2020 08 28

# Formulations used in « Gene Hull Sailboat 3.0 » for the hull generation

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As a complement to « Gene-Hull Sailboat 3.0 » application, its User Guide and its Examples, this document proposes the formulations involved for the Hull generation.

**Introduction :**

The approach is based on fit for purpose analytical functions defined directly in the usual cartesian coordinates x,y,z , with adimensional parameters dedicated for each purpose, in order to cover the widest possible variety of hull shapes for a sailboat.

The hull is mathematically described with :

Keel line : Zk(X)
Sheer line : Ysh(X), Zsh(X)
Sections : $Y_{VE}$(X,Z),

That allows to directly draw the traditional 3 views 2D with its waterlines and its buttock lines, an isometric perspective, and to compute the hull contribution to the usual ouput about hydrostatics, loading, stability, righting moment, ....

The needed input includes exactly :

13 geometrical data in metric
2 geometrical data in % :
1 angle (for the so-called alfa transformation)
21 adimensional parameters :

, in total : 37 input data

From a given set of input data, we can either :

- change only the geometrical data in metric : you build another project with a similar shape for the hull
- change only the adimensional parameters : you build another hull within the same geometrical constraints.
- change all the data, for a new project with a new shape of hull

I pay attention to use functions which keep a smooth evolution of the curvatures, which does not generate curvature evolution other than the ones necessary to shape the hull and to comply with the input geometrical constraints. It is a concern about the so-called « fairing » issue.

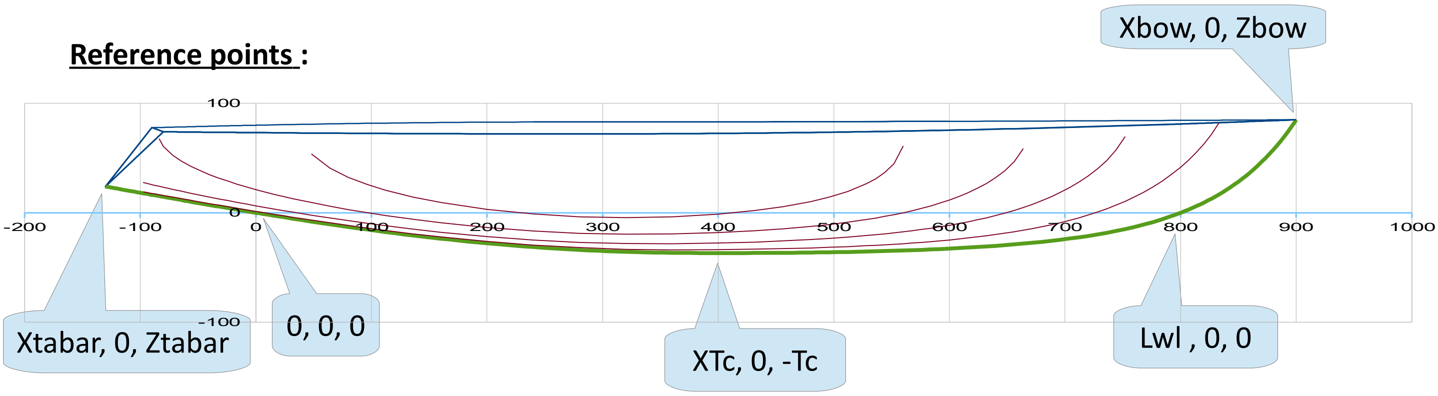
Finally, this mathematical approach can be implementated within a standard spreadsheet application, as proposed within « Gene-Hull Sailboat 3.0 » , which is an .ods file working under Open Office or Libre office.

**Coordinates system :**

x = 0 at section C0 ( = rear point of the waterline), x positive towards front
y = 0 in the symmetrical longitudinal plan,
z = 0 waterline surface, z positive towards up

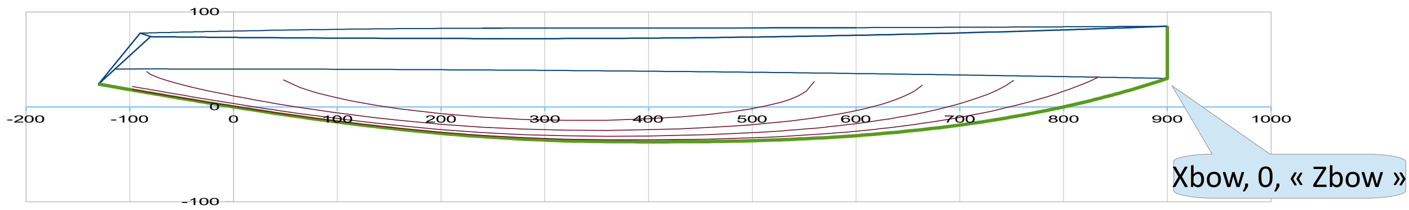
## 1. The keel line in the vertical plan of symmetry xz

**<u>Reference points</u> :**




The keel line (in green) is defined by 2 functions for respectively the fore part (when x > XTc) and the rear part (when x < XTc). (Nota : hull draft Tc is input positive, but -Tc is the coordinate) .

In case of vertical upper chine, « Zbow » considered for the computation nare at chine tip, i.e. « Zbow » = Zhcav




**Fore part of the keel line including the bow stem >>> for x such as XTc ≤ x < Xbow :**

The fore function, including the bow definition, involves the sum of 2 terms in order to cover a wide variety of possible bow line :

$$Z = Z1 + Z2$$

The second term Z2 is optional, to straighten the bow above the water and to accentuate the fore foot, its influence can be neutralized if the parameter Kbrion = 0.

**First term function is in the form of :**

$$Z1\,(X) = -Tc + (X - XTc)^{\wedge}[a + b\,(X\text{-}XTc)^{\wedge}Cet] \,/\, c$$

, where coefficients a, b, c are computed using the input adimensional parameters Puiqar and Puiqav and the geometrical data in order that the keel line can comply with the reference points showed here above.

Cet is an additional input adimensional parameter contibuting to shape the bow. Cet can usually be chosen in 0,1 to 100 range.

When applied within Gene-Hull notations, that gives :

$$Z1\ (X) = -Tc + (X - Xtc) \wedge [PuiZoav + CorPuiZoav\ .\ ((X - XTc)/(Lwl - XTc))\wedge Cet] / Kav$$

, where :

$Kav = (Lwl - XTc) \wedge Puiqav / Tc$

$CorPuiZoav = [Log(Kav\ .\ (Zbow + Tc))/Log(Xbow - XTc) - Puiqav]/[((Xbow - XTc)/(Lwl - XTc)) \wedge Cet - 1]$

$PuiZoav = Puiqav - CorPuiZoav$

The input data for this formulation :

Geometrical : Tc, Xtc, Lwl , Zbow, Xbow,

Parametric : Puiqav, Cet

**The second term of the function :**

$$Z2\ (X) = 0 \qquad \text{for } X < Xtc$$

$$Z2\ (X) = Kbrion\ .\ [- Xhbrionar\ .\ Sin(2\pi / Lamdar\ .\ X + Phiar)\ ]\ .\ (Z1 + Tc)\ .\ (X - XTc) / (Xbow - Xtc) \qquad \text{for } Xtc \leq X < Lwl$$

$$Z2\ (X) = Kbrion\ .\ [\ Sin(2\pi / Lamdav\ .\ X + Phiav)\ ]\ .\ (Z1 + Tc)\ .\ (X - XTc) / (Xbow - Xtc) \qquad \text{for } Lwl \leq X \leq Xbow$$

where :

$Z1 = Z1(X)$ previously defined

Kbrion = input data (usual values : 0 to 0,20) (when Kbrion = 0 >> the second term = 0)

$Xhbrionar = (Lwl - Xtc) / (Xbow - Lwl)$

$Lambdar = 2\ .\ (Lwl - Xtc)$

$Phiar = - 2\pi / Lamdar\ .\ Xtc$

$Lambdav = 2.\ (Xbow - Lwl)$

$Phiav = - 2\pi / Lamdav\ .\ Lwl$

The input data for this formulation :

