VI-III – SIMPLIFIED METHOD

The method of calculation that we have just studied is long, even if relatively precise, and this precision is not always necessary, the more so as the safety coefficients applied offer a rather broad guarantee. In fact, I considered developing it primarily to show, by progressive analysis, the factors that constitute the total stresses on the mast and it's rigging.

In practice it will generally not be necessary to resort to it or, in the extreme cases, it will be preferable to use the lenient method with the proviso of introducing the correct data with regard to the loads exerted by the sails.

In the majority of the cases one will use a simplified method, development by the office of Sparkman & Stephens and whose most up to date version was published in 1982 in the report of the 12e symposium of the AIAA [2].

This method is applied by many architects with some alterations resulting from personal experience. It is completely adapted to the traditional cruising yachts with, for the masthead rigging, a ratio of areas of the genoa to mainsail in the neighbourhood of 2. It calls for some comments, shown by an index, which we will develop here.

The process is as follows:

a) Loads in the mast and rigging

P_t = (MR_{30} x 1.5)/e with:
P_t: vertical effort on chainplates and the mast due to the transverse rigging;
MR_{30} (1): righting moment at 30° heel with mid-displacement;
e (2): chainplate distance from the axis of the mast;
1,5: coefficient for maximum heel.

b) Compression in the mast

P = 1,85 x P_t where: 1,85: coefficient to take account of the longitudinal rigging. (3)

c) Burden-sharing in the various elements of rigging **(Table VI-II)** (4)

d) The Safety Coefficients for the rigging

Cables: cap-shrouds: 2,5 - 2,75;
Lower shrouds and stays: 3.
Rods: shrouds and cap-shrouds: 2,25 - 2,5
Stays: 3.
Chainplates: 4.

e) Masts and Rigging of the mizzen

Pt = (MR_{30}/e) x 1,5 x C with:
C = 1/3 for the normal yawls and ketches;
C = 1/2 for the ketches with large mizzen.
P = Pt
Coefficients safety of rigging
Yawls: shrouds 1,5, chainplates 2;
Ketches: shrouds 2, chainplates 2,5.

f) Moment of inertia of the masts

$I_{cm}{}^4 = (Cs \times P \times L^2) / (K \times \pi^2 \times E) = C \times P \times L^2 \times 10^{-8}$ with
P: compression on the mast in daN;
L: length of the panel of mast in cm;
C: coefficient given in Table VI-III.

TABLEAU VI-II

RÉPARTITION DES EFFORTS DANS LE HAUBANNAGE TRANSVERSAL

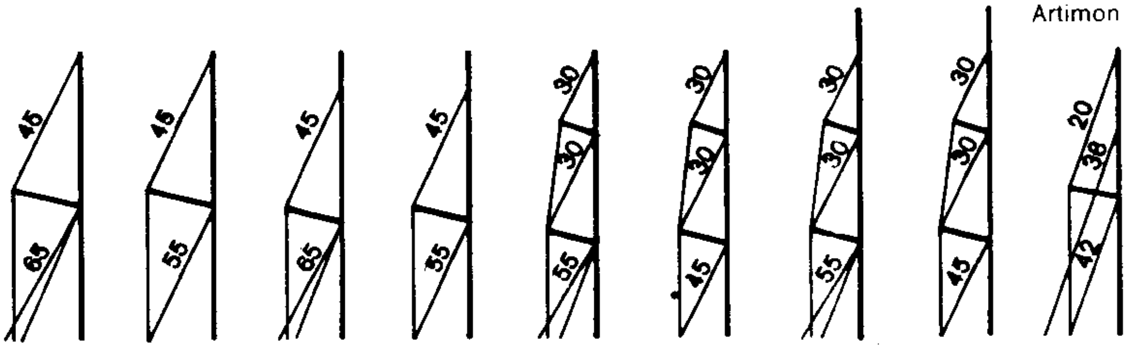

TABLE VI-III

VALUES OF ‘C’ FOR A LIGHT ALLOY MAST STEPPED ON THE KEEL

	Euler Coefficient, k	TRANS-VERSAL		LONGITUDINAL						
		Single Spreader	More than one Spreader	L to the stay			L to the stay			
				All Lengths $L_f < 10$ m with releasable baby stay			$10 \text{ m} < L_f < 14$ m			$L_f > 14$ m
				Mast Head		7/8	Mast Head		7/8	Mast Head
				Double Lowers	Single Lower and baby stay	Double Lowers	Double Lowers	Single Lower and baby stay	Double Lowers	Double Lowers
C_s		2.1	2,5	1,2		1,15	0,8		1,15	
Lower Panel	2,25	1,34	1,61	0,77	0,64	0,74	0,51	1 spreader 0.51	0,74	0,43
Upper Panels	1,5	2,16	2,4					2 spreader 0,43		

