is fixed, the appropriate length parameter for scaling is the cube root $L^*=D^{1/3}$ of the displacement. Results are presented in a non-dimensional manner as a function of the (volumetric) Froude number F_{nv}=U/sqrt(gL^*) based on that artificial length. In fact, were it not for scale (Reynolds number) effects, all results would be universal functions of this Froude number, and displacement would be irrelevant. For example, the final minimum total drag $R_t=R_v+R_w$ is expressed in terms of the coefficient

$C_t=R_t/(1/2*\mathrm{rho}*U^2L^{*2})$

which would be a function of F_{nv} alone were it not for the fact that the skin friction coefficient depends on Reynolds number.

In order to exhibit this scale effect of displacement, we carry out the optimisation at three fixed (dimensional) displacements of one, one hundred, and ten thousand tonnes. This large range of displacements means that in some cases the speeds are not realistic, but results are nevertheless provided for completeness in such cases. In fact we have also computed results for even larger vessels, up to one million tonnes.

For definiteness, we give most results for the fixed displacement of one tonne. Some such results have already been given in Figures 1.1 and 1.2. It is notable that for this displacement, L^*=1 metre, so that the non-dimensional length can also be interpreted as the actual length in metres. The volumetric Froude number is also uniquely proportional to the actual speed in metres/second or knots, and F_{nv}=1 occurs at 6.1 knots for a one-tonne vessel.

It is important to bear in mind that none of the Figures 2-4 to follow, where the total drag coefficient C_t is plotted against the volumetric Froude number F_{nv}, can be interpreted in the usual naval architectural manner as a graph of drag versus speed for a *given* ship. As F_{nv} varies, the ship itself changes its shape, and in particular its length, so as to keep the drag as small as possible.

4.2 Monohull without form drag

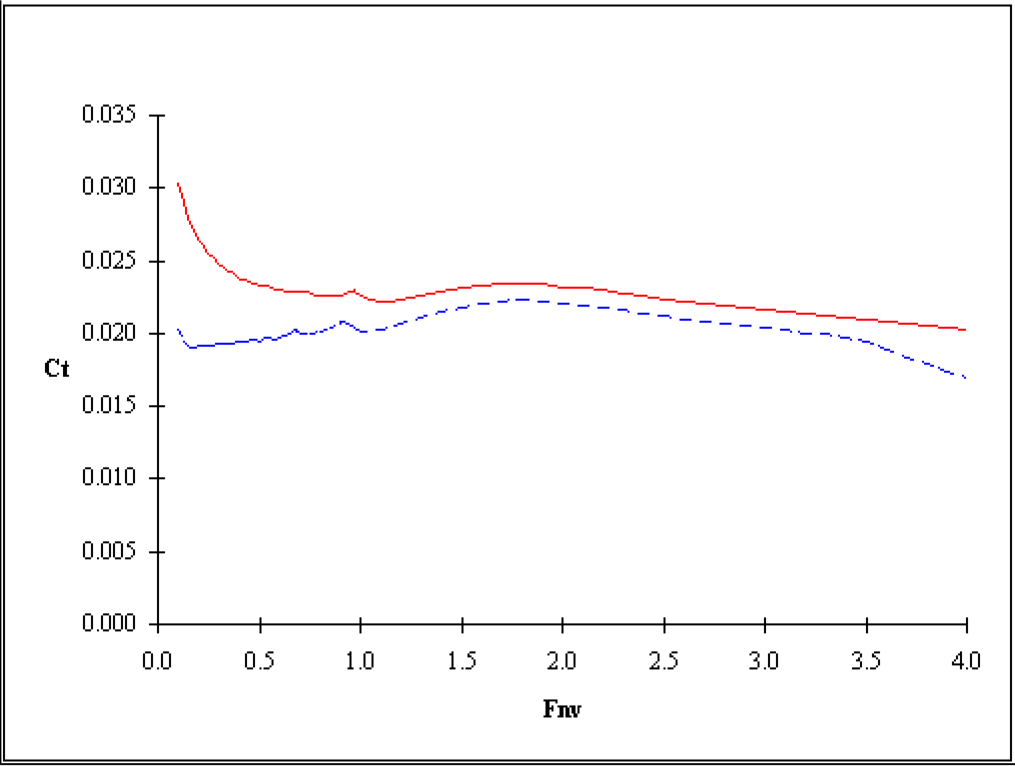

Fig. 2(a): The effect of form factor on the optimum total resistance of a one tonne monohull.

The dashed curve of Fig. 2(a) shows C_t as a function of volumetric Froude number F_{nv}, for a one-tonne "ship". This is the residual value of the total drag, after the ship's dimensions have been optimised to minimise C_t without any allowance being made for form drag. The hull parameters that produce these optimal C_t's are shown as the dashed curves in Figures 2(c)-2(e).

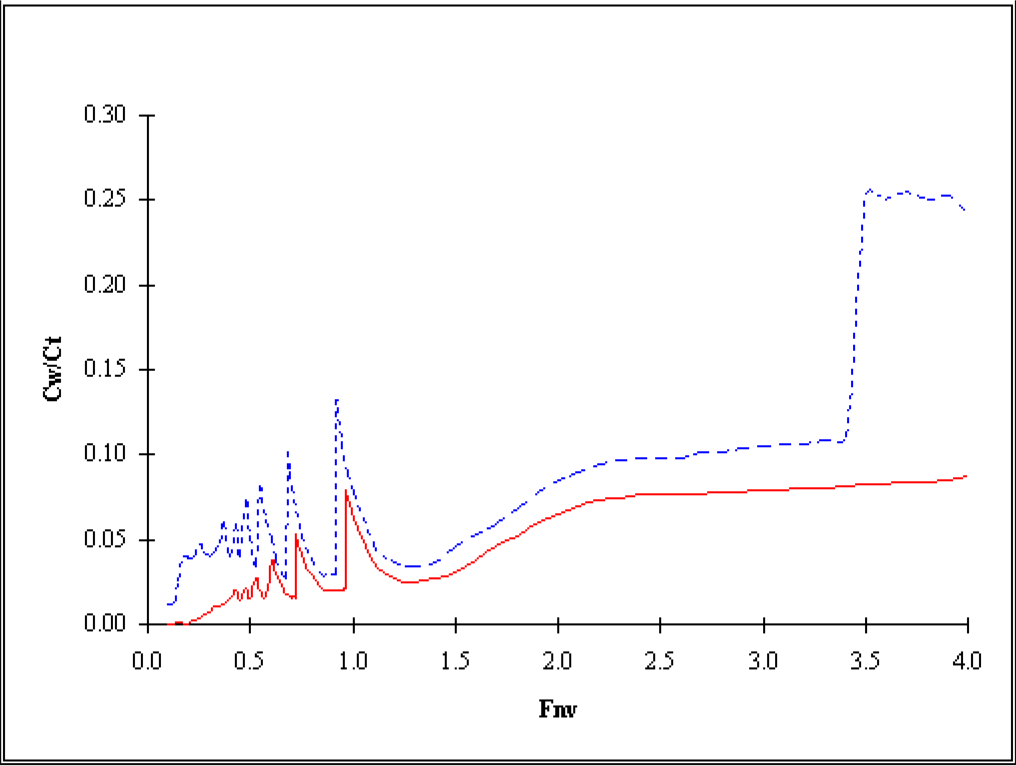

Fig. 2(b): The effect of form factor on the optimal proportion of wave resistance of a one tonne monohull.

Fig. 2(b) shows wave resistance as a fraction of the total drag. There is considerable scatter at low F_{nv}. This could be due to long shallow regions in the "fitness" landscape, where for example, one length is as good as another. Although C_t remains the same, C_w/C_t may vary. GODZILLA searches for the lowest total resistance and if it encounters two or more combinations of parameters with almost the same C_t, it cannot prefer one to the other. In these regions, it could be important to perform the integrations more accurately. In any case, wave resistance is less than 12% of the total for all F_{nv}<3.25.

