

## AN APPROXIMATE POWER PREDICTION METHOD

by

J. Holtrop\* and G.G.J. Mennen\*

## 1. Introduction

In a recent publication [1] a statistical method was presented for the determination of the required propulsive power at the initial design stage of a ship. This method was developed through a regression analysis of random model experiments and full-scale data, available at the Netherlands Ship Model Basin. Because the accuracy of the method was reported to be insufficient when unconventional combinations of main parameters were used, an attempt was made to extend the method by adjusting the original numerical prediction model to test data obtained in some specific cases. This adaptation of the method has resulted into a set of prediction formulae with a wider range of application. Nevertheless, it should be noticed that the given modifications have a tentative character only, because the adjustments are based on a small number of experiments. In any case, the application is limited to hull forms resembling the average ship described by the main dimensions and form coefficients used in the method.

The extension of the method was focussed on improving the power prediction of high-block ships with low  $L/B$ -ratios and of slender naval ships with a complex appendage arrangement and immersed transom sterns.

Some parts of this study were carried out in the scope of the NSMB Co-operative Research programme. The adaptation of the method to naval ships was carried out in a research study for the Royal Netherlands Navy. Permission to publish results of these studies is gratefully acknowledged.

## 2. Resistance prediction

The total resistance of a ship has been subdivided into:

$$R_{\text{total}} = R_F(1+k_1) + R_{APP} + R_W + R_B + R_{TR} + R_A$$

where:

- $R_F$  frictional resistance according to the ITTC-1957 friction formula
- $1+k_1$  form factor describing the viscous resistance of the hull form in relation to  $R_F$
- $R_{APP}$  resistance of appendages
- $R_W$  wave-making and wave-breaking resistance
- $R_B$  additional pressure resistance of bulbous bow near the water surface

$R_{TR}$  additional pressure resistance of immersed transom stern

$R_A$  model-ship correlation resistance.

For the form factor of the hull the prediction formula:

$$1+k_1 = c_{13} \{0.93 + c_{12}(B/L_R)^{0.92497} (0.95 - C_P)^{-0.521448} (1 - C_P + 0.0225 lcb)^{0.6906}\}$$

can be used.

In this formula  $C_P$  is the prismatic coefficient based on the waterline length  $L$  and  $lcb$  is the longitudinal position of the centre of buoyancy forward of  $0.5L$  as a percentage of  $L$ . In the form-factor formula  $L_R$  is a parameter reflecting the length of the run according to:

$$L_R/L = 1 - C_P + 0.06 C_P lcb / (4 C_P - 1)$$

The coefficient  $c_{12}$  is defined as:

$$c_{12} = (T/L)^{0.2228446} \quad \text{when } T/L > 0.05$$

$$c_{12} = 48.20(T/L - 0.02)^{2.078} + 0.479948 \quad \text{when } 0.02 < T/L < 0.05$$

$$c_{12} = 0.479948 \quad \text{when } T/L < 0.02$$

In this formula  $T$  is the average moulded draught. The coefficient  $c_{13}$  accounts for the specific shape of the afterbody and is related to the coefficient  $C_{\text{stern}}$  according to:

$$c_{13} = 1 + 0.003 C_{\text{stern}}$$

For the coefficient  $C_{\text{stern}}$  the following tentative guidelines are given:

Afterbody form	$C_{\text{stern}}$
V-shaped sections	- 10
Normal section shape	0
U-shaped sections with Hogner stern	+ 10

The wetted area of the hull can be approximated well by:

$$S = L(2T + B) \sqrt{C_M} (0.453 + 0.4425 C_B + 0.2862 C_M - 0.003467 B/T + 0.3696 C_{WP}) + 2.38 A_{BT}/C_B$$

In this formula  $C_M$  is the midship section coefficient,  $C_B$  is the block coefficient on the basis of the

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waterline length  $L$ ,  $C_{WP}$  is the waterplane area coefficient and  $A_{BT}$  is the transverse sectional area of the bulb at the position where the still-water surface intersects the stem.