Geomotrical : Xtc, Tc, Lwl , Xbow,

Parametric : Kbrion

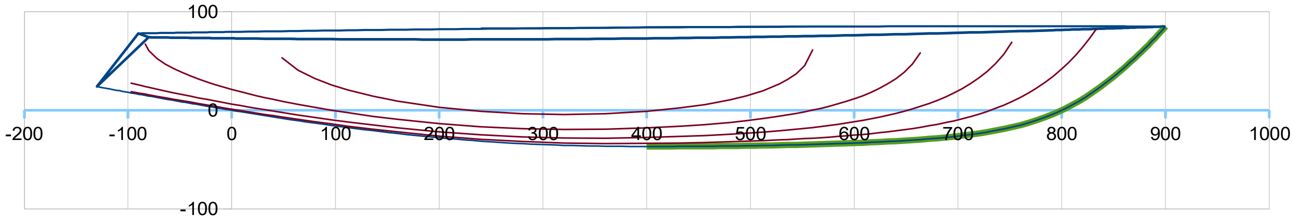
**Numerical check (geometrical data unit = cm) :**

| Input geometrical data : | Input adimensional parameters : |
|---|---|
| XTc = 400 | Puiquav = 2,45 |
| Tc = 36,58 | Puiqar = 2,35 |
| Lwl = 800 | Cet = 3,0 |
| Xbow = 900 | Kbrion = 0,1 |
| Zbow = 85 | |

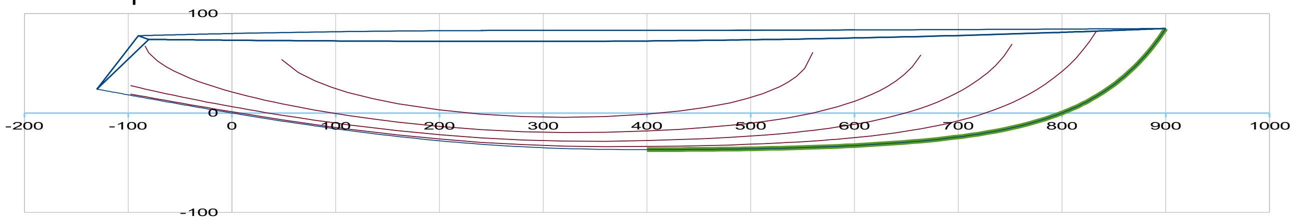
>>> computation of the coefficients :

| For the first term : | For the second term : |
|---|---|
| Kav = 64834,07 | Xhbrionar = 4,00 |
| CorPuiZoav = 0,11047 | Lambdar = 800,00 |
| PuiZoav = 2,3395 | Phiar = -3,14159 |
| | Lambdav = 800,00 |
| | Phiav = -25,13274 |

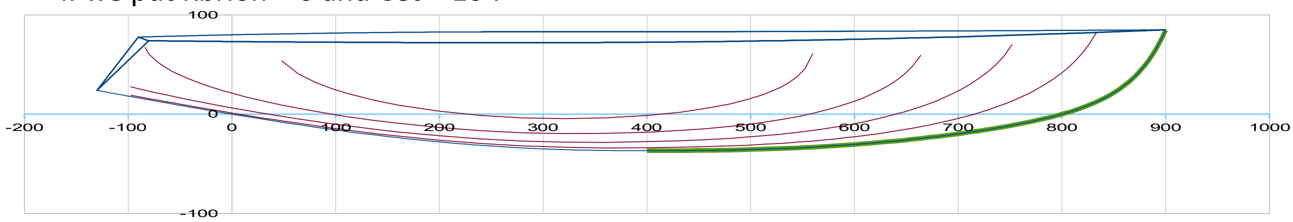
>>> keel line fore part with these input (in green) :




>>> if we put Kbrion = 0 :




>>> if we put Kbrion = 0 and Cet = 10 :



**Rear part of the keel line >>> for x such as Xtab ar < x < XTc :**

Z (X) = -Tc + (XTc - X) ^ [PuiZoar + CorPuiZoar . ((XTc – X) / XTc)] / Kar

, where :

Kar = XTc ^ Puiqar / Tc

CorPuiZoar = [Log(Kar . (Ztabar + Tc))/Log(XTc – Xtabar) – Puiqar]/[(XTc -Xtabar)/XTc - 1]

PuiZoar = Puiqar – CorPuiZoar

The input data for this formulation :

Geomotrical : Xtc, Tc, Xtabar, Ztabar

Parametric : Puiqar

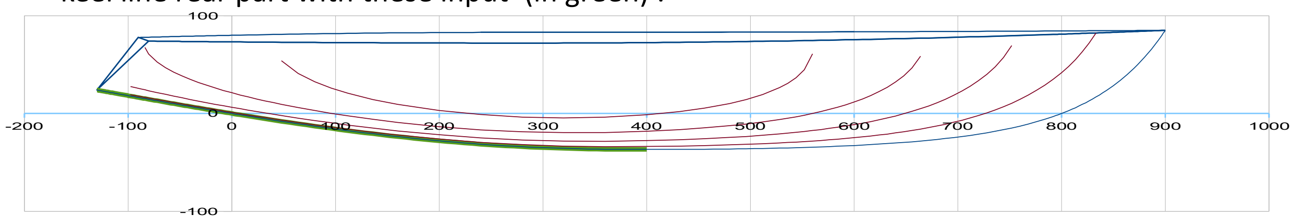
**Numerical check (geometrical data unit = cm) :**

| Input geometrical data : | Input adimensional parameters : |
|---|---|
| XTc = 400 | Puiquar = 2,35 |
| Tc = 36,58 | |
| Xtabar = -130 | |
| Ztabar = 24 | |

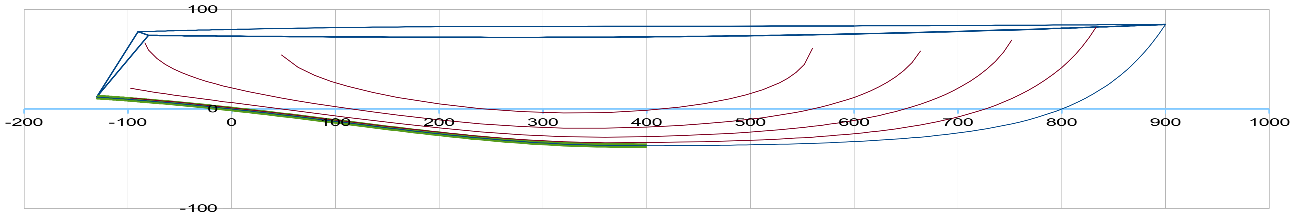
>>> computation of the coefficients :

| |
|---|
| Kar = 35612,07 |
| CorPuiZoar = -0,07694 |
| PuiZoar = 2,4269 |

>>> keel line rear part with these input (in green) :