Note:

- For rigging with only one lower in the plane of the mast and without baby stay, to add 10 % to the longitudinal moment of inertia calculated for double lower-shrouds.
- For a yacht of more than 14 m of waterline length with a releasable baby stay take L of the rigging to the baby stay with C = 0,45.
- For a deck stepped mast, one uses an Euler coefficient lower by 20 %:

C deck stepped = 1,25 x C keel stepped.

- For a mast in spruce $I_{SP} = I_{alu}/0,139$. (5)

g) Forestay (6)

The graph of Figure VI-23 directly gives the diameter of the forestay according to MR_{30}/h where 'h' is the vertical distance between centre of sail and centre of the immersed canoe body, this being taken to be 40 % of the draft.

The strength of the cable or the rod is counted to 125 daN/mm^2 [=1250 MPa = to Dyform] on the nominal diameter.

A reduction is not envisaged for a material of higher strength.

The ends must have a strength 5 % greater than that of the cable.

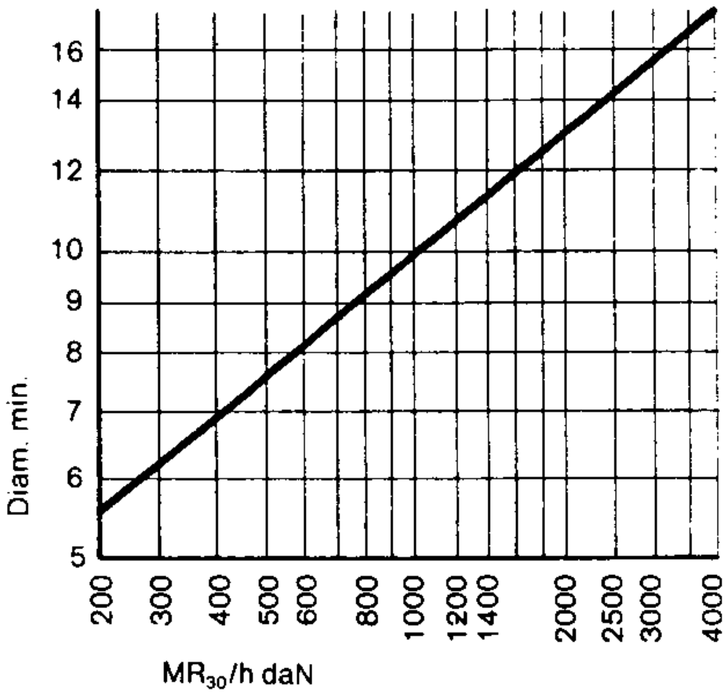

Fig. VI-23. Diamètre de l'étai en fonction de MR_{30}/h, basé sur une résistance à la rupture R_r = 125 daN/mm², la section étant calculée à partir du diamètre extérieur, qu'il s'agisse de câble 1 × 19 ou de monofil.

h) Running Rigging

For the halyards of the jib and mainsail, Stephens gives a table of correspondences with the dimensioning of the forestay. Table VI-IV is the metric adaptation to which were added the sheets of the genoa and of the mainsail as well as the halyards, spinnaker brace and sheets. It is valid for cruising boats having unit surfaces of mainsail, maximum genoa and spinnaker in ratios close to 1, 2 and 4.

The breaking strengths in daN/mm^2 indicated correspond to the Sarma cables, the ropes Glenstein Cup for the halyards and the spinnaker braces, with the ropes Glenstein Gemini for sheets.

One will be able to also refer to the tables of the pages 346 and 347 of Sails and Rigging (éd. of 1982), in particular for the correspondence with Kevlar ropes.

For the ropes, strength alone is not taken into account because one cannot go down below a certain diameter if one wants to preserve a correct handhold; the diameter of 8 mm is then an extreme minimum. As for the diameter of the sheet of the mainsail it depends of course on the purchase of the tackle; the diameter indicated here would be that of a direct load (1:1), without purchase.