The appendage resistance can be determined from:

$$R_{APP} = 0.5 \rho V^2 S_{APP} (1 + k_2)_{eq} C_F$$

where  $\rho$  is the water density,  $V$  the speed of the ship,  $S_{APP}$  the wetted area of the appendages,  $1 + k_2$  the appendage resistance factor and  $C_F$  the coefficient of frictional resistance of the ship according to the ITTC-1957 formula.

In the Table below tentative  $1 + k_2$  values are given for streamlined flow-oriented appendages. These values were obtained from resistance tests with bare and appended ship models. In several of these tests turbulence stimulators were present at the leading edges to induce turbulent flow over the appendages.

Approximate $1 + k_2$ values	
rudder behind skeg	1.5 – 2.0
rudder behind stern	1.3 – 1.5
twin-screw balance rudders	2.8
shaft brackets	3.0
skeg	1.5 – 2.0
strut bossings	3.0
hull bossings	2.0
shafts	2.0 – 4.0
stabilizer fins	2.8
dome	2.7
bilge keels	1.4

The equivalent  $1 + k_2$  value for a combination of appendages is determined from:

$$(1 + k_2)_{eq} = \frac{\Sigma(1 + k_2)S_{APP}}{\Sigma S_{APP}}$$

The appendage resistance can be increased by the resistance of bow thruster tunnel openings according to:

$$\rho V^2 \pi d^2 C_{BTO}$$

where  $d$  is the tunnel diameter.

The coefficient  $C_{BTO}$  ranges from 0.003 to 0.012. For openings in the cylindrical part of a bulbous bow the lower figures should be used.

The wave resistance is determined from:

$$R_W = c_1 c_2 c_5 \nabla \rho g \exp\{m_1 F_n^d + m_2 \cos(\lambda F_n^{-2})\}$$

with:

$$c_1 = 2223105 c_7^{3.78613} (T/B)^{1.07961} (90 - i_E)^{-1.37565}$$

$$c_7 = 0.229577 (B/L)^{0.33333} \quad \text{when } B/L < 0.11$$

$$c_7 = B/L \quad \text{when } 0.11 < B/L < 0.25$$

$$c_7 = 0.5 - 0.0625 L/B \quad \text{when } B/L > 0.25$$

$$c_2 = \exp(-1.89 \sqrt{c_3})$$

$$c_5 = 1 - 0.8 A_T / (BT C_M)$$

In these expressions  $c_2$  is a parameter which accounts for the reduction of the wave resistance due to the action of a bulbous bow. Similarly,  $c_5$  expresses the influence of a transom stern on the wave resistance. In the expression  $A_T$  represents the immersed part of the transverse area of the transom at zero speed.

In this figure the transverse area of wedges placed at the transom chine should be included.

In the formula for the wave resistance,  $F_n$  is the Froude number based on the waterline length  $L$ . The other parameters can be determined from:

$$\lambda = 1.446 C_p - 0.03 L/B \quad \text{when } L/B < 12$$

$$\lambda = 1.446 C_p - 0.36 \quad \text{when } L/B > 12$$

$$m_1 = 0.0140407 L/T - 1.75254 \nabla^{1/3}/L + 4.79323 B/L - c_{16}$$

$$c_{16} = 8.07981 C_p - 13.8673 C_p^2 + 6.984388 C_p^3 \quad \text{when } C_p < 0.80$$

$$c_{16} = 1.73014 - 0.7067 C_p \quad \text{when } C_p > 0.80$$

$$m_2 = c_{15} C_p^2 \exp(-0.1 F_n^{-2})$$

The coefficient  $c_{15}$  is equal to  $-1.69385$  for  $L^3/\nabla < 512$ , whereas  $c_{15} = 0.0$  for  $L^3/\nabla > 1727$ .