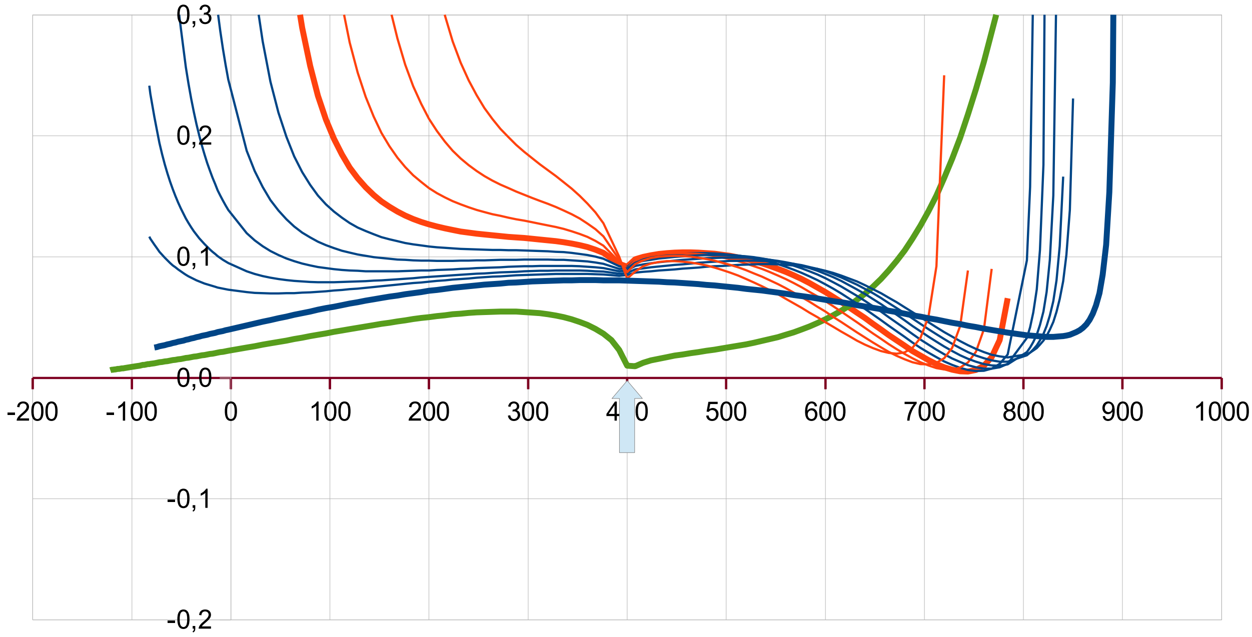

>>> if we put Ztabar = 12 (i.e. lower rear vault end)



**About the connection of the two half parts :** there is of course tangential continuity and curvature continuity at the connection point Xtc. The two functions behaves as polynomes of degrees Pui q av (fore) / Pui q ar (rear) in the vicinity of Xtc, and with usual values of 2,2 to 2,5, the curvature 1/R curve shows a downward oriented peak towards zero at Xtc (the green line here below) : <u>it is an analytical result, not an anomaly</u>.

Curvatures 1/R

Red : waterlines below H0 (thick red = H0) ; Blue : waterlines above H0 (thick blue = sheer line)
green : keel and buttock lines (thick green = keel line)

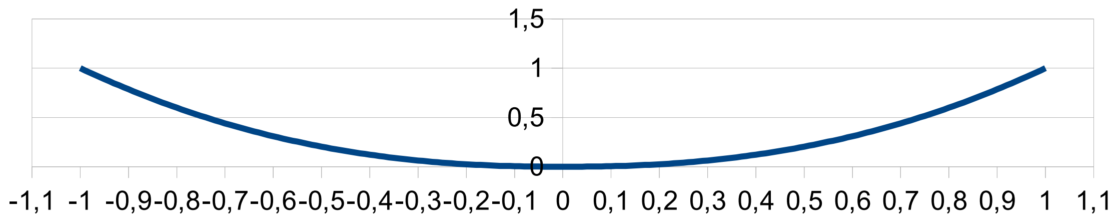



Demonstration with a polynome of degree 2,3 :

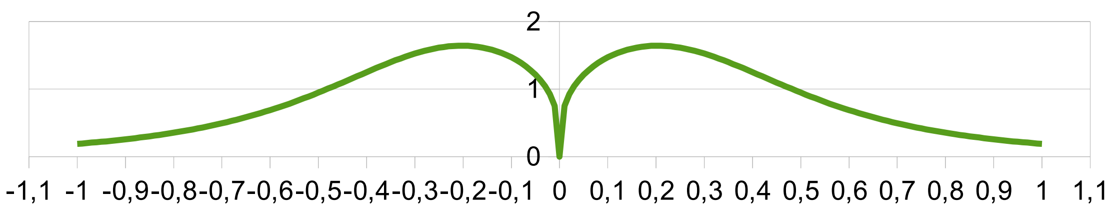
Z = X^2,3 >> Z' = 2,3 . X^1,3 & Z'' = 2,3 . 1,3 . X^0,3

>>> curvature 1/R = IZ''I / (1 + Z'^2)^(3/2) = 2,3 . 1,3 . X^0,3 / (1 + 2,3^2 . X^2,6)^1,5

Polynome X^2,3



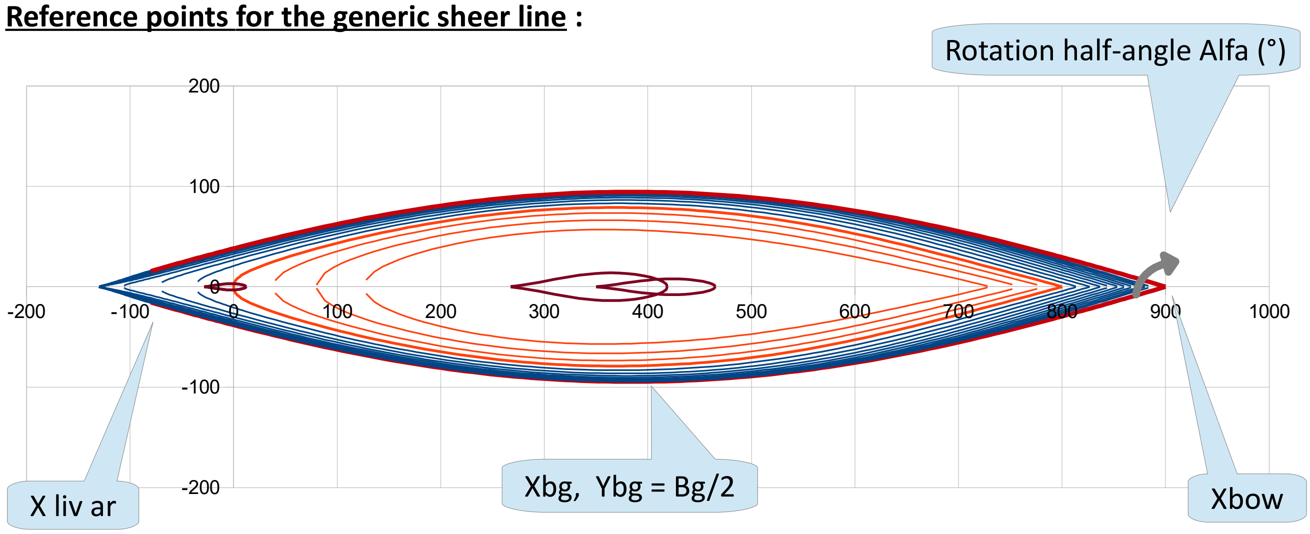

Curvature 1/R of the polynome X^2,3



**2. The sheer line, in its horizontal projection xy**

The sheer line is computed through a 3 steps method : 1) sheer line of a generic hull, 2) the real sheer line by « opening » the generic one of an half angle alfa with the bow end as the center of rotation (exactly, only the y values issued from the rotation are taken into account, the x values are kept unchanged), 3) in option, introduction of a scow shape influence.

**Reference points for the generic sheer line** :




**2,1) The generic sheer line** Yg is defined by this function valid for all Xlivar ≤ X ≤ Xbow :

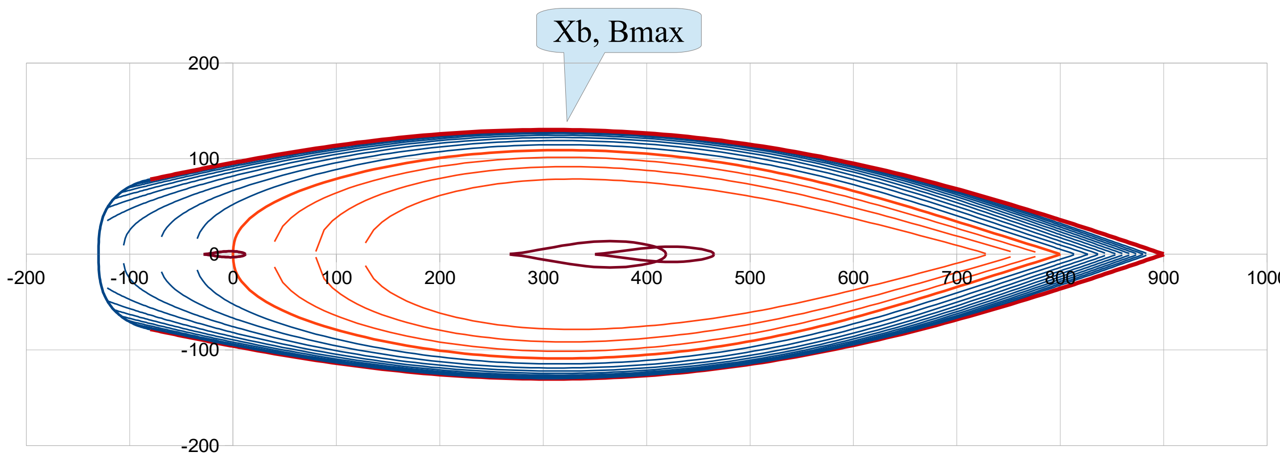
Yg (X) = Bg/2 – Abs(X - Xbg)^(Puilivy - Corpuiliv . (Abs(X -Xbg)/(Xbow - Xbg))^Puicorpui) / c