TABLEAU VI-IV
CORRESPONDANCES BETWEEN STAYS, HALYARDS AND SHEETS

Stay - 1 x 19		Genoa and Spinnaker Halyard		Mainsail Halyard		Genoa sheet	Mainsail sheet	Spinnaker sheet	Spinnaker Brace	
∅	R_f	7 X 19	Cup	7 X 19	Cup				7 x 19	Cup
19	27000	11,2								
15,9	22000	9,6	18	8	16	16	16	16	9,6	18
14,3	17800	8	16	8	16	16	16	16	8	18
12,7	13600	8	16	8	16	14	16	16	8	18
11,2	10800	8	16	6,4	16	14	14	14	8	16
9,6	8200	6,4	14	5,6	14	12	14	14	-	16
8	5800	5,6	14	4,8	14	12	12	12	-	14
7	4650	4,8	12	4	12	10	10	10	-	12
6,4	3650	4	10	3,2	10	8	8	8	-	10
5,6	2850	4	10	3,2	8	8	8	8	-	10
4,8	2050	3,2	8	2,4	8	8	8	8	-	8
4	1450	3,2	8	2,4	8	8	8	8	-	8
3,2	900	2,4	8	2,4	8	8	8	8		

BREAKING STRENGTH OF THE CABLES AND ROPES in daN/mm^2

Cables mm	2,4	3,2	4	4,8	5,6	6,4	8	9,6
Cable 7 x 19	350	660	950	1350	1900	2500	3900	5700

Cordages mm	6	8	10	12	14	16	18
Glenstein Cup	565	1040	2040	3120	3920	5780	6900
Glenstein Gemini	755	1160	2270	3140	4000	4890	6200

[

VI-III - 1. - REMARKS ON SIMPLIFIED METHOD (1)

(1) We know that the righting moment is equal to the moment of capsizing, itself produced from F_y x H, lateral force of the sails by the vertical distance of the centres of the sails and immersed canoe body. However, the moment of the rigging compared to the horizontal plane of deck collar is F_y x h', h' being the height of the centre of the sails above the deck collar. One can think that there is the need to introduce a certain safety coefficient or to compensate for other additional components increasing the moment of the rigging loaded on the chainplates, such as the moment exerted by the vertical component of the jib sheet load, or the weight of the mast.

(2) The cap-shrouds and the lower shrouds do not always lead onto isolated chainplates on the side of the hull. In this case it is necessary to calculate an 'e' intermediate such as:

$e = (\sum C_{cap}\ e') + (C_{low}.\ e'')] / [(\sum C_{cap} + C_{low})]$ where

C is the percentage of P_T corresponding to the stay concerned and given in table VI-II;
e' and e'' are the respective spacings of the chainplates of the cap-shroud and the lower shroud.
For example if, in a rig with two sets of spreaders, the chainplate of lower is e'' = 1,25 m and that of the cap-shroud is with e' = 1,20 m

e = [(30 + 30) x 1,20 + 45 x 1,25] / (30 + 30 + 45) = 1,22 m

(3) The coefficient 1,85 appears a little small even for yachts where one seeks a minimum angle of the forestay and it does not seem to take account of the load of the return of the halyards. One can thus change it to 2. In the example analysed previously it reached 2,05 by including the returns of the halyards.

(4) For the lower shroud, it should not be forgotten that it acts off the vertical component of the load on the chainplate, the resultant in the stays will be thus F/cos α, α being the angle of the stay with the mast in the transverse plan.
In the case of double lowers, the forward lower has a higher load than that of the aft lower since it balances the aft load of the sail, a role filled alternatively by the baby stay. Nevertheless as the safety coefficient of the aft lower must be higher, one will also share the load on the two lowers. The larger coefficient compared to the single lower probably takes account of the increase due to their angulation in the longitudinal plane and to the greater variations due to the dynamic effects.
The table does not give the loads in the intermediate fore stay. One will be able to estimate that the longitudinal loading that they have to balance will be equal to:

- for the baby stay: 1,3 % of the load of the lower shroud, -
- for the intermediate stay : 2,4 % of the load of the intermediate shroud.