For values of  $512 < L^3/\nabla < 1727$ ,  $c_{15}$  is determined from:

$$c_{15} = -1.69385 + (L/\nabla^{1/3} - 8.0)/2.36$$

$$d = -0.9$$

The half angle of entrance  $i_E$  is the angle of the waterline at the bow in degrees with reference to the centre plane but neglecting the local shape at the stem. If  $i_E$  is unknown, use can be made of the following formula:

$$i_E = 1 + 89 \exp\{-(L/B)^{0.80856} (1 - C_{WP})^{0.30484} (1 - C_p - 0.0225 lcb)^{0.6367} (L_R/B)^{0.34574} (100 \nabla/L^3)^{0.16302}\}$$

This formula, obtained by regression analysis of over 200 hull shapes, yields  $i_E$  values between  $1^\circ$  and  $90^\circ$ . The original equation in [1] sometimes resulted in negative  $i_E$  values for exceptional combinations of hull-form parameters.

The coefficient that determines the influence of the bulbous bow on the wave resistance is defined as:

$$c_3 = 0.56 A_{BT}^{1.5} / \{BT(0.31 \sqrt{A_{BT}} + T_F - h_B)\}$$

where  $h_B$  is the position of the centre of the transverse area  $A_{BT}$  above the keel line and  $T_F$  is the forward draught of the ship.

The additional resistance due to the presence of a bulbous bow near the surface is determined from:

$$R_B = 0.11 \exp(-3 P_B^{-2}) F_{ni}^3 A_{BT}^{1.5} \rho g / (1 + F_{ni}^2)$$

where the coefficient  $P_B$  is a measure for the emergence of the bow and  $F_{ni}$  is the Froude number based on the immersion:

$$P_B = 0.56 \sqrt{A_{BT}} / (T_F - 1.5 h_B)$$

and

$$F_{ni} = V / \sqrt{g(T_F - h_B - 0.25 \sqrt{A_{BT}}) + 0.15 V^2}$$

In a similar way the additional pressure resistance due to the immersed transom can be determined:

$$R_{TR} = 0.5 \rho V^2 A_T c_6$$

The coefficient  $c_6$  has been related to the Froude number based on the transom immersion:

$$c_6 = 0.2(1 - 0.2 F_{nT}) \quad \text{when } F_{nT} < 5$$

or

$$c_6 = 0 \quad \text{when } F_{nT} \geq 5$$

$F_{nT}$  has been defined as:

$$F_{nT} = V / \sqrt{2 g A_T / (B + B C_{WP})}$$

In this definition  $C_{WP}$  is the waterplane area coefficient.

The model-ship correlation resistance  $R_A$  with

$$R_A = \frac{1}{2} \rho V^2 S C_A$$

is supposed to describe primarily the effect of the hull roughness and the still-air resistance. From an analysis of results of speed trials, which have been corrected to ideal trial conditions, the following formula for the correlation allowance coefficient  $C_A$  was found:

$$C_A = 0.006(L + 100)^{-0.16} - 0.00205 + 0.003 \sqrt{L/7.5} C_B^4 c_2 (0.04 - c_4)$$

with

$$c_4 = T_F / L \quad \text{when } T_F / L \leq 0.04$$

or

$$c_4 = 0.04 \quad \text{when } T_F / L > 0.04$$

In addition,  $C_A$  might be increased to calculate e.g. the effect of a larger hull roughness than standard. To this end the ITTC-1978 formulation can be used from which the increase of  $C_A$  can be derived for roughness values higher than the standard figure of  $k_s = 150 \mu\text{m}$  (mean apparent amplitude):

$$\text{increase } C_A = (0.105 k_s^{1/3} - 0.005579) / L^{1/3}$$

In these formulae  $L$  and  $k_s$  are given in metres.

### 3. Prediction of propulsion factors

The statistical prediction formulae for estimating the effective wake fraction, the thrust deduction fraction and the relative-rotative efficiency as presented in [1] could be improved on several points.