, where c = (Xbow – Xbg) ^(Puilivy – Corpuiliv) / (Bg/2)

**2,2) Then the real sheer line** Y is computed through :

Y (X) = Yg(X) + (Xbow – X) . sin(alfa) + Yg(X) . (1- cos(alfa))

**After this reformulation of Y, from which the maximum beam Bmax and its postion Xb can be computed. An example of the view after the opening alfa :**



**2,3) Introduction of a scow influence**, the Y of the sheer line becomes Ys as :

Ys(X) = Y (X) + Scow(X) . [Bmax/2 – Y(X)] . [(Xbow – X)/(Xbow – Xb)]^Puiscow

where Scow(X) = Scow . [1 – 0,5 . ((Xbow – X)/(Xbow – Xtabar))^2]

Nota : these scow influence does not changed Bmax and its location Xb

**The input data for these formulations :**

Geometrical data : Bg, Xbg, Xbow, Xtabar
Alfa rotation : alfa (°)
Adimensional parameters : Puilivy, Corpuiliv, Puicorpui, Scow, Puiscow

Remind : Bmax and its Xb position should be computed from the Y(X) function (search of a maximum and its location)

**Numerical check (geometrical data unit = cm) :**

| Input geometrical data : | Input adimensional parameters : |
|---|---|
| Bg = 189 | Puilivy = 2 |
| XBg = 384 | Corpuiliv = 0,02 |
| Xbow = 900 | Puicorpui = 2 |
| Xtabar = -130 | Scow = 0,6 |
| Input rotation Alfa (°) = 3,66 | Puiscow = 0,35 |

>>> computation of the coefficients :

| |
|---|
| c = 2410,19 |

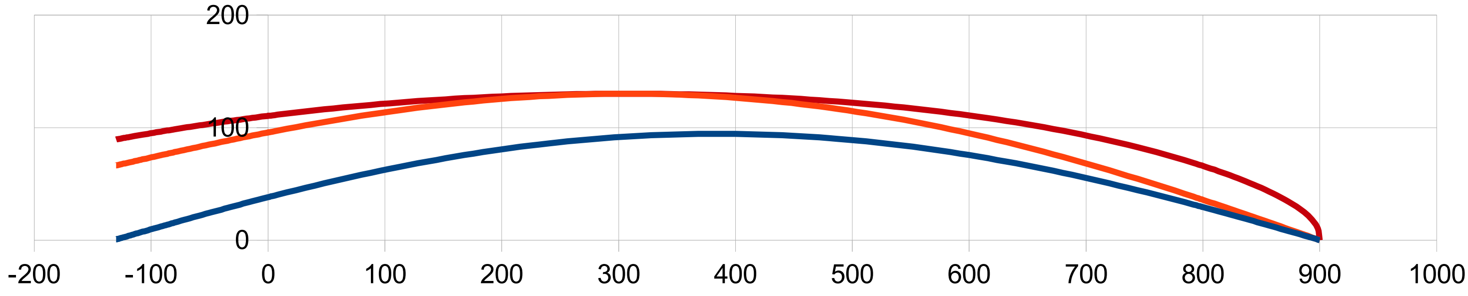
>>> computation of Bmax and Xb

| |
|---|
| Bmax = 260 |
| Xb = 304 |

>>> the sheer line is showed at each step : generic, after alfa rotation, after scow influence

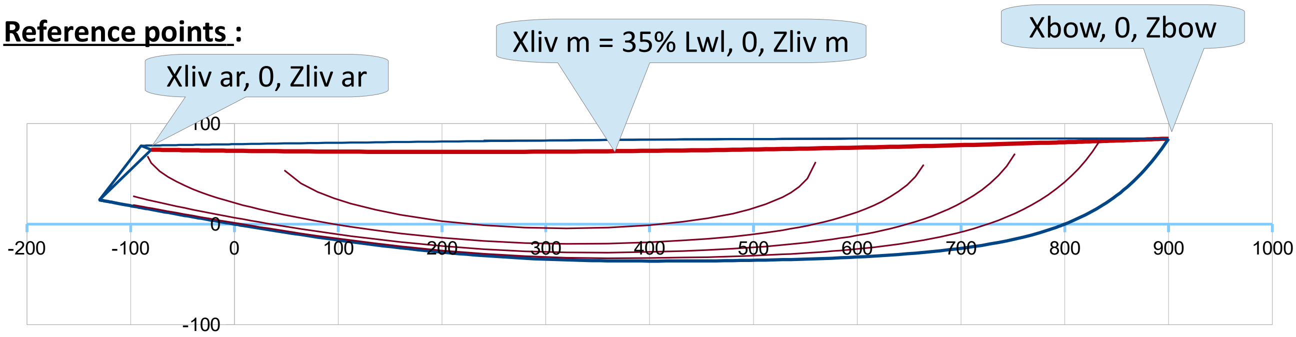
Sheer line (in projection in plan xy)

Blue : generic ; Red ; after alfa rotation ; Brown : with scow influence option



## 3. Sheer line / in its vertical projection xz

**Reference points :**




The sheer line in its vertical projection is defined by 3 reference points and a degree 2 polynome :

$$Z(x) = a \cdot X^2 + b \cdot X + c$$

Where

a = [(Zlivm – Zbox) . (Xlivar – Xlivm) – (Zlivar – Zlivm) . (Xlivm – Xbow)] / [(Xlivm^2 – Xbow^2) . (Xlivar – Xlivm) – (Xlivar^2 – Xlivm^2) . (Xlivm-Xbow)]
b = [(Zlivm – Zbow) – a . (Xlivm^2 – Xbow^2)] / (Xlivm - Xbow)
c = Zlivm – a . Xlivm^2 – b . Xlivm

The input data for this formulation :
Geometrical data : Xlivar, Zlivar, Xlivm, Zlivm, Xbow, Zbow

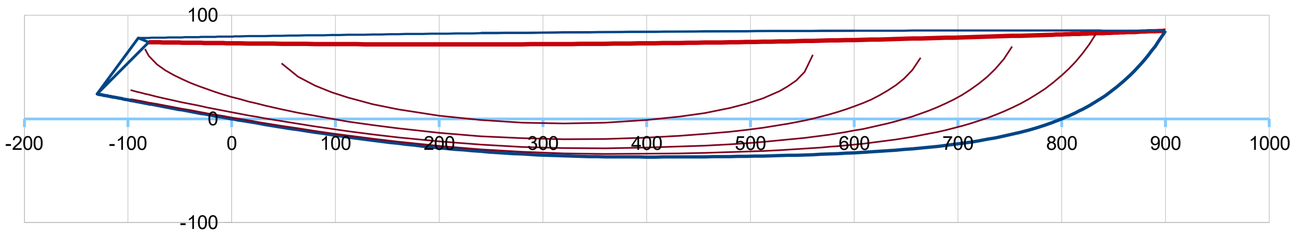
**Numerical check (geometrical data unit = cm) :**

| | | |
|---|---|---|
| Xlivar = - 80 | Xlivm = 280 | Xbow = 900 |
| Zlivar = 74 | Zlivm = 72 | Zbow = 85 |