If there is only one baby stay, between the two sets of spreaders, as in our example, one will take the sum of the two preceding loads.
Finally it should be noted that these coefficients do not take account of the respective lengths of the spreaders and the spacing of the chainplates. One can thus think that one considers these a constant dimension. Under other conditions one indeed notes transfers of the load of one shroud to the next, which partly explains the differences in the coefficients between the cap-shroud and the lower shroud while there was not much in the example studied, the undervaluation of the proportion of the loads due to the forestay representing the balance of the difference.

(5) For a material of a modulus of elasticity E' different from light alloy one has

$I_{E'} = I_{alu}$ x 7000 / E'

Here, the modulus of elasticity of the spruce is taken for 975 daN/mm^2.

(6) The value of the load is not given for the backstay. One will be able to use 70 % of the forestay, in the same way for runner balancing the forestay in a 7/8 rig. For this one will be able to take account of the difference in angle.

VI-III - 2. - APPLICATION TO THE PRECEDING EXAMPLE

Let us point out the characteristics necessary for the simplified calculation and compare with the preceding results.
RM_{30} = 3360 daN-m;
H = 6,61 m;
e = 1,26 m

a) P_T = 3360 x 1,5 / 1,26 = 4000 daN

b) P = 4000 x 1,85 = 7400 daN against 11859 is, for this last an initial safety coefficient of 1,6 compared to the simplified method.

c) Load in the stays:
Cap-shroud: (2,75 x 4000 x 30) / (100 cos 13,74°) = 3397 daN, against 4103,4 daN
Intermediate Shroud: (2,75 x 4000 x 30) / (100 cos 14,58°) = 3410 daN, against 4010,4 daN
Lower-shroud: (3,00 x 4000 x 45) / (100 cos 14,03°) = 5566 daN, against 4689,6 daN
The differences, less on the upper shrouds and more on the lower shroud, correspond to the transfers of load that occurs with the progressive shortening of the spreaders.
The totals are appreciably equal with: 12373 daN against 12803,4 daN.
Fore Stay:
RM_{30} / 6,61 = 508,72 giving, according to graph VI-23, a diameter of 7,7 correspondent to a load of 5820 daN, against 5834 daN.
Baby Stay:
[3 x 4000 x 45 x 1,3 / 100^2] + [2,5 x 4000 x 30 x 2,4 / 100^2] = 142,2 / sin 15° = 549,4 daN, against 827 daN
The diameter of the most similar 1 x 19 cable will be 8 mm for the whole of the shrouds and the forestay, 7 mm for the backstay and 3,2 for the baby stay.

d) Moments of inertia of the mast (Table VI-III)
Transverse:
Lower panel: 1,61 x 7770 x 430^2 x 10^{-8} x 1,25 = 289 cm^4
Higher Panels: 2,4 x 7770 x 415^2 x 10^{-8} = 321 cm^4
Longitudinal:
Lower panel: 0,64 x 7770 x 1280^2 x 10^{-8} x 1,25 = 1018 cm^4
Against 380 and 730 for the selected profile.
It is noted that the longitudinal moment of inertia is stronger than for the selected profile. But a coefficient 0,64 relates to only the rigging with one set of spreaders, but we here have two of them. If we take, as for the boats from 10 to 14 m, a coefficient of 0,51, the moment of inertia falls to 811 cm^4, which is much closer. The difference confirms the higher stress in the longitudinal direction already noted.
In fact one notes that the majority of the profiles of the trade have a ratio of the moments of inertia in the neighbourhood of 2.
For flexible masts working in compression and inflection, and either in buckling, the moments of inertia of the profiles should be closer to a ratio = 1.

e) Running Rigging
From the diameter of the forestay (8 mm), table VI-IV gives us:

Genoa and spinnaker halyards:	cable 7 x 19 - 5,6∅,	and rope - 14∅;
Mainsail halyard:	cable 7 x 19 - 4,8∅,	and rope - 12∅;
Sheets:		12∅;
Spinnaker Brace:		14∅.

It is quite certain that these diameters of the sheets correspond to the maximum of the heaviest sails and that other sizes must be planned for the lighter sails.