For single-screw ships with a conventional stern arrangement the following adapted formula for the wake fraction can be used:

$$w = c_9 C_V \frac{L}{T_A} \left( 0.0661875 + 1.21756 c_{11} \frac{C_V}{(1 - C_{P1})} \right) + 0.24558 \sqrt{\frac{B}{L(1 - C_{P1})}} - \frac{0.09726}{0.95 - C_P} + \frac{0.11434}{0.95 - C_B} + 0.75 C_{\text{stern}} C_V + 0.002 C_{\text{stern}}$$

The coefficient  $c_9$  depends on a coefficient  $c_8$  defined as:

$$c_8 = BS / (LDT_A) \quad \text{when } B/T_A < 5$$

or

$$c_8 = S(7B/T_A - 25) / (LD(B/T_A - 3)) \quad \text{when } B/T_A > 5$$

$$c_9 = c_8 \quad \text{when } c_8 < 28$$

or

$$c_9 = 32 - 16 / (c_8 - 24) \quad \text{when } c_8 > 28$$

$$c_{11} = T_A / D \quad \text{when } T_A / D < 2$$

or

$$c_{11} = 0.0833333(T_A / D)^3 + 1.33333 \quad \text{when } T_A / D > 2$$

In the formula for the wake fraction,  $C_V$  is the viscous resistance coefficient with  $C_V = (1 + k) C_F + C_A$ . Further:

$$C_{P1} = 1.45 C_P - 0.315 - 0.0225 lcb$$

In a similar manner the following approximate formula for the thrust deduction for single-screw ships with a conventional stern can be applied:

$$t = 0.001979 L / (B - BC_{P1}) + 1.0585 c_{10} + 0.00524 - 0.1418 D^2 / (BT) + 0.0015 C_{\text{stern}}$$

The coefficient  $c_{10}$  is defined as:

$$c_{10} = B/L \quad \text{when } L/B > 5.2$$

or

$$c_{10} = 0.25 - 0.003328402 / (B/L - 0.134615385) \quad \text{when } L/B < 5.2$$

The relative-rotative efficiency can be predicted

well by the original formula:

$$\eta_R = 0.9922 - 0.05908 A_E/A_O + \\ + 0.07424(C_P - 0.0225 lcb)$$

Because the formulae above apply to ships with a conventional stern an attempt has been made to indicate a tentative formulation for the propulsion factors of single-screw ships with an open stern as applied sometimes on slender, fast sailing ships:

$$w = 0.3 C_B + 10 C_V C_B - 0.1$$

$$t = 0.10 \quad \text{and} \quad \eta_R = 0.98.$$

These values are based on only a very limited number of model data. The influence of the fullness and the viscous resistance coefficient has been expressed in a similar way as in the original prediction formulae for twin-screw ships. These original formulae for twin-screw ships are:

$$w = 0.3095 C_B + 10 C_V C_B - 0.23 D/\sqrt{BT}$$

$$t = 0.325 C_B - 0.1885 D/\sqrt{BT}$$

$$\eta_R = 0.9737 + 0.111(C_P - 0.0225 lcb) + \\ - 0.06325 P/D$$

#### 4. Estimation of propeller efficiency

For the prediction of the required propulsive power the efficiency of the propeller in open-water condition has to be determined. It has appeared that the characteristics of most propellers can be approximated well by using the results of tests with systematic propeller series. In [2] a polynomial representation is given of the thrust and torque coefficients of the B-series propellers. These polynomials are valid, however, for a Reynolds number of  $2.10^6$  and need to be corrected for the specific Reynolds number and the roughness of the actual propeller. The presented statistical prediction equations for the model-ship correlation allowance and the propulsion factors are based on Reynolds and roughness corrections according to the ITTC-1978 method, [3]. According to this method the propeller thrust and torque coefficients are corrected according to:

$$K_{T\text{-ship}} = K_{T\text{-B-series}} + \Delta C_D 0.3 \frac{P c_{0.75} Z}{D^2}$$

$$K_{Q\text{-ship}} = K_{Q\text{-B-series}} - \Delta C_D 0.25 \frac{c_{0.75} Z}{D}$$

Here  $\Delta C_D$  is the difference in drag coefficient of the profile section,  $P$  is the pitch of the propeller and

$c_{0.75}$  is the chord length at a radius of 75 per cent and  $Z$  is the number of blades.

$$\Delta C_D = (2 + 4(t/c)_{0.75}) \{0.003605 - (1.89 + 1.62 \\ \log(c_{0.75}/k_p))^{-2.5}\}$$

In this formula  $t/c$  is the thickness-chordlength ratio and  $k_p$  is the propeller blade surface roughness.