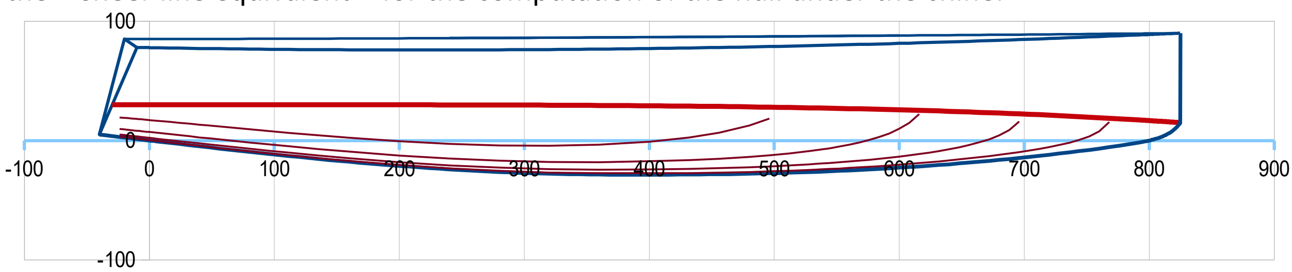
\>>> parameters :

| | | |
|---|---|---|
| a = 2,7065 E-05 | b = -1,0968 E-02 | c = 7,2949 E+01 |

\>>> sheer line with these data :




Nota : in case of a vertical chine as adopted in most modern designs, the hard chine line becomes the « sheer line equivalent » for the computation of the hull under the chine.

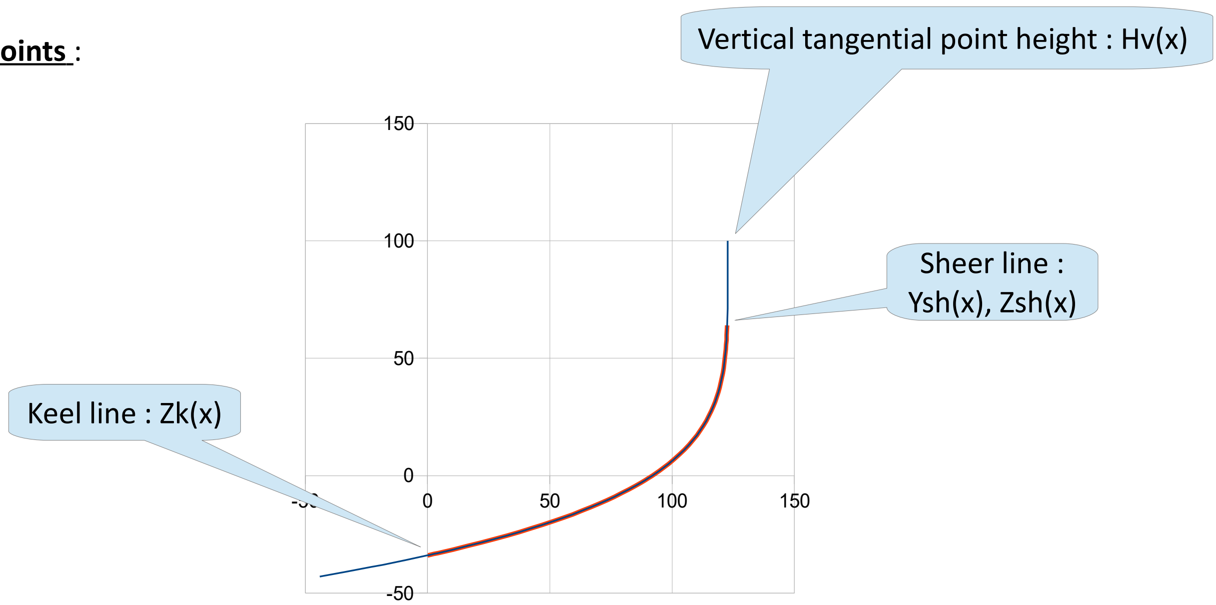


## 4. Sections

2 types of elementary sections are defined, « V » shape and « E » shape, and then a combination of V and E shapes is operated to define the real sections.

### 4.1 « V » shape sections

**Reference points** :




For any section at X, the reference points are the ones provided by the keel line (here renamed Zk (X)) and the sheer line (here renamed Ysh(x),Zsh(x)) which are the functions previously defined + an Hv(X) function, Hv being the tangential point of the V shape polynome.

The formulation is in the form of :

$$Yv(X,Z) = A(X) - Abs(Hv(X) - Z)^{Puiv(X)} / B(X)$$

, where :

$Hv(X) = [CHvm + (CHvar - CHvm)*[(Lwl/2 - X)/(Lwl/2 - Xtabar)]^{PuiHv}] . Zsh(X)$
for $Xtabar \leq X < Lwl/2$

$Hv(X) = [CHvm + (CHvav - Chvm)*[(X - Lwl/2) / (Xbow - Lwl/2)]^{PuiHv} ] . Zsh(X)$
for $Lwl/2 \leq X \leq Xbow$

$Puiv(X) = Puivar + (Puivav - Puivar)*[(X - Xtabar)/(Xbow - Xtabar)]^{[PuiPuiv - CorPuiPuiv*(X - Xtabar)/(Xbow - Xtabar)]}$
for $Xtabar \leq X < Xbow$

$B(x) = [Abs(Hv(x) - Zk(x))^{Puiv(x)} - Abs(Hv(x) - Zsh(x))^{Puiv(x)}] / Y1(x)$

$A(x) = [Abs(Hv(x) - Zk(x))^{Puiv(x)}] / B(x)$

**The input data for these formulations :**

Functions of X previously defined : Zk(X), Ysh(X) , Zsh (X)

Geometrical data : Lwl, Xtabar, Xbow

Adimensional parameters : CHvav, CHvm, CHvar, PuiHv, Puivav, Puivar, PuiPuiv, CorPuiPuiv

**Numerical check (geometrical data unit = cm) :**

Zk(X) : as defined in 1.

Ysh (X) : as defined in 2. , but with Scow = 0,03 and Puiscow = 0,25

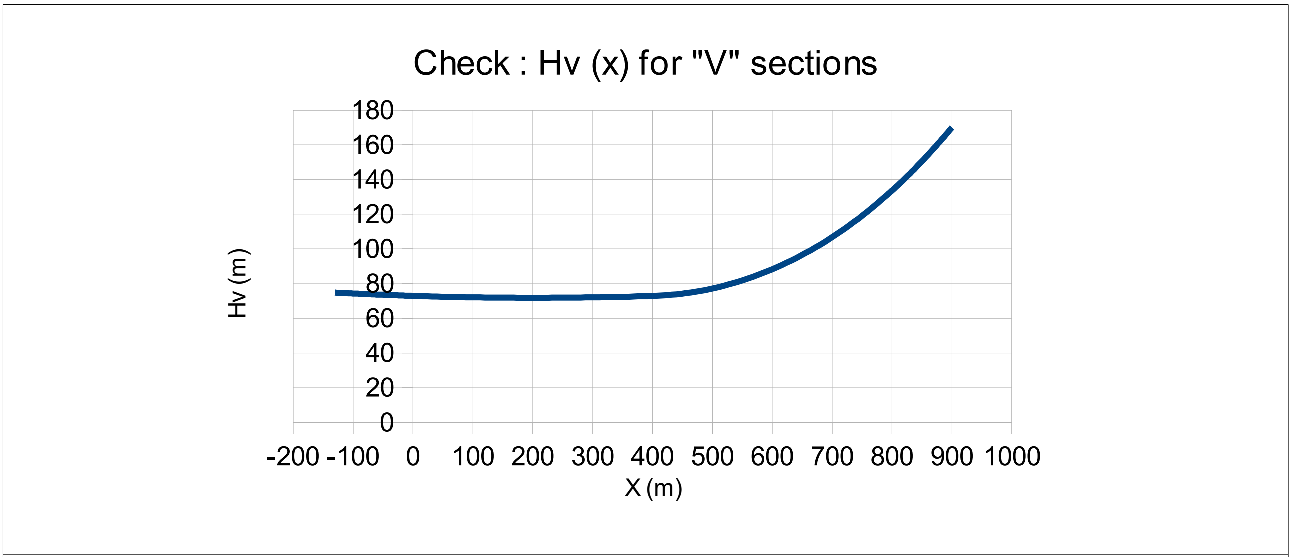
Zsh (X) : as defined in 3.