The most obvious feature of the dashed curve in Figure 2(b), however, is the sudden increase in the proportion of wave resistance for F_{nv}>3.25, a rather high speed (of the order of 20 knots for a 1-tonne vessel) near the upper end of the range being considered in this study. Figure 2(c) shows that the optimum (non-dimensional) length also drops sharply to a very low level at this speed. This discontinuity is essentially an interchange in the roles of two local minima, as in Figure 1.1. For F_{nv}<3.25, the longer ship is best; for F_{nv}>3.25 the shorter ship is best, and in the present case, the shorter ship is so short as to be quite unrealistic. Indeed, this "ship" almost eliminates its wave resistance by going to a very high rather than a very low conventional Froude number. Minimum viscous drag dictates minimum surface area, and that inevitably pushes the optimum toward a hemispherical geometry. In the present case, other constraints prevent this hemisphere being achieved exactly, but this class of "optimum" ship does tend to have length comparable to the beam and draft. Clearly this is not a realistic conclusion, and in particular would lead us to question the validity of neglecting form drag.

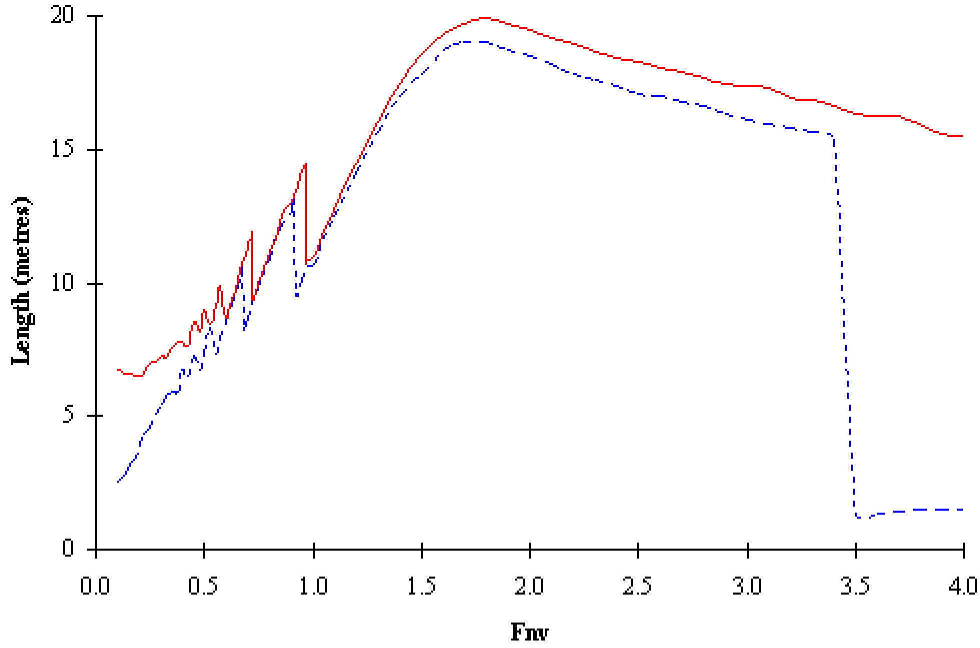

Fig. 2(c): The effect of form factor on the optimal length of a one tonne monohull.

Returning to the "realistic" ships produced for lower speeds, with F_{nv}<3.25, as the "design speed" increases from zero in that range, the optimum length L/L^* (same as length in metres for a one-tonne ship) shown in Figure 2(c) increases to a maximum of about 22 at a F_{nv} value of about 1.8 before decreasing slowly as the speed increases further. This volumetric Froude number of 1.8 corresponds to a conventional (actual length-based) Froude number of 0.38, or a speed of 11 knots for a one-tonne vessel. At speeds below this value, the usual very dramatic large rise in wave resistance occurs as the length-based Froude number increases. Not surprisingly, longer ships are then preferred as the speed rises.

This trend cannot continue for ever. Eventually, the optimal shiplength reaches a maximum, and further increases in speed can no longer be met by increasing length to keep operating well below the wave resistance main peak. Instead, the length-based Froude number passes (quite rapidly) through the value where wave resistance is maximal, but the proportion of wave resistance is nevertheless kept sufficiently low to achieve an optimal design because of the large shiplength. Eventually as the speed increases further, the optimal shiplength starts to decrease again, since we are now operating at a length-based Froude number above the main wave resistance peak. Then the wave resistance decreases with Froude number, and hence shorter ships have less rather than more wave resistance at any given speed, and are preferred in the optimisation.

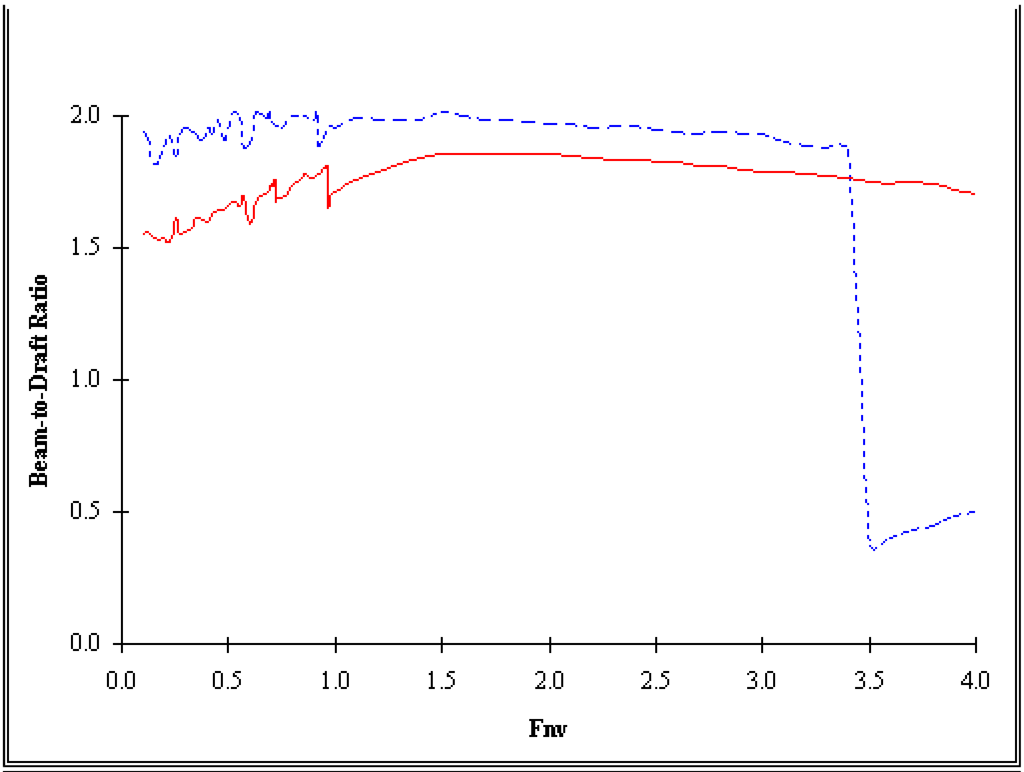

Fig. 2(d): The effect of form factor on the optimal beam-to-draft ratio of a one tonne monohull.

When the length is so great, the surface area strongly controls the optimisation, and to minimise the increase in frictional resistance, semi-circular sections tend to be preferred. This is clear in Figure 2 (d), where it can be seen that the beam-to-draft ratio B/T stays at a value of roughly 2 for F_{nv} between 1.0 and 2.5.