For this roughness the value of  $k_p = 0.00003$  m is used as a standard figure for new propellers.

The chord length and the thickness-chordlength ratio can be estimated using the following empirical formulae:

$$c_{0.75} = 2.073(A_E/A_O) D/Z$$

and

$$(t/c)_{0.75} = (0.0185 - 0.00125 Z) D/c_{0.75}.$$

The blade area ratio can be determined from e.g. Keller's formula:

$$A_E/A_O = K + (1.3 + 0.3 Z) T/(D^2(p_o + \rho gh - p_v))$$

In this formula  $T$  is the propeller thrust,  $p_o + \rho gh$  is the static pressure at the shaft centre line,  $p_v$  is the vapour pressure and  $K$  is a constant to which the following figures apply:

$K = 0$  to  $0.1$  for twin-screw ships

$K = 0.2$  for single-screw ships

For sea water of 15 degrees centigrade the value of  $p_o - p_v$  is  $99047$  N/m<sup>2</sup>.

The given prediction equations are consistent with a shafting efficiency of

$$\eta_S = P_D/P_S = 0.99$$

and reflect ideal trial conditions, implying:

— no wind, waves and swell,

— deep water with a density of  $1025$  kg/m<sup>3</sup> and a temperature of 15 degrees centigrade and

— a clean hull and propeller with a surface roughness according to modern standards.

The shaft power can now be determined from:

$$P_S = P_E / (\eta_R \eta_o \eta_S \frac{1-t}{1-w})$$

#### 5. Numerical example

The performance characteristics of a hypothetical single-screw ship are calculated for a speed of 25 knots. The calculations are made for the various resistance components and the propulsion factors, successively.

The main ship particulars are listed in the Table on the next page:

## Main ship characteristics

length on waterline	$L$	205.00 m
length between perpendiculars	$L_{pp}$	200.00 m
breadth moulded	$B$	32.00 m
draught moulded on F.P.	$T_F$	10.00 m
draught moulded on A.P.	$T_A$	10.00 m
displacement volume moulded	$\nabla$	37500 m <sup>3</sup>
longitudinal centre of buoyancy		2.02% aft of $\frac{1}{2}L_{pp}$
transverse bulb area	$A_{BT}$	20.0 m <sup>2</sup>
centre of bulb area above keel line	$h_B$	4.0 m
midship section coefficient	$C_M$	0.980
waterplane area coefficient	$C_{WP}$	0.750
transom area	$A_T$	16.0 m <sup>2</sup>
wetted area appendages	$S_{APP}$	50.0 m <sup>2</sup>
stern shape parameter	$C_{stern}$	10.0
propeller diameter	$D$	8.00 m
number of propeller blades	$Z$	4
clearance propeller with keel line		0.20 m
ship speed	$V$	25.0 knots

## References

- Holtrop, J. and Mennen, G.G.J., 'A statistical power prediction method', International Shipbuilding Progress, Vol. 25, October 1978.
- Oosterveld, M.W.C. and Oossanen, P. van, 'Further computer analyzed data of the Wageningen B-screw series', International Shipbuilding Progress, July 1975.
- Proceedings 15th ITTC, The Hague, 1978.

The calculations with the statistical method resulted into the following coefficients and powering characteristics listed in the next Table:

$F_n$	= 0.2868	$F_{nT}$	= 5.433
$C_P$	= 0.5833	$R_{TR}$	= 0.00 kN
$L_R$	= 81.385 m	$c_4$	= 0.04
$lcb$	= -0.75% (relative to $\frac{1}{2}L$ )	$C_A$	= 0.000352
$c_{12}$	= 0.5102	$R_A$	= 221.98 kN
$c_{13}$	= 1.030	$R_{total}$	= 1793.26 kN
$1 + k_1$	= 1.156	$P_E$	= 23063 kW
$S$	= 7381.45 m <sup>2</sup>	$C_V$	= 0.001963
$C_F$	= 0.001390	$c_9$	= 14.500
$R_F$	= 869.63 kN	$c_{11}$	= 1.250
$1 + k_2$	= 1.50	$C_{P1}$	= 0.5477
$R_{APP}$	= 8.83 kN	$w$	= 0.2584
$c_7$	= 0.1561	$c_{10}$	= 0.15610
$i_E$	= 12.08 degrees	$t$	= 0.1747
$c_1$	= 1.398	$T$	= 2172.75 kN
$c_3$	= 0.02119	$A_E/A_O$	= 0.7393
$c_2$	= 0.7595	$\eta_R$	= 0.9931
$c_5$	= 0.9592	$c_{0.75}$	= 3.065 m
$m_1$	= -2.1274	$t/c_{0.75}$	= 0.03524
$c_{15}$	= 1.69385	$\Delta C_D$	= 0.000956
$m_2$	= -0.17087		
$\lambda$	= 0.6513	From the B-series polynomials:	
$R_W$	= 557.11 kN	$K_{Ts}$	= 0.18802
$P_B$	= 0.6261	$n$	= 1.6594 Hz
$F_{ni}$	= 1.5084	$K_{Qo}$	= 0.033275
$R_B$	= 0.049 kN	$\eta_o$	= 0.6461
		$P_S$	= 32621 kW

## A STATISTICAL RE-ANALYSIS OF RESISTANCE AND PROPULSION DATA

by

J. Holtrop\*

### 1. Introduction

In a recent publication [1] a power prediction method was presented which was based on a regression analysis of random model and full-scale test data. For several combinations of main dimensions and form coefficients the method had been adjusted to test results obtained in some specific cases. In spite of these adaptations the accuracy of the method was found to be insufficient for some classes of ships. Especially for high speed craft at Froude numbers above 0.5 the power predictions were often wrong. With the objective to improve the method the data sample was extended covering wider ranges of the parameters of interest. In this extension of the data sample the published results of the Series 64 hull forms [2] have been included. The regression analyses were now based on the results of tests on 334 models. Beside these analyses of resistance and propulsion properties a method was devised by which the influence of the propeller cavitation could be taken into account. In addition some formulae are given by which the effect of a partial propeller submergence can tentatively be estimated. These formulae have been derived in a study carried out in a MARIN Co-operative Research programme. Permission to publish these results is gratefully acknowledged.

### 2. Re-analysis of resistance test results

The results were analysed using the same sub-division into components as used in [1]:

$$R_{\text{Total}} = R_F(1+k_1) + R_{APP} + R_W + R_B + R_{TR} + R_A$$

where:

$R_F$  = frictional resistance according to the ITTC-1957 formula

$1+k_1$  = form factor of the hull

$R_{APP}$  = appendage resistance

$R_W$  = wave resistance

$R_B$  = additional pressure resistance of bulbous bow near the water surface

$R_{TR}$  = additional pressure resistance due to transom immersion

$R_A$  = model-ship correlation resistance.

A regression analysis provided a new formula for the form factor of the hull:

$$1+k_1 = 0.93 + 0.487118 c_{14} (B/L)^{1.06806} (T/L)^{0.46106}$$

$$(L/L_R)^{0.121563} (L^3/\nabla)^{0.36486} (1-C_p)^{-0.604247}$$

In this formula  $B$  and  $T$  are the moulded breadth and draught, respectively.  $L$  is the length on the waterline and  $\nabla$  is the moulded displacement volume.  $C_p$  is the prismatic coefficient based on the waterline length.

$L_R$  is defined as:

$$L_R = L(1 - C_p + 0.06 C_p lcb / (4 C_p - 1))$$

where  $lcb$  is the longitudinal position of the centre of buoyancy forward of  $0.5L$  as a percentage of  $L$ .

The coefficient  $c_{14}$  accounts for the stern shape. It depends on the stern shape coefficient  $C_{\text{stern}}$  for which the following tentative figures can be given:

Afterbody form	$C_{\text{stern}}$	$c_{14} = 1 + 0.011 C_{\text{stern}}$
Pram with gondola	-25	
V-shaped sections	-10	
Normal section shape	0	
U-shaped sections with Hogner stern	10	

As regards the appendage resistance no new analysis was made. For prediction of the resistance of the appendages reference is made to [1].