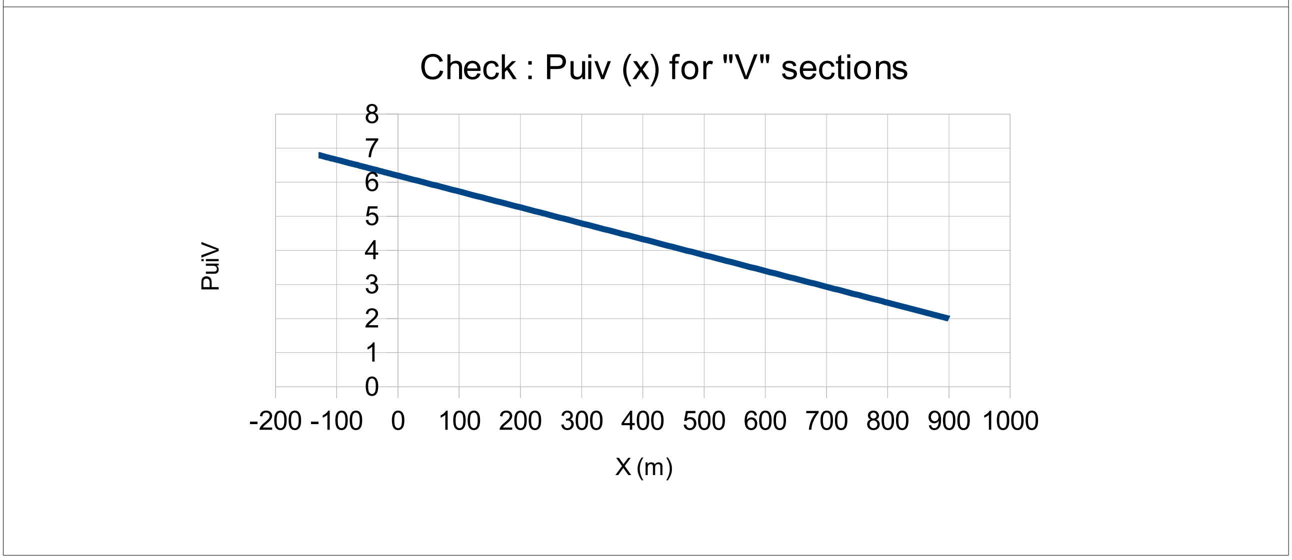
Lwl = 800 ;  Xtabar = -130 ;  Xbow = 900

Adimensional parameters :

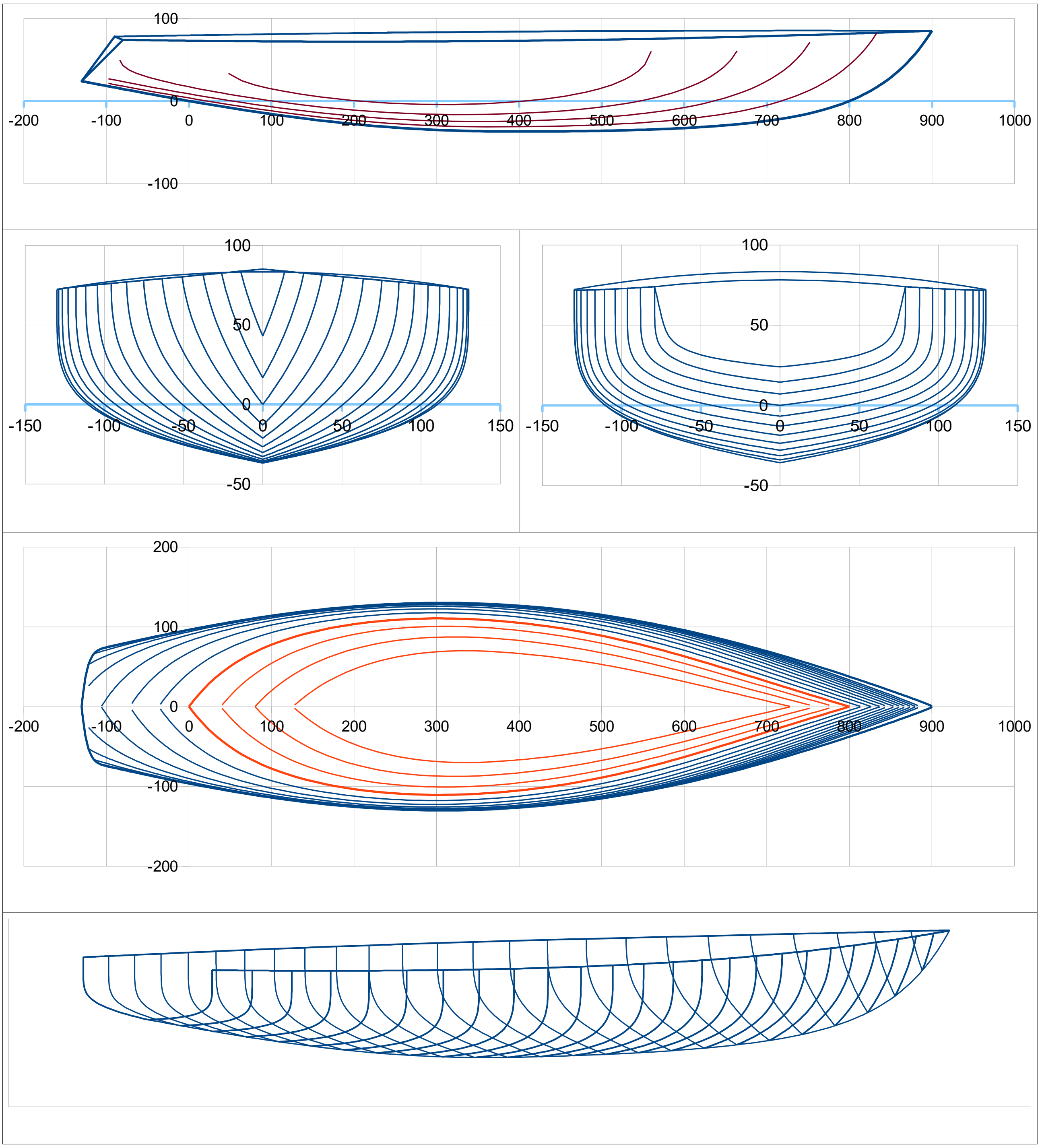
| | |
|---|---|
| **C Hv av** | **2,00** |
| **C Hv m** | **1,00** |
| **C Hv ar** | **1,00** |
| **Pui Hv** | **2,00** |
| **Pui V av** | **2,00** |
| **Pui V ar** | **6,80** |
| **Pui Pui V** | **1,00** |

Computation >>> Hv(X) and PuiV(X)



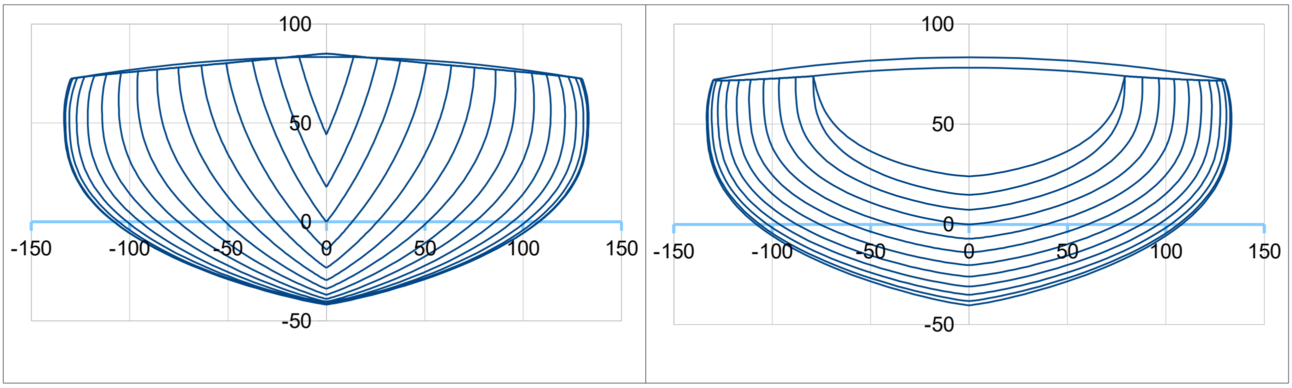



>>> 3 views 2D and isometric perspective of the hull with such full « V » sections :