VI-III - 3. - PROCTOR GRAPHIC

The mast manufacturer, Proctor, established a graph allowing the quick determination of the profile type to use according to two characteristics of the yacht: the floatation length and the height of the fore-triangle. I recalled it as from [Je l'ai retracé à??] the transverse moments of inertia. The longitudinal moment of inertia is almost double the transverse (fig. VI-24).
This graph is practical for a quick determination, at the level of the draft for example, but is not sufficient, in my opinion, when one requires a precision or better-defined safety.

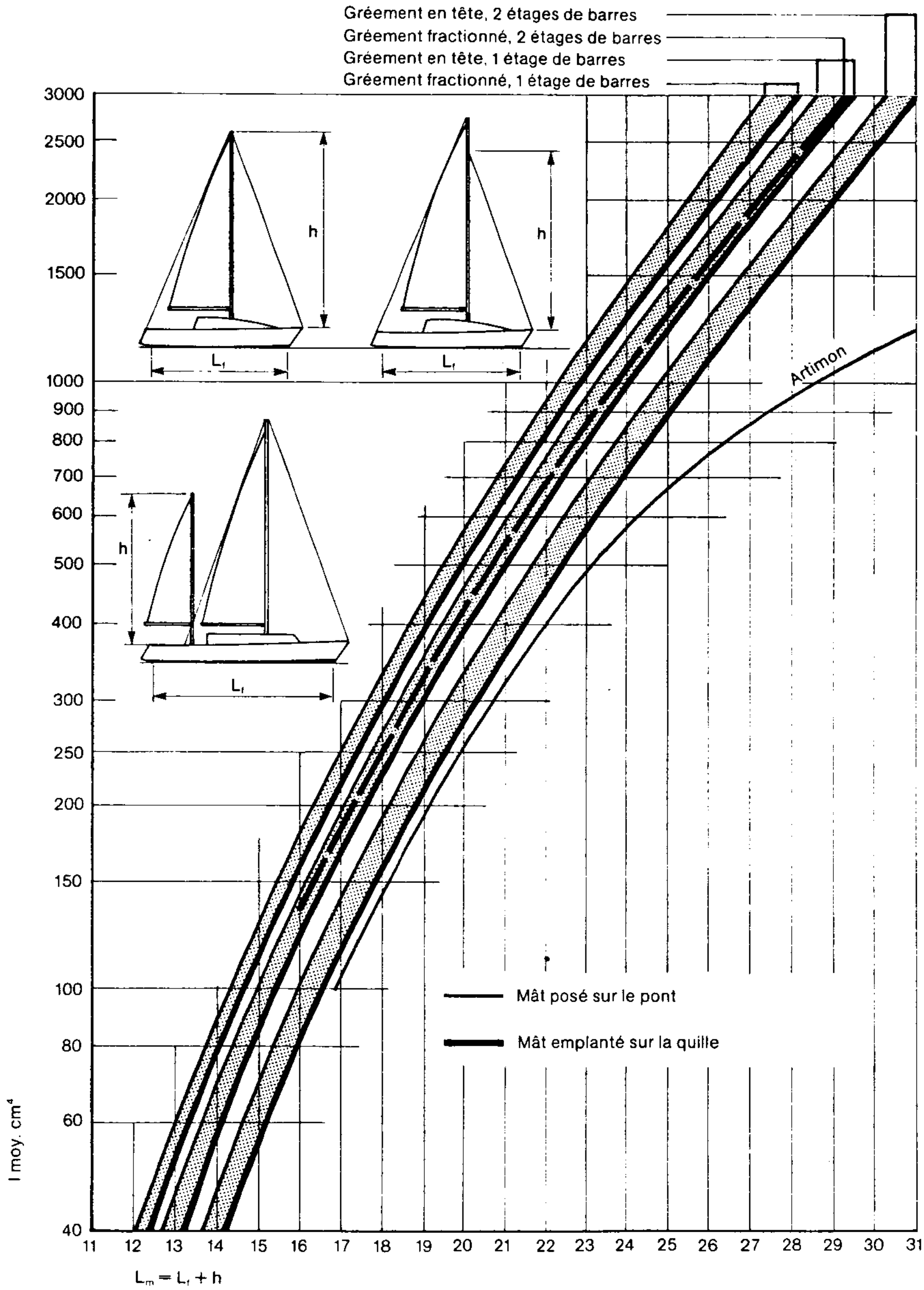

Fig. VI-24. Graphique donnant une première approximation du moment d'inertie moyen $I_{moy} = \frac{I_x + I_y}{2}$, *en fonction de la somme de la longueur de flottaison et de la hauteur du capelage de foc ou du mât d'artimon.*

VI-III - 4. - MIZZEN MAST

The force developed by the sail having been determined from the distribution coefficients defined at the beginning of this chapter the calculation is the same as the one of a big mast without a jib. In the simplified method one has

$P_T = [(MR_{30} \times 1{,}5) / (e \times C)]$ with

C = 3 for a yawl; C = 2 for a ketch with a big mizzen.