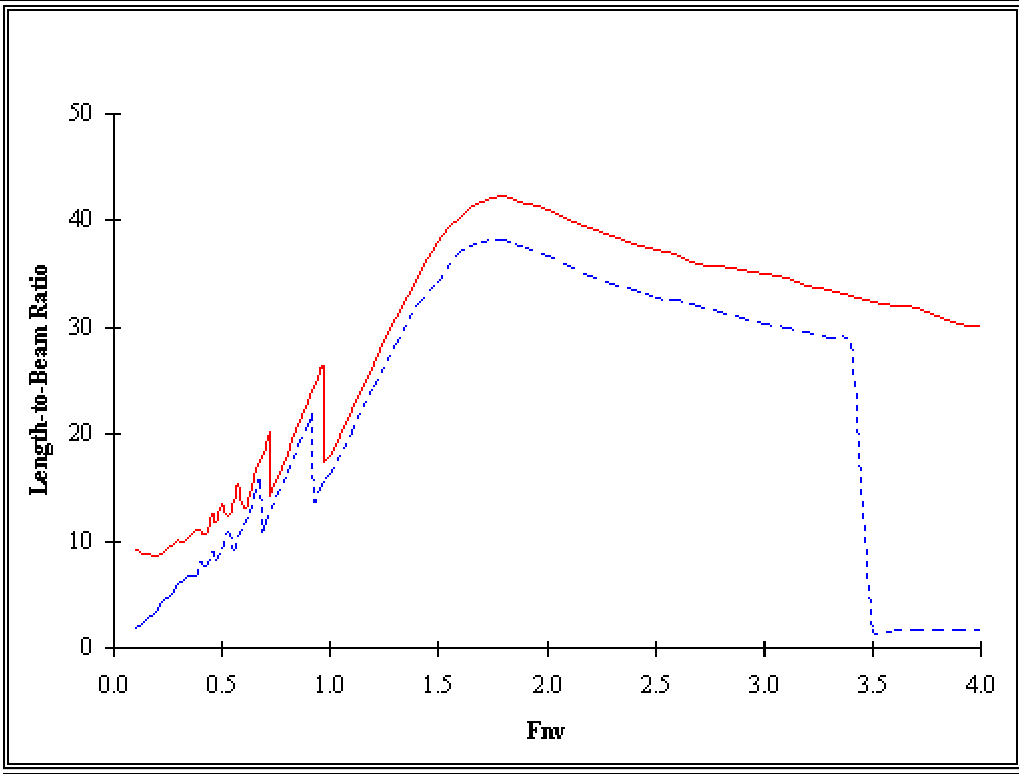

Fig. 2(e): The effect of form factor on the optimal length-to-beam ratio of a one tonne monohull.

The optimum ships are very slender. Figure 2(e) shows the length/beam ratio, which is very high indeed (reaching a maximum of about 42 at F_{nv}=1.8) by conventional ship standards, though not entirely unreasonable for rowing shells.

4.3 Monohull with form drag

Figures 2(a) - 2(e) also show (solid curves) the same monohull calculations as in the previous section for a one-tonne ship, but here the total resistance now includes Scragg and Nelson's (1993) form factor.

Figure 2(a) indicates that there is only a quite small increase in the residual total drag C_t for all speeds, consistent with the fact that the form drag is small, especially for the present very fine hulls. The greatest impact of form effects on the optimisation process occurs at very low and very high F_{nv}. The solid-line C_t curves of Figure 2(a) are smoother at low F_{nv} than the dashed curves, and the ultimate decrease in C_t at high F_{nv} is no longer as rapid.

Figure 2(b) shows that with form drag included, the proportion of wave resistance now remains below 10% for all speeds and all displacements. The scatter at low F_{nv} is not so pronounced as in the optimisations without form effects. Most important of all, however, is that there is no longer a sudden discontinuous increase in the proportion of wave resistance for F_{nv}>3.25. We have already anticipated this, since the very short ships that were suggested at high speeds by the optimisation without form drag are now heavily penalised by their large entrance and exit angles, and fail in total drag competition with a local minimum corresponding to a longer ship.

Figure 2(c) confirms this point, indicating that the optimum ship stays "long" for all speeds, with no discontinuity at any high-end speed. Indeed, with the inclusion of form effects, there is a tendency towards slightly longer optimum ships. The beam-to-draft ratios shown in Figure 2(d) are generally about 10% smaller with form drag included. For our canoe body, small entrance and exit angles can only be achieved by reducing the beam, so there is a slight tendency toward non-circular cross-sections, with B/T<2.

At the intermediate speeds which are of the greatest practical interest, there is only a small effect of the form factor on all outputs, and the qualitative discussion in the previous section about transition through the speeds where the wave resistance and hence the optimum length is maximal applies equally with or without form factor. Nevertheless, because as we have seen, inclusion of a form factor makes for a smoother and more realistic optimisation process at all speeds, such a factor is included in all of the remaining computations presented here.

4.4 Variation in Displacement

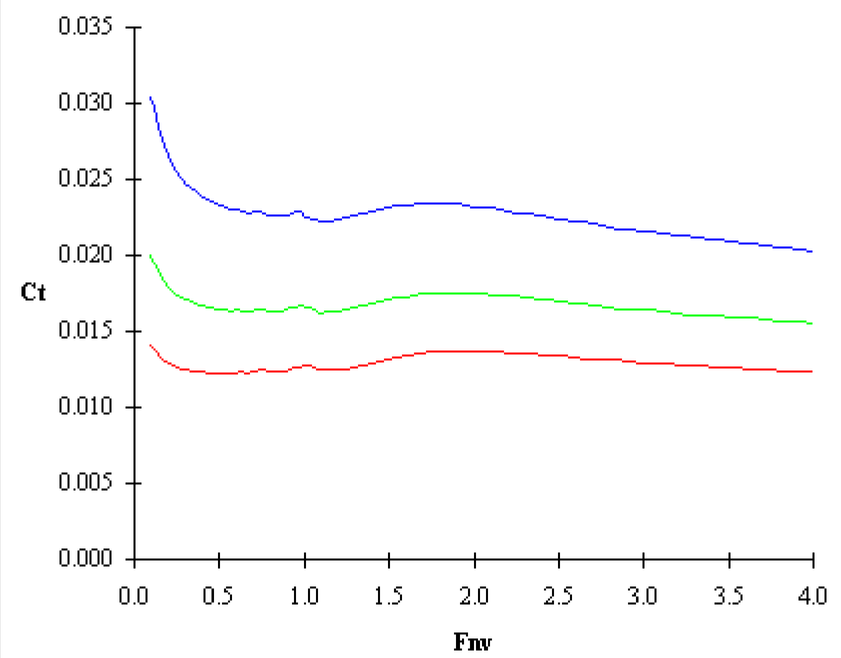

Fig. 3(a): The effect of displacement on the optimal total resistance of a monohull.

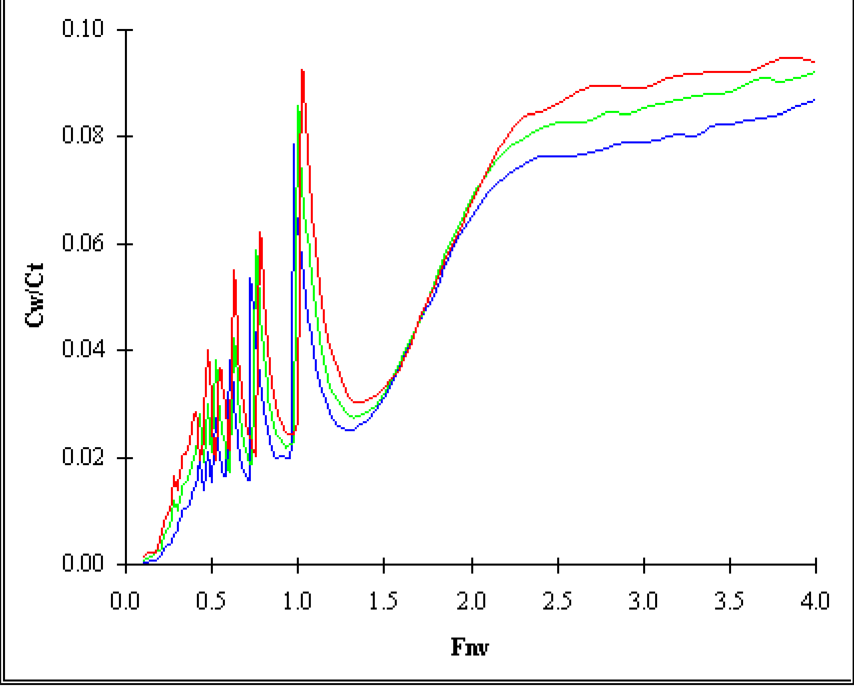

Fig. 3(b): The effect of displacement on the optimal proportion of wave resistance of a monohull.