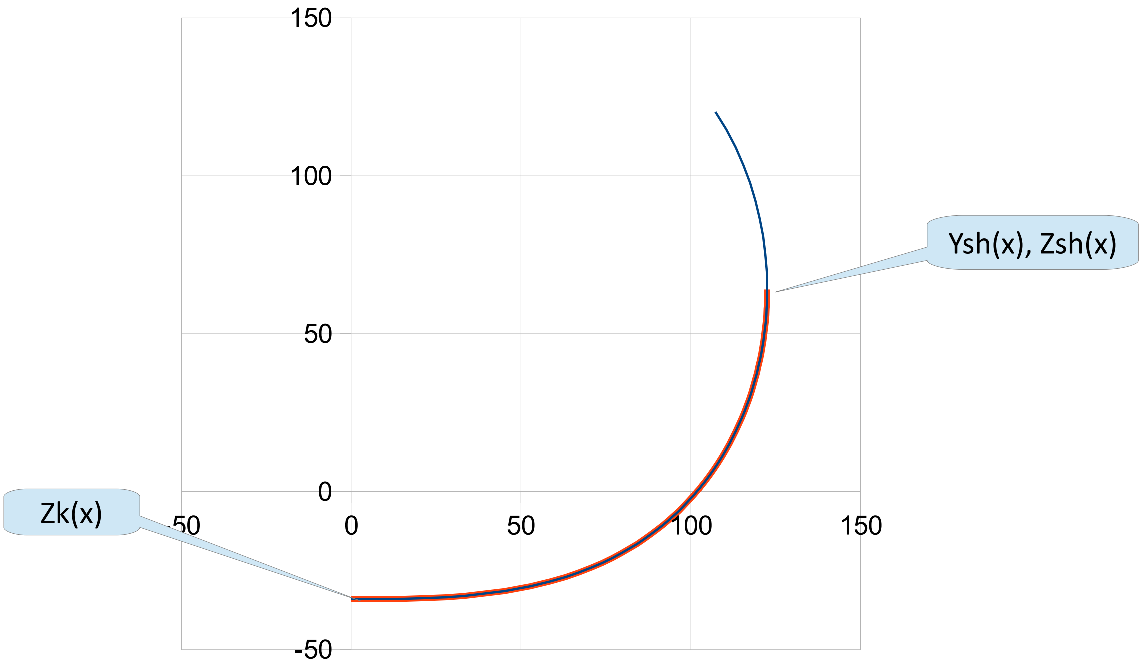

To note that the tangential point Hv(X) can be lower than the sheer point Zsh(X) by using Chv's parameters <1 (especially Chvm) , that introduces a tumblehome influence on the sections.

Example :




### 4.2 « E » shape sections

**Reference points :**




The formulation is in the form of :

$$Y_E(X,Z) = Y_{sh}(X) - [1- ((Z_{sh}(X) - Z)/(Z_{sh}(X) - Z_k(X))\text{^}PuiE1]\text{^}(1/PuiE2)$$

, where the new input adimensional parameters are **PuiE1** and **PuiE2**

**Numerical check (geometrical data unit = cm) :**

Zk(X) : as defined in 1.

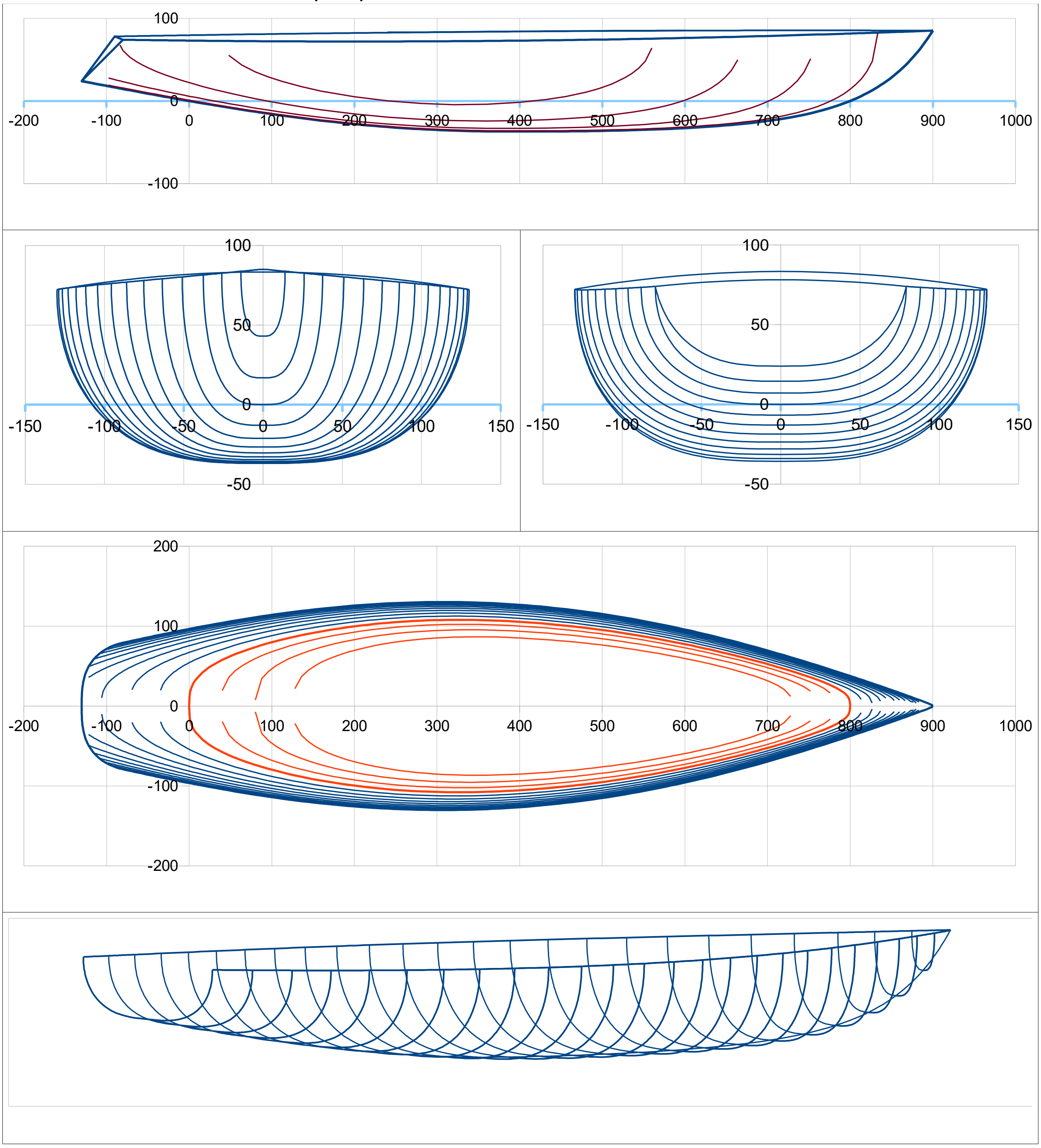
Ysh (X) : as defined in 2. , but with Scow = 0,03 and Puiscow = 0,25

Zsh (X) : as defined in 3.