A re-analysis was made of the wave resistance. A new general formula was derived from the data sample of 334 models but calculations showed that this new prediction formula was not better in the speed range up to Froude numbers of about  $F_n = 0.5$ . The results of these calculations indicated that probably a better prediction formula for the wave resistance in the high speed range could be devised when the low speed data were left aside from the regression analysis. By doing so, the following wave resistance formula was derived for the speed range  $F_n > 0.55$ .

$$R_{W-B} = c_{17} c_2 c_5 \nabla \rho g \exp \{ m_3 F_n^d + m_4 \cos(\lambda F_n^{-2}) \}$$

where:

$$c_{17} = 6919.3 C_M^{-1.3346} (\nabla/L^3)^{2.00977} (L/B-2)^{1.40692}$$

$$m_3 = -7.2035(B/L)^{0.326869} (T/B)^{0.605375}$$

The coefficients  $c_2$ ,  $c_5$ ,  $d$  and  $\lambda$  have the same definition as in [1]:

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$$\begin{aligned}
c_2 &= \exp(-1.89\sqrt{c_3}) \\
c_5 &= (1 - 0.8A_{T'}^1)/(BTC_M) \\
\lambda &= 1.446 C_p - 0.03 L/B \\
&\quad \text{when } L/B < 12 \\
\lambda &= 1.446 C_p - 0.36 \\
&\quad \text{when } L/B > 12 \\
d &= -0.9 \\
c_3 &= 0.56 A_{BT}^{1.5} / \{BT(0.31\sqrt{A_{BT}} + T_F - h_B)\} \\
m_4 &= c_{15} 0.4 \exp(-0.034 F_n^{-3.29}) \\
c_{15} &= -1.69385 \\
&\quad \text{when } L^3/\nabla < 512 \\
c_{15} &= -1.69385 + (L/\nabla^{1/3} - 8)/2.36 \\
&\quad \text{when } 512 < L^3/\nabla < 1726.91 \\
c_{15} &= 0 \\
&\quad \text{when } L^3/\nabla > 1726.91
\end{aligned}$$

The midship section coefficient  $C_M$  and the transverse immersed transom area at rest  $A_T$  and the transverse area of the bulbous bow  $A_{BT}$  have the same meaning as in [1]. The vertical position of the centre of  $A_{BT}$  above the keel plane is  $h_B$ . The value of  $h_B$  should not exceed the upper limit of  $0.6 T_F$ .

Because attempts to derive prediction formulae for the wave resistance at low and moderate speeds were only partially successful it is suggested to use for the estimation of the wave resistance up to a Froude number of 0.4 a formula which closely resembles the original formula of [1]. The only modification consists of an adaptation of the coefficient that causes the humps and hollows on the resistance curves. This formula, which is slightly more accurate than the original one reads:

$$\begin{aligned}
R_{W-A} &= c_1 c_2 c_5 \nabla \rho g \exp\{m_1 F_n^d + m_4 \cos(\lambda F_n^{-2})\} \\
&\quad \text{with:} \\
c_1 &= 2223105 c_7^{3.78613} (T/B)^{1.07961} (90 - i_E)^{-1.37565} \\
c_7 &= 0.229577 (B/L)^{0.33333} \\
&\quad \text{when } B/L < 0.11 \\
c_7 &= B/L \\
&\quad \text{when } 0.11 < B/L < 0.25 \\
c_7 &= 0.5 - 0.0625 L/B \\
&\quad \text{when } B/L > 0.25 \\
m_1 &= 0.0140407 L/T - 1.75254 \nabla^{1/3} / L - \\
&\quad \quad \quad 4.79323 B/L - c_{16} \\
c_{16} &= 8.07981 C_p - 13.8673 C_p^2 + 6.984388 C_p^3 \\
&\quad \text{when } C_p < 0.8 \\
c_{16} &= 1.73014 - 0.7067 C_p \\
&\quad \text{when } C_p > 0.8
\end{aligned}$$

$m_4$ : as in the  $R_W$  formula for the high speed range.