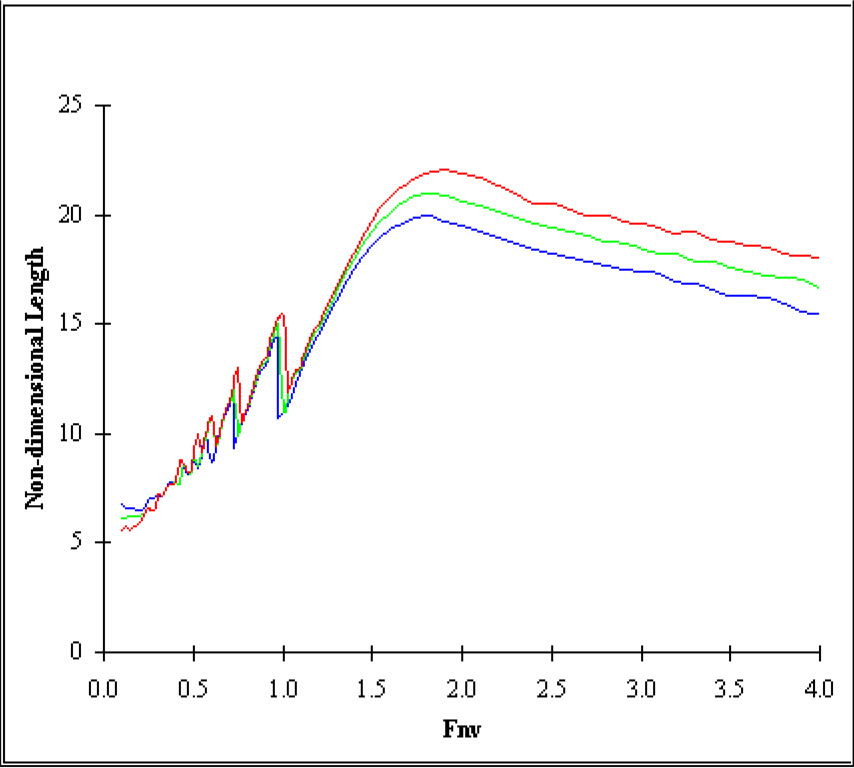

Fig. 3(c): The effect of displacement on the optimal length of a monohull.

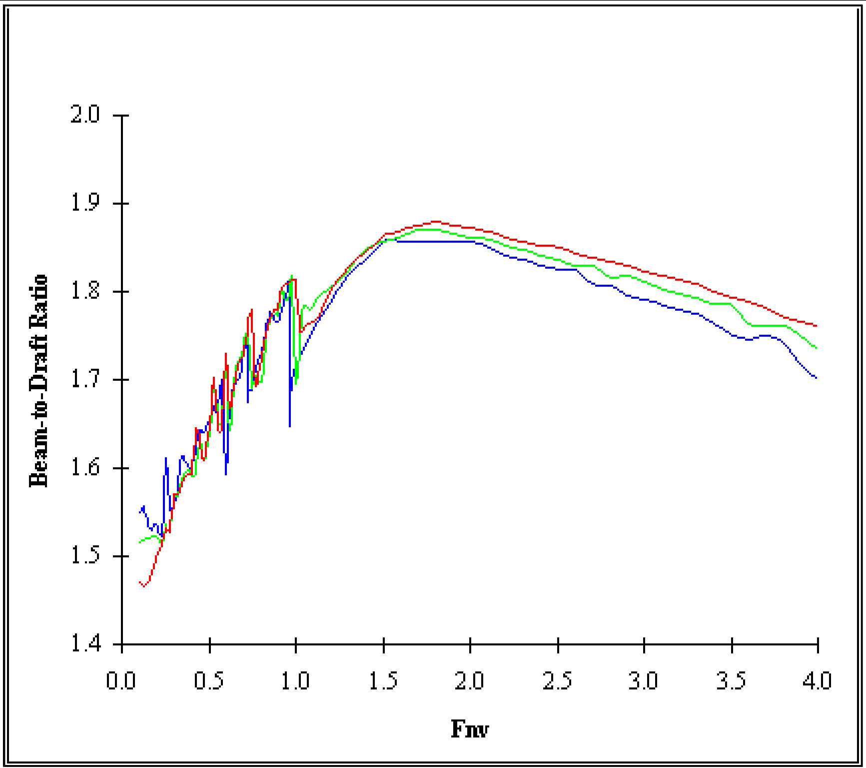

Fig. 3(d): The effect of displacement on the optimal beam-to-draft ratio of a monohull.

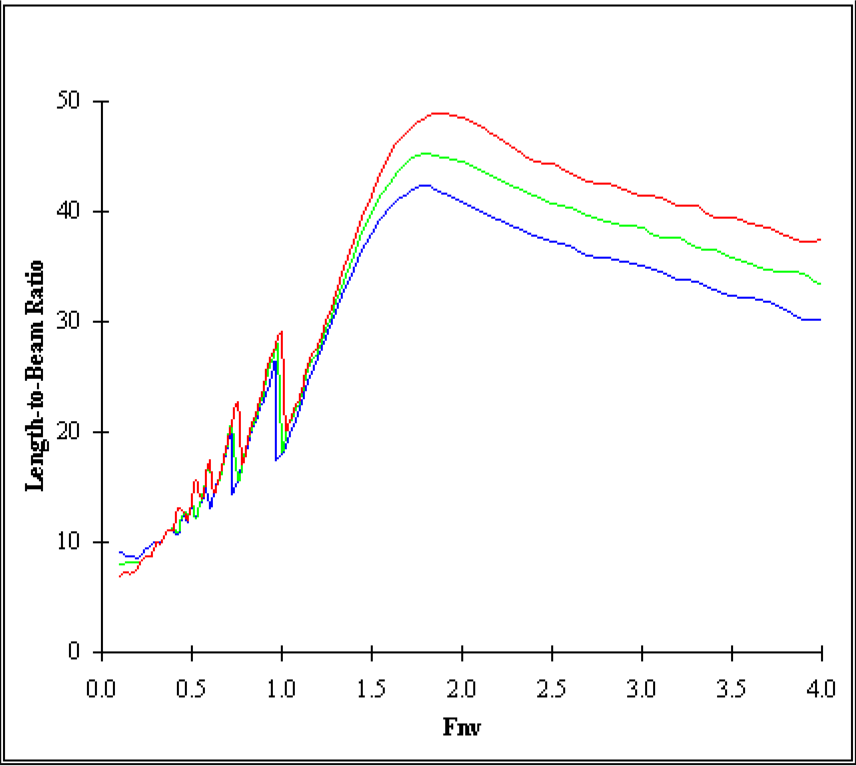

Fig. 3(e): The effect of displacement on the optimal length-to-beam ratio of a monohull.

In Figures 3(a)-(e) and 4(a)-(c), the blue, green and red curves correspond to displacements of 1, 100, and 10,000 tonnes respectively.

Figures 3(a)-(e) indicate variation with displacement of the same quantities that were discussed earlier for the one-tonne ship. Note that the proportionality constant relating actual speed to volumetric Froude number varies as the one-sixth power of displacement. Specifically, the actual speed at F_{nv}=1 is 6.1 knots for a 1-tonne vessel, 13.1 knots for a 100-tonne vessel, and 28.3 knots for a 10,000-tonne vessel.

Smaller ships have larger C_t because they are shorter, their Reynolds numbers are smaller, and consequently the skin friction coefficient is larger. Of course the actual total drag R_t is much larger for larger ships, once we multiply C_t by 1/2*rho*U*D$^{2/3}$.

In the most important middle range of speeds, the dependence of the results on displacement is quite smooth and predictable by interpolation within the curves presented here.

4.5 Catamarans

We now give results for catamarans where the two demihulls are identical and their centreplanes are spaced W apart. GODZILLA attempts to choose W (as well as L and T) optimally, noting that W can only affect the wave resistance part of the total drag. The effect on the optimisation process of including form drag is similar for catamarans and monohulls. Hence results are presented here for catamarans only with form drag included.