The safety coefficients are the following:

Yawls: shrouds 1,5; chainplates 2;

Ketches: shrouds 2, chainplates 2,5.

The loads in the shrouds are valued at:

20% of P_T for the Cap-shroud;

42% of P_T for the lower shroud;

38% of P_T for the intermediate fore stay.

VI-III - 5. - CALCULATION OF THE BOOM

The boom is held at the gooseneck, to the point of attachment of the mainsheet and to the point of attachment of the vang.

The first analysis suggested is that the normal effort to the boom followed a triangulated distribution with a maximum at the point at the clew and 0 to the tack. The cutting of the modern large sails moves the load distribution a lot more towards the clew. On the other hand, the main sheet constitutes a more and more important fixing point especially, of course, as soon as it is separated from clew.

To take account of these considerations one will define the boom as a beam working in bending under the effort of a resultant situated 4/5 of its length from the gooseneck (0,8 E) and subjected to a compression axial force applied at the clew. We will estimate that the moment generated by this force is balanced instrumentally by the moments generated by the mainsheet and the vang.

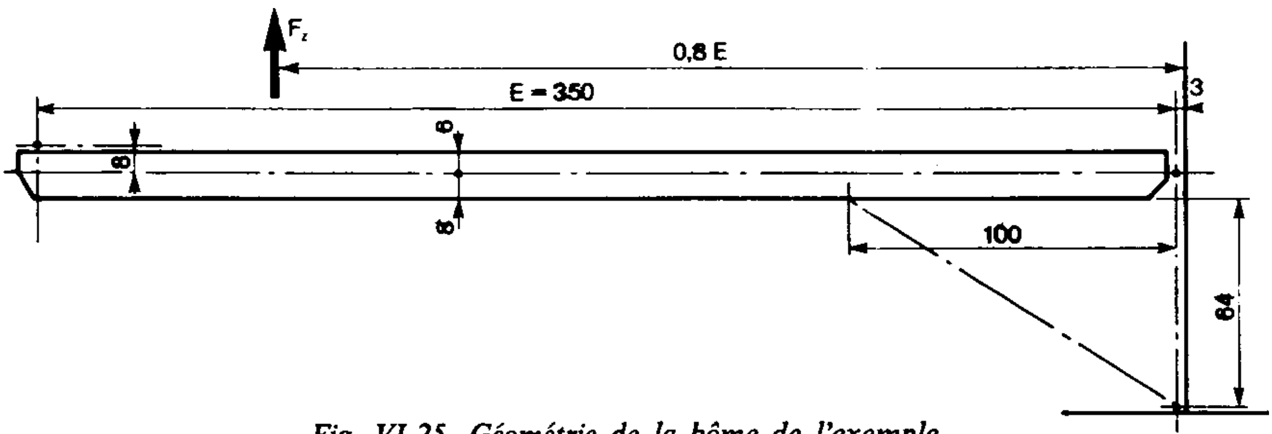

Fig. VI-25. Géométrie de la bôme de l'exemple.

In certain cases these conditions can be again harder, as for example for a big transocean sailer, for which one will be able to consider that the mainsheet can be brought to bear the entire the effort of the sail.

Having determined the value of F_y of the main sail by the method established in section V-II-3, we estimate the forces at:

$3\ F_{yg\text{-}v} \cos \alpha = F_z$

$3\ F_{yg\text{-}v} \sin \alpha = F_x$

with $\alpha = \tan^{-1}(E/P)$

For our example (fig. VI-25) will have us with

$E = 353$ cm, $e = 100$ cm and $\alpha = 17{,}07°$

$F_z = 3 \times 236{,}6 \cos 17{,}07° = 678{,}5$ daN

$F_x = 3 \times 236{,}6 \sin 17{,}07° = 194{,}5$ daN

The moment of F_z in comparison with the gooseneck will be:

$M = F_z \times 0{,}8 \times E = 678{,}5 \times 0{,}8 \times 350 = 189980$ cm-daN

and the vertical load on the vang:

$F_{h\text{-}b} = 189980 / (2 \times 100) = 950$ daN

This effort will generate an additional compression in the forepart the boom:

$C = 950 \times 100 / 64 = 1484$ daN

Under the effect of the load of the vang, the boom will bend. The maximum moment bending will be situated in the point of attachment of the vang and will be:

$M_{f\,max} = F_{h\text{-}b}\ (0{,}8\ E - e) \times e / (0{,}8\ E)$

$= 950\ (0{,}8 \times 350 - 100) \times 100 / 0{,}8 \times 350$

$= 61071{,}4$ cm-daN

With $R_p = 2400$ daN/cm^2 one will have to have:

$I/v \geq 61071{,}4 / 2400 = 25{,}45$ cm^3.

The profile F 340 of Francespar will give us the following characteristics:

- dimensions: 101 x 140 mm
- section: 13,7 cm^2
- Iy = 150 cm^4
- Iz = 370 cm^4

The clew attachment will be at about 8 cm above the centreline, giving an $I_z/v = 370/8 = 46{,}25$ cm^3 for the underside, working in tension, and $I_z/v = 370/6 = 61{,}67$ cm^3 for the topside, working in compression.

The critical load that the boom can bear will be:

$P_{cri} = \pi^2 \times 700000 \times 370/350^2 = 20867$ daN

The bending moment applied to the boom will be the sum of the due moments to the vertical load and to the off centre position of the compression load.

This off centre position will be equal to the distance between the point of clew and the horizontal axis of the gooseneck, in our case about 8 cm. One will therefore have:

$M = 61071{,}4 + (194{,}5 \times 8) = 62627{,}4$ cm-daN

Using the same calculation method as for the mast, one will have:

$\sigma_{max} = [194{,}5 / 13{,}7] + [62627{,}4 / [46{,}25\ (1 - 194{,}5 / 20867)]] = 1381$ daN/cm^2 for the underside and

$\sigma_{max} = [194{,}5 / 13{,}7] + [62627{,}4 / [61{,}67\ (1 - 194{,}5 / 20867)]] = 1039{,}3$ daN/cm^2 for the topside, with at least a coefficient of 1,9 in comparison with elastic limit of the alloy.

The loads on the boom situated between the point of attachments of the vang and gooseneck is subjected to an additional compression.

The point of application of this load being situated underneath the section will induce a bending moment opposite to the direction of the compression from the clew. The section concerned being, moreover, rather short, the generated stresses will be compression only and equal to P/S, that is to say in our example:
$\sigma c = 1484 / 13,7 = 108,3$ daN/cm^2
that be added to add to σ_{max} of the top of the boom to give 1039,3 + 108,3 = 1147,6 daN/cm^2 while the stress on the bottom part will be reduced to be: 1381 - 108,3 = 1272,7 daN/cm^2.

In the transverse direction the normal efforts on the boom are weak but can accidentally become large, for example, with the aft wind, if it engages on a roll and the mainsheet withholds it. These loads being difficult to determine, one will take for the transverse moment of inertia of the boom between 0,5 and 0,6 of the vertical moment of inertia.
The bending moment diminishes almost linearly from the point of attachment of the vang towards the clew; it is tempting to reduce the boom by the cutting of openings in its sides. These will have to be progressively increased in width and the remaining alternate braces will have to be inclined to 45° to withstand the shearing loads (fig. VI-26).

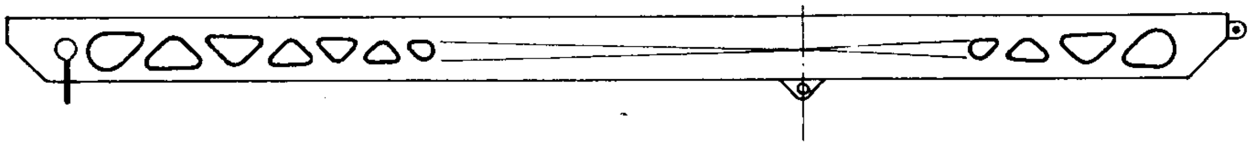

Fig. VI-26. Allègement de la bôme.