For the speed range  $0.40 < F_n < 0.55$  it is suggested to use the more or less arbitrary interpolation formula:

$$R_W = R_{W-A_{0.4}} + (10F_n - 4)(R_{W-B_{0.55}} - R_{W-A_{0.4}})/1.5$$

Here  $R_{W-A_{0.4}}$  is the wave resistance prediction for  $F_n = 0.40$  and  $R_{W-B_{0.55}}$  is the wave resistance for  $F_n = 0.55$  according to the respective formulae.

No attempts were made to derive new formulations for the transom pressure resistance and the additional wave resistance due to a bulb near the free surface. The available material to develop such formulae is rather scarce. As regards the height of the centre of the transverse bulb area  $h_B$  it is recommended to obey the upper limit of  $0.6 T_F$  in the calculation of the additional wave resistance due to the bulb.

### 3. Re-analysis of propulsion data

The model propulsion factors and the model-ship correlation allowance were statistically re-analysed using the extended data sample. This data sample included 168 data points of full-scale trials on new built ships. In the analysis the same structure of the wake prediction formulae in [1] was maintained. By the regression analyses new constants were determined which give a slightly more accurate prediction.

A point which has been improved in the wake prediction formula is the effect of the midship section coefficient  $C_M$  for full hull forms with a single screw. The improved formula for single screw ships with a conventional stern reads:

$$\begin{aligned}
w &= c_9 c_{20} C_V \frac{L}{T_A} \left( 0.050776 + 0.93405 c_{11} \frac{C_V}{(1 - C_{p1})} \right) \\
&\quad + 0.27915 c_{20} \sqrt{\frac{B}{L(1 - C_{p1})}} + c_{19} c_{20}
\end{aligned}$$

The coefficient  $c_9$  depends on the coefficient  $c_8$  defined as:

$$c_8 = BS/(L D T_A) \quad \text{when } B/T_A < 5$$

or

$$c_8 = S(7B/T_A - 25)/(LD(B/T_A - 3)) \quad \text{when } B/T_A > 5$$

$$c_9 = c_8 \quad \text{when } c_8 < 28$$

or

$$c_9 = 32 - 16/(c_8 - 24) \quad \text{when } c_8 > 28$$

$$c_{11} = T_A/D \quad \text{when } T_A/D < 2$$

or

$$c_{11} = 0.0833333(T_A/D)^3 + 1.33333 \quad \text{when } T_A/D > 2$$

$$c_{19} = 0.12997/(0.95 - C_B) - 0.11056/(0.95 - C_p) \quad \text{when } C_p < 0.7$$

or

$$c_{19} = 0.18567 / (1.3571 - C_M) - 0.71276 + 0.38648 C_p$$

when  $C_p > 0.7$

$$c_{20} = 1 + 0.015 C_{\text{stern}}$$

$$C_{p1} = 1.45 C_p - 0.315 - 0.0225 lcb .$$

The coefficient  $C_V$  is the viscous resistance coefficient with

$$C_V = (1 + k) C_F + C_A$$

As regards the thrust deduction of single screw ships a new formula was devised of comparable accuracy:

$$t = 0.25014 (B/L)^{0.28956} (\sqrt{BT}/D)^{0.2624} /$$

$$/(1 - C_p + 0.0225 lcb)^{0.01762} + 0.0015 C_{\text{stern}}$$

For the relative-rotative efficiency an alternative prediction formula was derived but because its accuracy is not better than that of the original one it is suggested to use the prediction formula of [1]:

$$\eta_R = 0.9922 - 0.05908 A_E/A_O +$$

$$+ 0.07424 (C_p - 0.0225 lcb)$$

For multiple-screw ships and open-stern single-screw ships with open shafts the formulae of [1] were maintained.

The model-ship correlation allowance was statistically analysed. It appeared that for new ships under ideal trial conditions a  $C_A$ -value would be applicable which is on the average 91 per cent of the  $C_A$ -value according to the statistical formula of [1]. Apparently, the incorporation of more recent trial data has reduced the average level of  $C_A$  somewhat. It is suggested, however, that for practical purposes the original formula is used.