From a survey of modern high speed catamaran dimensions, Insel and Molland (1991) concluded that the general range of parameters was: L/B=6 to 12, L/L*=6 to 9, B/T=1.0 to 3.0 and C_b=0.33 to

0.45. Our optimum hulls are very much longer; however C_b and B/T are within the above range.

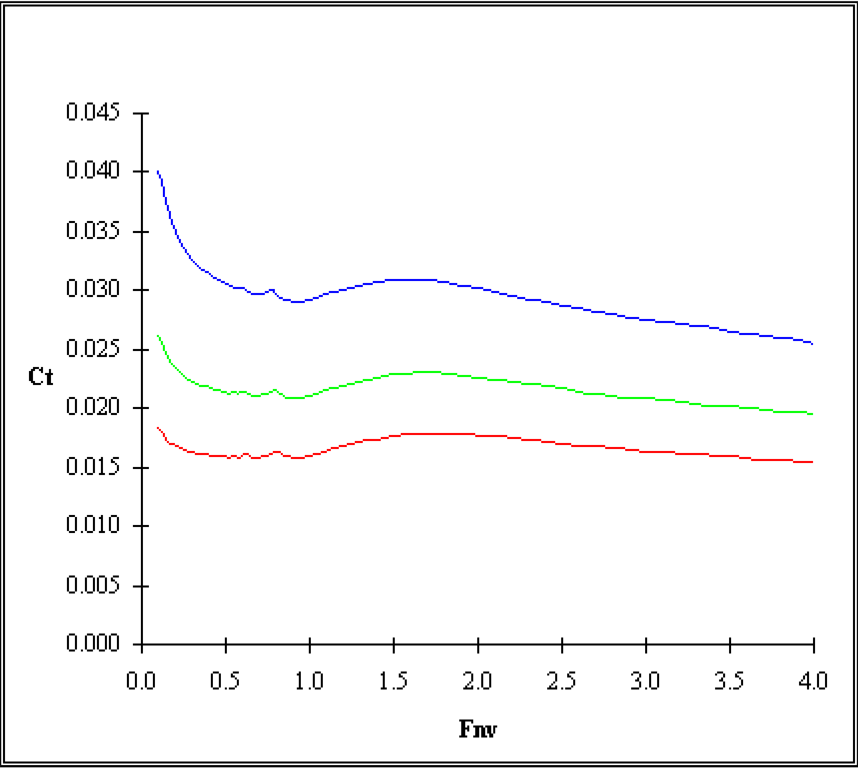

Fig. 4(a): The effect of displacement on the optimal total resistance of a catamaran.

The C_t curves in Figure 4(a) are similar in general character to those for the monohull in Figure 3(a). It is obvious that there is no speed or displacement at which a catamaran has lower total resistance than an optimum monohull of the same displacement. The proportion of wave resistance for optimum catamarans is generally similar to that for optimum monohulls as given in Figure 3(b), and is always less than 10%.

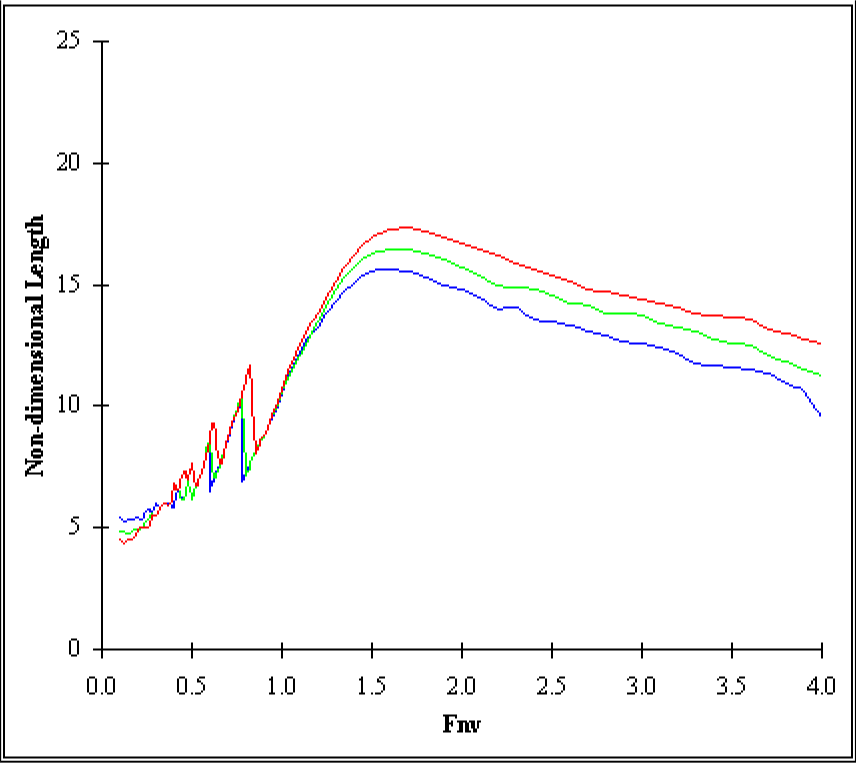

Fig. 4(b): The effect of displacement on the optimal length of a catamaran.

A comparison of the optimum length results in Figures 4(b) and 3(c) shows that at low speeds, catamarans have an optimal length roughly the same as the equivalent monohulls. At higher speeds around F_{nv}=1.6 (namely 10 knots for a one-tonne vessel), where the optimal length of a catamaran reaches its maximum a little earlier than a monohull, optimal catamarans tend to be roughly 25% shorter than the optimal monohull of the same displacement. Length-to-beam ratios for the demihulls of optimal catamarans are similar to that for monohulls (Figure 2(e)) at all speeds. For example, each demihull beam is also about 25% less than that of the full equivalent monohull at about F_{nv}=1.6 when L/B takes its maximum value of about 41. That is, each demihull of an optimum catamaran is approximately as slender as the optimum monohull, and is much more slender than conventional catamaran hulls. Beam-to-draft ratios are also similar to those of monohulls, and nearly semi-circular sections are preferred.

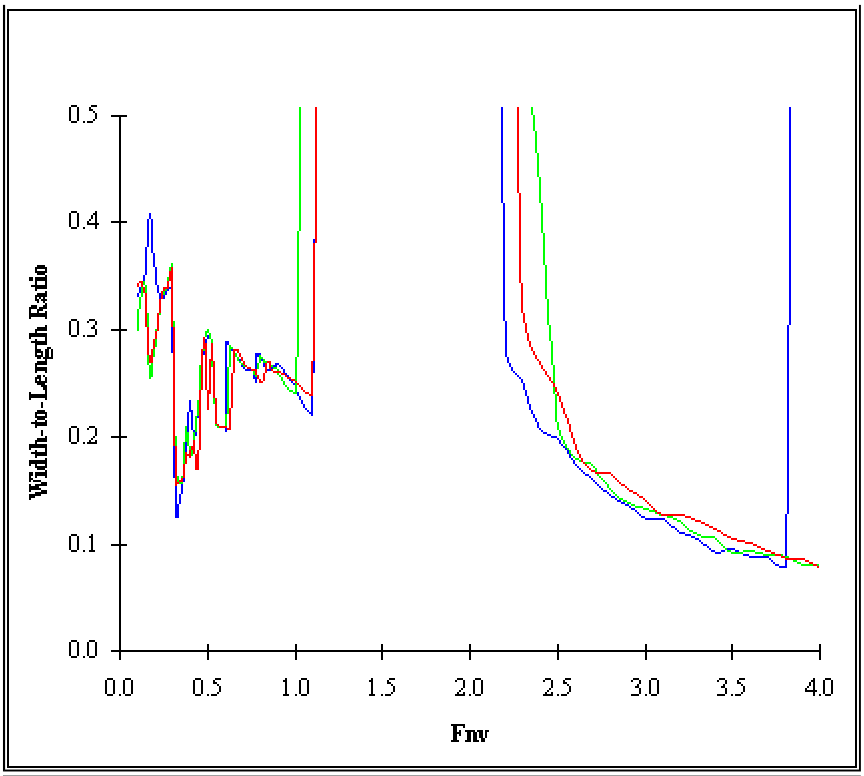

Fig. 4(c): The effect of displacement on the optimal width-to-length ratio of a catamaran.

Figure 4(c) shows the optimum hull separation W/L. It can be seen that for F_{nv} between 0.2 and 1.1 the optimum separation is roughly 20%-30% of the length of the catamaran. For F_{nv} between 1.1 and 2.2 there is no optimum finite spacing. To all intents and purposes, if one only wishes to minimise total drag, there is no reason why the two hulls need to be close to each other, because they cannot favourably interfere with each other to reduce wave resistance in that speed range. Indeed, they must interfere with each other unfavourably, and our conclusion is that the further they are apart the better from the drag point of view. This speed range is one of considerable practical interest, and it is one in which the decision on choice of hull separation distance must be made on grounds other than drag minimisation. The existence of a speed range where there is no best separation distance is in rough accord with the results of Turner and Taplin (1968), wherein it was pointed out that this conclusion tends toward catamarans that are impractically wide for all but sailing boats. At still higher speeds, there again seems to be a band of F_{nv} (say between 2.3 and 3.4) where there is an optimum finite hull separation, again of about 20% of the length. For F_{nv}>3.4, again there is no best separation distance.