Adimensional parameters :

| | |
|---|---|
| **Pui E1** | **2,100** |
| **Pui E2** | **3,00** |

>>> 3 views 2D and isometric perspective of the hull with such full « E » sections :




#### 4.3 Combination of shapes

The combination in function of X of the « V » and the « E » shapes is possible, giving « VE » sections through :

$$\text{VE sections :} \quad Y_{VE}(x,z) = Y_V(x,z)^{\wedge mix(x)} * Y_E(x,z)^{\wedge(1-mix(x))}$$

, with :

$$mix(x) = mixVEav + (mixVEar - mixVE_{av}) * [(Xbow - X)/(Xbow - Xtabar)]^{\wedge PuimixVE}$$

, where the new input data are the adimensional parameters :

mixVEav =  the fore end value, to input within (0,1)   1 means full V, 0 means full E

mixVEar = the rear end value,  to input within (0,1)

PuimixVE = the degree of the polynome

**Numerical check (geometrical data unit = cm) :**

Zk(X) : as defined in 1.

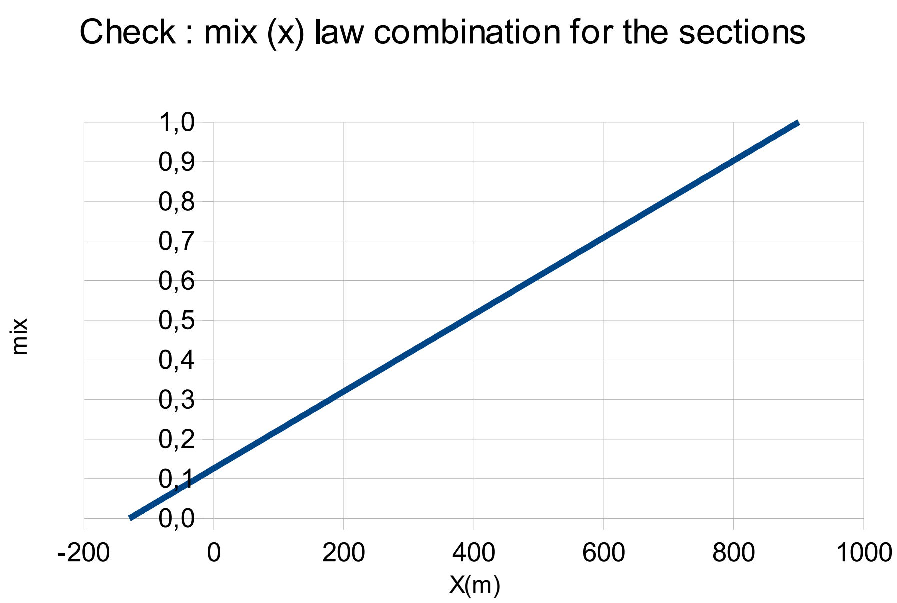
Ysh (X) : as defined in 2. , but with Scow = 0,03 and Puiscow = 0,25

Zsh (X) : as defined in 3.

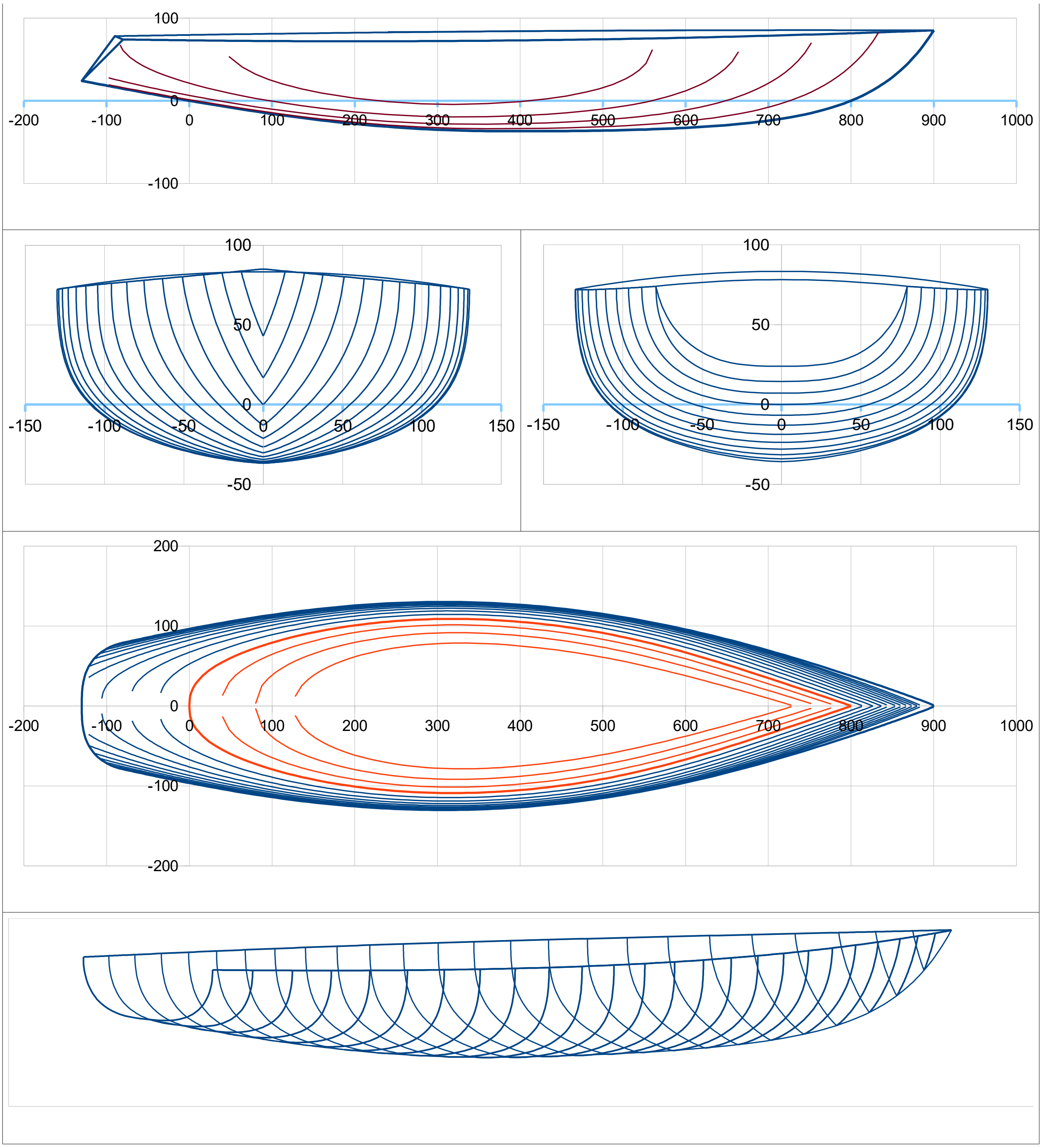
Adimensional parameters for the sections and their combination VE :

| | |
|---|---|
| **Sections V :** | |
| **C Hv av** | **2,00** |
| **C Hv m** | **1,00** |
| **C Hv ar** | **1,00** |
| **Pui Hv** | **2,00** |
| **Pui V av** | **2,00** |
| **Pui V ar** | **6,80** |
| **Pui Pui V** | **1,00** |
| **Sections E and combination VE :** | |
| **Pui E1** | **2,100** |
| **Pui E2** | **3,00** |
| **mix VE av** | **1,00** |
| **mix VE ar** | **0,00** |
| **Pui mix VE** | **1,00** |

>>> mix (X) :



>>> 3 views 2D and isometric perspective of the final hull with such combination « VE »
(it is the hull of the V1 boat given in the Examples document and in place in Gene-Hull Sailboat 3.0)




**In conclusion**, the required functions are defined :

Keel line : Zk(X)

Sheer line : Ysh(X), Zsh(X)

Sections : $Y_{VE}(X,Z)$

, from which it is possible to compute, e.g. within a spreasheet, all the hull usual contributions to the sailboat hydrostatics (Displacement, LCB, Cp, Swaterplane, Swetted, Shull, …), the equilibrium with a loading and with heel angle (Sinkage, Trim, Gz, Righting moment, Minimum free board, …).