Insel and Molland (1991) comment on some aspects of this phenomenon, stating that "The wave interference can effectively be neglected above a particular speed which is both separation and L/B dependent. This is an interesting and important result since it suggests that, for higher speed designs, the choice of hull spacing may be based on other requirements such as seakeeping performance without incurring significant penalties in calm water resistance." However, they do not seem to have observed the second range of speeds where destructive interference again becomes useful.

4.6 Trimarans

The trimarans of interest here consist of a main hull together with two side hulls placed parallel to each other, with their centres at a distance a aft of the centre of the main hull and at distances b abeam of it. The side hulls can have different displacements, lengths and drafts from the main hull, but otherwise have the same shape as the main hull. We carry out a six-parameter optimisation, for each separate value of the ratio sigma between the (sum of two) side-hull displacement and the total (main plus two side hulls) displacement.

In order to reduce the amount of output data that has to be presented for trimarans, we select a relatively small set of speeds (displacement-based Froude numbers of 1.3,1.7 and 2.1), and plot results at each fixed speed versus the above-defined displacement ratio sigma. The speeds chosen are in the "interesting" range, namely speeds above those where discontinuities occur, but below those where planing and other presently-neglected flow phenomena might be important. This range (say 8-10-13 knots for a 1-tonne vessel) is also the competition range for some sporting applications.

Plotting trimaran results versus the displacement ratio sigma has the feature that the monohull results are reproduced when sigma=0 and the catamaran results when sigma=1, which is a useful check. The trimaran thus interpolates between monohulls and catamarans, for 0<sigma<1.

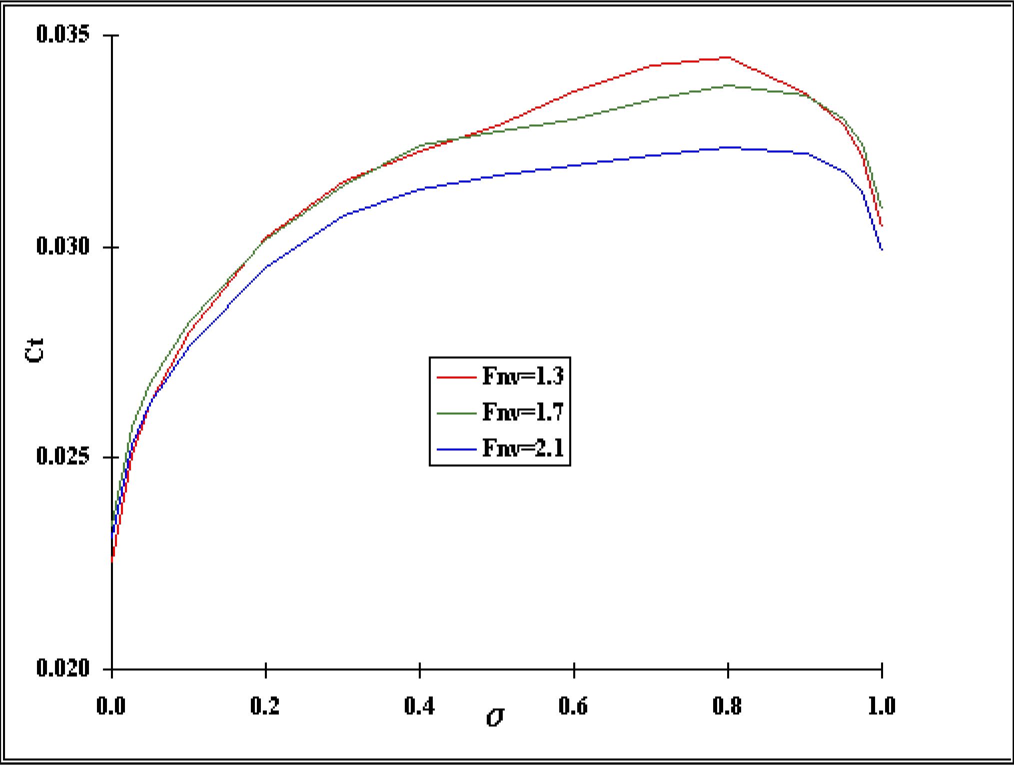

Fig. 6(a): Optimum total resistance of one tonne generalised trimarans.

Figure 6(a) gives the residual total drag as a function of sigma and shows that trimarans are never competitive with the best monohull. As sigma increases from the monohull value of 0, the drag rises to a maximum at about sigma=0.8 before falling again toward the catamaran limit at sigma=1, which (as already discussed) is inferior to the monohull, and also to any trimaran with sigma less than about 0.2.

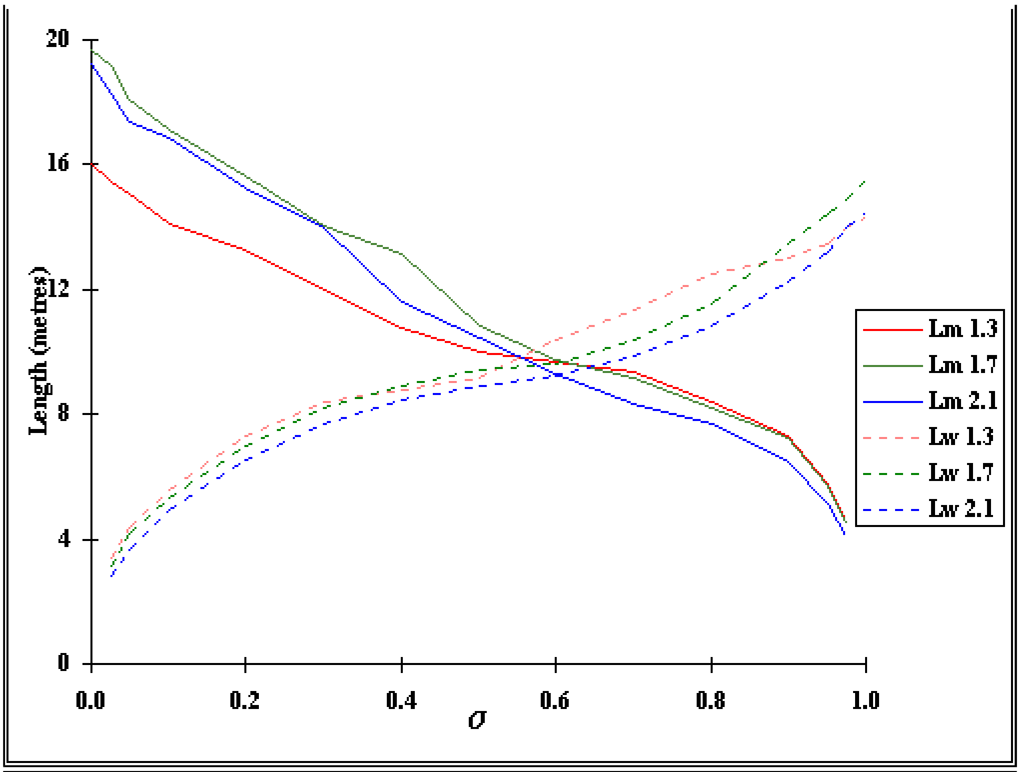

Fig. 6(b): Optimum lengths of one tonne generalised trimarans.

Figure 6(b) gives the optimum lengths of the main and wing hulls. The former decreases steadily and the latter increases steadily with increasing sigma. The three hulls are all of the same length at about sigma=0.6.

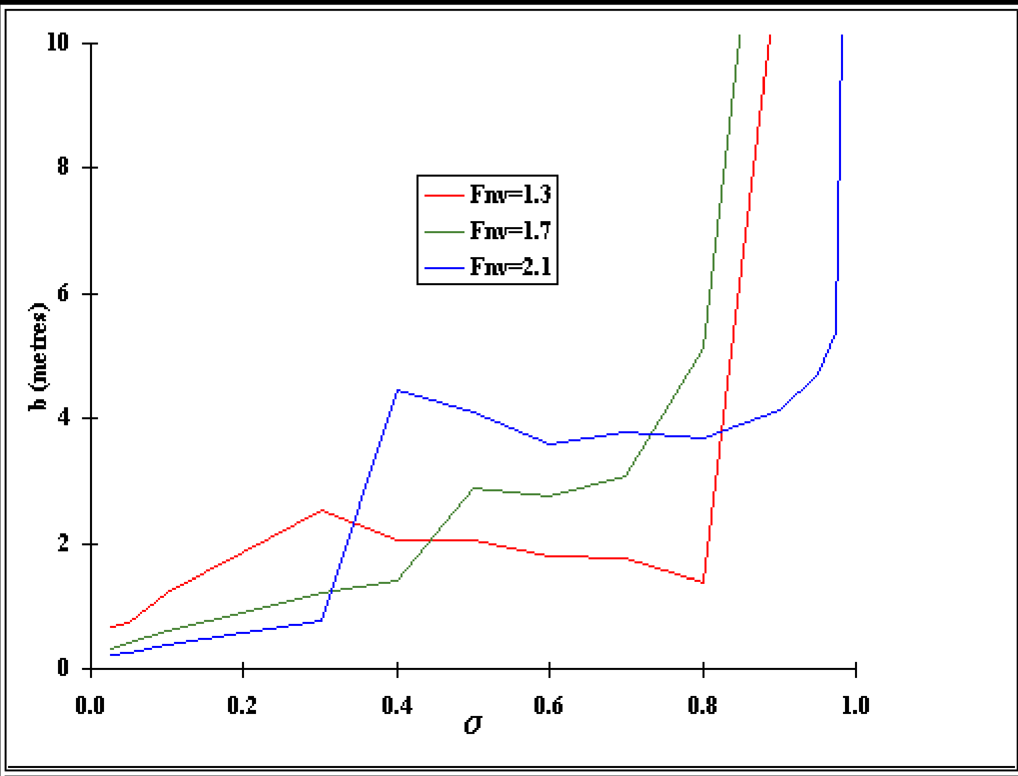

Fig. 6(c): Optimum lateral separation of one tonne generalised trimarans.

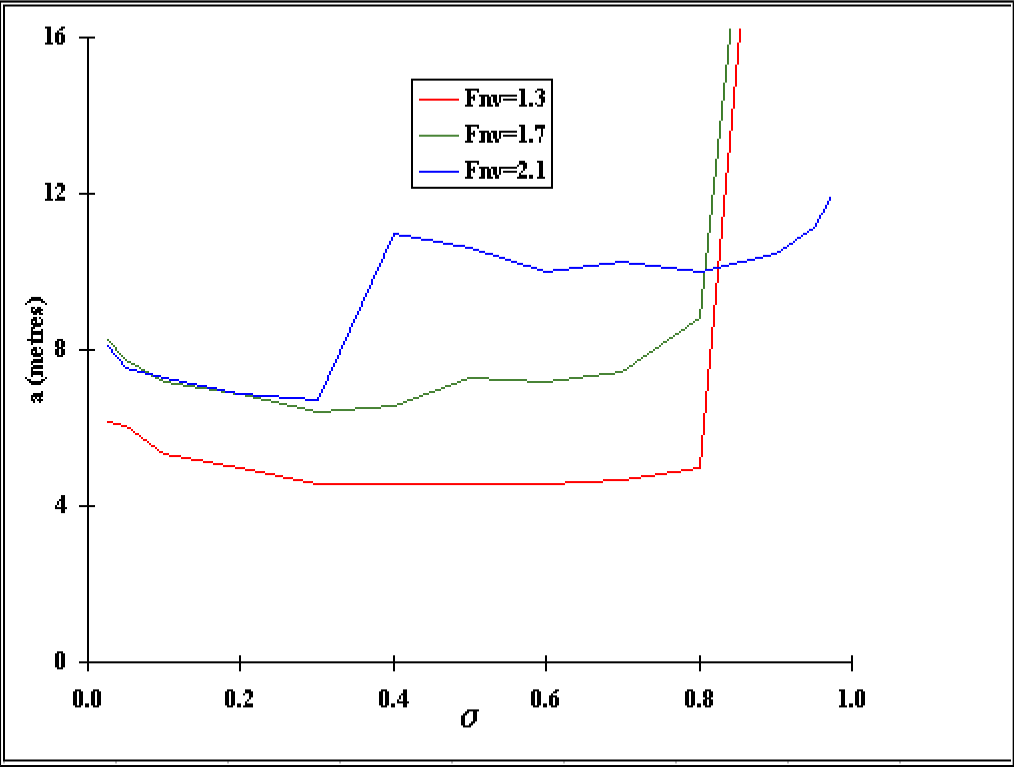

Fig. 6(d): Optimum longitudinal separation of one tonne generalised trimarans.

Figures 6(c) and 6(d) give the optimum lateral and longitudinal offsets of the wing hulls. The lateral offset b would approach W/2 in the catamaran limit sigma=1, where W is as before the lateral separation of the catamaran demihulls. However, in the speed range being examined, the optimum value of W for a catamaran is actually infinite. Thus as sigma increases from the monohull value of zero, the lateral offset remains relatively small at less than 4 metres until sigma exceeds about 0.8, then rises rapidly as the trimaran turns into a catamaran.

The optimum longitudinal offset a given in Figure 6(d) displays a somewhat complicated variation with sigma. The limiting value at the monohull end sigma=0 seems to be about one-half of the main hull length, and since the wing hull lengths are tending to zero in this limit with zero lateral separation, the wing hulls simply "tuck in" at the stern of the main hull. At the other extreme, in the catamaran limit sigma=1, there is again a tendency for a to increase rapidly but of course the "main" hull then becomes a hydrodynamically insignificant "dagger board" far ahead of the dominating wing hulls.

The isosceles triangle formed by the centres of the three hulls of the trimaran has a half-angle that is quite small for near-monohull cases with sigma<0.3, but tends to range between 10 and 15 degrees for larger sigma. This is consistent with estimates of the optimum half angle for minimum wave resistance (references?), noting that it implies that the wing hulls lie just inside the Kelvin angle of the wave pattern of the main hull.

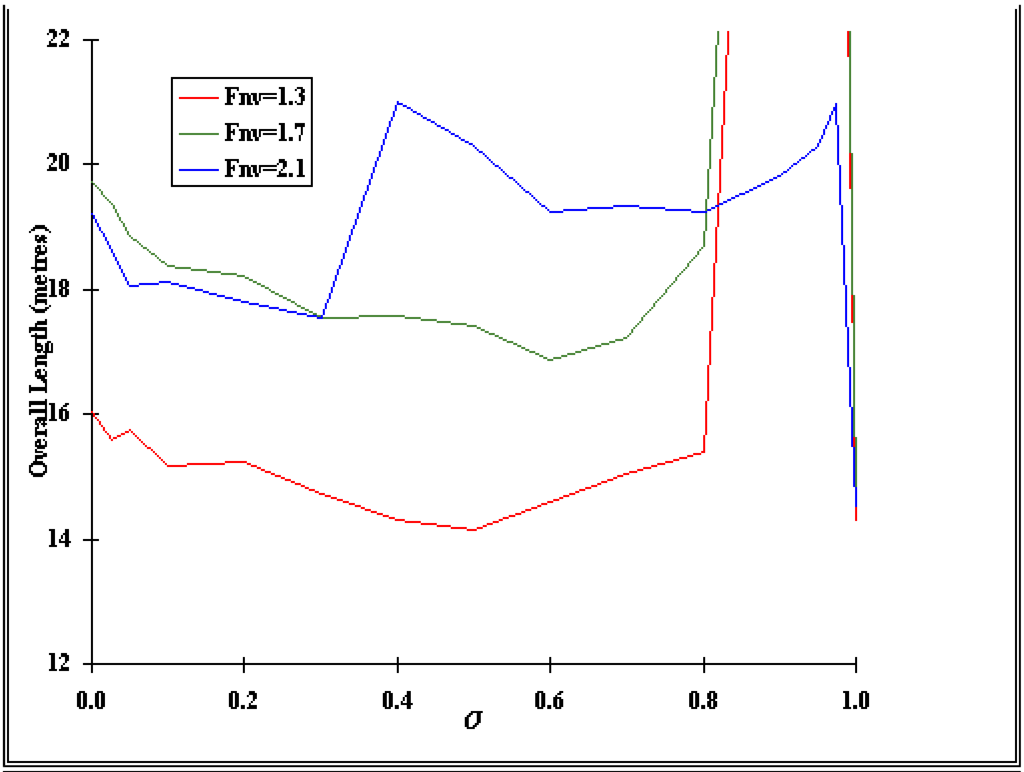

Fig. 6(e): Optimum overall length of one tonne generalised trimarans.

Figure 6(e) gives the overall length of the trimaran. This is the sum of the main and wing hull half-lengths, plus the longitudinal separation a. For near-monohulls, because the wing hulls are short and the longitudinal separation a is less than half the main hull length, the overall length is close to that of the monohull, decreasing slightly as sigma increases from zero. For the lower speeds, the overall length reaches a minimum of about 10% less than the monohull length at about sigma=0.5-0.6; there is a more complicated variation at the higher speed. The overall length becomes large as the trimaran approaches a catamaran, but loses meaning as the "main" hull becomes of vanishing size and significance relative to the wing hulls.

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5. CONCLUSION

We have found optimum ships for minimum total drag over a large range of speeds and displacements. Results were obtained both with and without form drag corrections, and for monohulls, catamarans and trimarans. Although the net contribution of form drag is small, it can nevertheless be important in determining the optimum. The optimum ships tend to be longer and have a lower wave resistance relative to viscous resistance than conventional ships. The genetic algorithm tool GODZILLA has proved useful in searching for the global minimum in the presence of two or more local minima, and will be essential in extended work involving shape variations and other constraints. Optimum (long) monohulls are always better than optimum catamarans or trimarans of the same total displacement, from the point of view of total calm-water drag alone, unless there are restrictions on the ship geometry.

ACKNOWLEDGEMENTS

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REFERENCES

Bramlette, Mark F. and Bouchard, Eugene E., "Genetic Algorithms in Parametric Design of Aircraft", in "Handbook of Genetic Algorithms", Lawrence Davis (ed.), 1991, pp. 109-123.

Chapman R.B. "Hydrodynamic design of semisubmerged ships", J. Basic Eng. ASME, pp 874-884, 1972.

De Jong, K. A., "Genetic Algorithms are NOT Function Optimizers", in "Foundations of Genetic Algorithms:2", San Mateo, CA, Morgan Kauffmann, 1993.

Dhingra, A. K., and Lee, B.H., "A Genetic Algorithm Approach to Single and Multiobjective Structural Optimization with Discrete Continuous Variables", Int. J. Num. Meth. Eng., Vol. 37, pp. 4059-4080, 1994.

Goldberg, David E., "Genetic Algorithms in Search, Optimization and Machine Learning", Addison-Wesley, 412 pages, 1989.

Eshelman, Larry J., "The CHC Adaptive Search Algorithm: How to Have Safe Search When Engaging in Nontraditional Genetic Recombination ", Foundations of Genetic Algorithms, Rawlins, G. J. E. (ed.), San Mateo, CA, Morgan Kaufmann, 1991, pp. 265-283.

Eshelman, Larry J. and Schaffer, J. David, "Preventing Premature Convergence in Genetic Algorithms by Preventing Incest", Proc. Fourth Int. Conf. on Genetic Algorithms and their Applications, Belew, R. and Booker, %L. (eds), Morgan Kaufmann, 1991, pp. 115-122.

Gerritsma, J., Onnink, R. and Versluis, A., "Geometry, Resistance and Stability of the Delft Yacht Hull Series", Int. Shipb. Prog. Vol 28, 1981, pp. 276-297.

Gerritsma, J., Keunig, J. A. and Versluis, A., "Sailing Yacht Performance in Calm Water and in Waves", 11th Chesapeake Sailing Yacht Symposium, 1993, pp. 233-245.

Heitkoetter, Joerg and Beasley, David, eds. "The Hitch-Hiker's Guide to Evolutionary Computation: A List of Frequently Asked Questions (FAQ)", USENET: {\it comp.ai.genetic}. Available via anonymous FTP from *rtfm.mit.edu:/pub/usenet/news.answers/ai-faq/genetic/* About 90 pages.

Holland, J. H., "Adaptation in Natural and Artificial Systems ", Ann Arbor: The University of Michigan Press, 1975.

Holtrop, J. and Mennen, G. G. J., "A Statistical Power Predicition Method", Int. Shipb. Prog., Vol. 25, No. 290, 1978, pp. 253-256.

Hotan, Edgar, "Speedy ships for Asia fast-food routes", The Advertiser, Wed. 27 Dec. 1995, Adelaide, Australia, p. 50.

Insel, M. and Molland, A. F., "An Investigation into the Resistance Components of High Speed Displacement Catamarans"", RINA 1991, pp. 1-20.

Proceedings of the 8th ITTC, Madrid, Spain 1957, published by Canal de Experiencias Hidrodinamicas, El Pardo, Madrid.

Lazauskas L. "User's Guide for GODZILLA", The University of Adelaide, Applied Mathematics Report L9601, 1996, in preparation.

Lin W.-C., "The force and moment on a twin hull ship in steady potential flow", 10th O.N.R. Symp. Naval Hydro., MIT, pp.493-516, 1974.

Michell J.H., "The wave resistance of a ship", Phil. Mag. (5) vol 45, pp. 106-123, 1898.

Millward, A., "The Effect of Hull Separation and Restricted Water Depth on Catamaran Resistance", RINA, 1992, pp. 341-349.

Miyata, H., Nogami, H., Shirai, M. and Shirose, Y., "Fast Ferry by Super-Slender Twin Hull", ???

Powell, David J., Tong, S. S. and Skolnick, Michael M., " EnGENEous: domain independent, machine learning for design optimization", Proc. Third Int. Conf. on Genetic Algorithms and their Applications, J. D. Shaffer, ed., Morgan Kaufmann, 1989, pp.151-159.

Salvesen N., von Kerczek C., Scragg C.A., Cressy C.P. and Meinhold M.J., "Hydrodynamic design of SWATH ships", Trans SNAME vol 93, pp. 325-348, 1985.

Schmid, L. J., Sved, G. and Simpson, A.R., "Genetic Algorithms Applied to Structural Engineering Problems", in ed. K. H. Phua, C. M. Wang et al., "Optimization Techniques and Applications ", vol. 2, World Scientific, 1992, pp. 844-849.

Scragg, Carl A. and Nelson, Bruce D. "The Design of an Eight-Oared Rowing Shell", Marine Technology, Vol. 30, No. 2, April 1993, pp. 84-99.

Syswerda, Gilbert, "Uniform Crossover in Genetic Algorithms ", Proc.Third Int. Conf. on Genetic Algorithms and their Applications, J. D. Shaffer, ed., Morgan Kaufmann, 1989, pp. 2-9.

Tuck, E. O., "Wave Resistance of Thin Ships and Catamarans", The University of Adelaide, Applied Mathematics Report T8701, Jan. 1987.

Tuck, E. O., "The Wave Resistance Formula of J.H. Michell (1898) and its Significance to Recent Research in Ship Hydrodynamics", Journal of the Australian Mathematical Society, Series B, Vol. 30, 1989, pp. 365-377.

Turner, H. and Taplin, A., "The Resistance of Large Powered Catamarans", Trans. SNAME, 1968, pp. 180-213.

Wehausen, John V., "Wave Resistance of Thin Ships", Symp. on Naval Hydrodynamics, ONR, Sept 24-28 1956, Washington D. C.

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7. SYMBOLS

We will use a somewhat cumbersome set of symbols and pseudo-code in this note until more browsers support mathematical equations and notation.

Symbol	Description	Units
B	Beam	metres
C_b	Block coefficient	-
C_f	Frictional resistance coefficient	-
C_p	Prismatic coefficient	-
C_t	Total resistance coefficient	-
C_v	Viscous resistance coefficient	-
C_w	Wave resistance coefficient	-
D	Displacement	cubic metres, tonnes
F_n	Length-based Froude number	-
F_{nv}	Volumetric Froude number	-
g	Gravitational acceleration	metres/sec/sec
$theta_{entry}$	Entrance half-angle	degrees
$theta_{exit}$	Exit half-angle	degrees
1+k	Hughes form factor	-
L	Length	metres
L^*	Cube root of displacement	metres
nu	Water kinematic viscosity	square metres/sec
R	Reynolds number	-
R_t	Total resistance	kN
R_v	Viscous resistance	kN
R_w	Wave resistance	kN
S	Wetted Area	square metres
T	Draft	metres
U	Ship speed	metres/sec, knots
W	Catamaran hull separation	metres
rho	Water density	kg/cubic metres