

9 Example: Computing the mechanical properties of a composite (by hand)

In this section we present an example of the computation of the mechanical properties of a monolithic laminate, using glass fiber E, formed by MAT 300 and Roving 800, on an epoxy matrix. The content of fibre in mass of the MAT 300 fibre is $\psi^M = 0,30$, and the content of fibre in volume of the ROV 800 is $\psi^R = 0,478$. The calculations are performed using the formulae in the ISO 12215-5. A diagram indicating the different parameters that are computed and the dependencies between them is shown in Figure 31.

		E glass	...	Matrices (Polyester/epoxy)
Specific gravity (ρ_f or ρ_m)	t/m^3	2,56	1,2
Elastic Modulus, E	N/mm^2	73000	...	3300
Shear modulus, G	N/mm^2	30000	...	1222
Poisson's ratio ν	1	0,22	...	0,32

Table 2: Values in Table C.4 of the ISO 12215-5 Norm for the E glass fiber and the epoxy matrix.

Table 2 presents some of the values given by Table C.4 of the ISO 12215-5 Norm. In this table we give the mechanical properties of the glass fiber E and the epoxy matrix. For any of these parameters, the sub-index “f” will refer to “fiber”, and the sub-index “m” will refer to matrix. Let us recall that the fiber is the fiber glass E and the matrix is the epoxy resin, in this example. Also, the student will note that the ISO 12215-5 Norm distinguishes between E_{f1} and E_{f2} . This differentiation serves to take into account the cases when different mechanical properties have to be considered depending on whether the direction is axial or transverse. When there is no such difference, as it is the case in this example, these sub-indexes will be avoided.

These values will be used in the calculations that follow.

9.1 Mechanical properties of MAT 300

We will first obtain the mechanical properties of the MAT 300 ($w = 0,3 \text{ kg/m}^2$). Once the values of (ρ , E , G , ν) have been obtained from Table C.4 of the ISO 12215-5 Norm, the **volumetric amount of fiber in the laminate** ϕ , the **composite density**, ρ_c and the t/w ratio are obtained from Table C.2 of the Norm. That table provides the following formulae for these parameters:

$$\phi = \frac{\psi}{\psi + (1 - \psi) \frac{\rho_f}{\rho_m}}, \quad (17)$$

$$\rho_c = \rho_f \phi + \rho_m (1 - \phi), \quad (18)$$

and

$$\frac{t}{w} = \frac{1}{\phi \rho_f}. \quad (19)$$

Applying these equations with ψ_M we get

$$\begin{aligned} \phi^M &= 1,67; \\ \rho_c^M &= 1,427 \text{ kg/m}^3; \\ t/w^M &= 2,335 \text{ mm/(kg/m}^2) \end{aligned}$$

Now, these values are used to obtain some intermediate parameters, prior to the calculation of the mechanical properties of the material. To do so, the formulae in Table C.5 of the Norm is applied. In particular, we use the

following formulae, considering $\zeta = 1$:

$$E_{UD1} = 0,975 (E_{f1} \phi + E_m(1 - \phi)), \quad (20)$$

$$E_{UD2} = E_m \frac{1 + \zeta \eta_E \phi}{1 - \eta_E \phi}, \quad (21)$$

$$\eta_E = \frac{E_{f2}/E_m - 1}{E_{f2}/E_m + \zeta}, \quad (22)$$

$$\eta_G = \frac{G_f/G_m - 1}{G_f/G_m + \zeta}, \quad (23)$$

$$G_{UD12} = G_m \frac{1 + \zeta \eta_G \phi}{1 - \eta_G \phi}, \quad (24)$$

$$\nu_{UD} = \nu_f \phi + \nu_m (1 - \phi) \quad (25)$$

$$\nu_{UD21} = \nu_{UD12} \frac{E_{UD2}}{E_{UD1}} \quad (26)$$

$$\tau_{UIL} = 22,5 - \frac{33\phi}{\phi + 0,89} \quad (27)$$

The parameters E_{f1} and E_{f2} are the same in our example, corresponding to $E_{f1} = E_{f2} = E_f$. Considering these formulas together with the values that we have calculate so far for the MAT 300, we get:

$$E_{UD1} = 14580 \text{ N/mm}^2;$$

$$E_{UD2} = 4490 \text{ N/mm}^2;$$

$$\eta_E = 0,913;$$

$$\eta_G = 0,922;$$

$$G_{UD12} = 1667 \text{ N/mm}^2;$$

$$\nu_{UD} = 0,303;$$

$$\nu_{UD21} = 0,0934;$$

$$\tau_{UIL} = 17,28.$$

After obtaining these values, we need to have a look at Table C.5-2 of the Norm and apply the formulas presented there. In particular, we need to use:

$$E_{CSM} = \frac{3}{8} E_{UD1} + \frac{5}{8} E_{UD2}, \quad (28)$$

$$G_{CSM} = \frac{1}{8} E_{UD1} + \frac{1}{4} E_{UD2}, \quad (29)$$

$$\nu_{CSM} = \frac{E_{CSM}}{2G_{CSM}} - 1. \quad (30)$$

With the values given above we finally get the mechanical properties of the fiber MAT 300:

$$E_{CSM} = 8276 \text{ N/mm}^2;$$

$$G_{CSM} = 2946 \text{ N/mm}^2;$$

$$\nu_{CSM} = 0,405.$$

At this point, we get the fracture strains from Table C.6 of the ISO 12215-5 Norm (see table 4):

$$\varepsilon_{ut} = 1,35\% = 0,0135;$$

$$\varepsilon_{uc} = 1,70\% = 0,0170;$$

$$\varepsilon_{uf} = 1,88\% = 0,0188;$$

$$\gamma_u = 2,00\% = 0,0200.$$

9.2 Mechanical properties of ROV 800

Now, let us repeat the process for the roving 800 ($w = 0,8 \text{ kg/mm}^2$). First, taking into account the density of the fiber: $\rho_f = 2,56 \text{ t/m}^3$.

$$\begin{aligned}\psi &= 0,478; \\ w &= 0,8 \text{ kg/mm}^2; \\ \rho_f &= 2,56 \text{ t/m}^3.\end{aligned}$$

From the Table C.4 of the ISO 12215-5 Norm, we get:

$$\begin{aligned}E_{f1} &= 73000 \text{ N/mm}^2; \\ E_{f2} &= 73000 \text{ N/mm}^2; \\ G_f &= 30000 \text{ N/mm}^2; \\ \nu_f &= 0,22.\end{aligned}$$

Now, using again expressions (17), (18) and (19) we get

$$\begin{aligned}\phi &= 0,300; \\ \rho_c &= 1,608 \text{ kg/m}^3; \\ t/w &= 1,300 \text{ mm/(kg/m}^2\text{)}.\end{aligned}$$

Taking $\zeta = 1$, and applying expressions from (20) to (27), taking from Table C.5 of the ISO 12215-5 Norm, we get

$$\begin{aligned}E_{UD1} &= 23627 \text{ N/mm}^2; \\ E_{UD2} &= 5795 \text{ N/mm}^2; \\ \eta_E &= 0,913; \\ \eta_G &= 0,922; \\ G_{UD12} &= 2157 \text{ N/mm}^2; \\ \tau_{UIL} &= 14,17.\end{aligned}$$

With this parameters, we evaluate the formulae from Table C.5 of the ISO 12215-5 Norm:

$$E_{BD+} = \frac{1}{2} (E_{UD1} + E_{UD2}), \quad (31)$$

$$G_{BD+} = G_{UD12}, \quad (32)$$

$$\nu_{BD+} = \nu_{UD12} \frac{E_{UD2}}{E_{BD+}} \quad (33)$$

This leads to the following values for the Roving 800:

$$\begin{aligned}E_{BD+} &= 14711 \text{ N/mm}^2; \\ G_{BD+} &= 2157 \text{ N/mm}^2; \\ \nu_{BD+} &= 0,114.\end{aligned}$$

Finally, from Table C.6 of the ISO 12215-5 Norm (see table 4), we get:

$$\begin{aligned}\varepsilon_{ut} &= 1,55\% = 0,0155; \\ \varepsilon_{uc} &= 1,40\% = 0,0140; \\ \varepsilon_{uf} &= 1,84\% = 0,0184; \\ \gamma_u &= 1,70\% = 0,0170.\end{aligned}$$

Property	Units	MAT 300		ROV 800	
ψ , content of fiber in terms of mass	%	ψ_1	3 0	ψ_2	4 7 , 8
Thickness of dry fiber	mm	t_1	0 , 7	t_2	1 , 0 4
Weight	Kg/m^2	w_1	0,3	w_2	0,8
Young's Modulus	N/mm^2	E_1	8276	E_2	14711
Shear Modulus	N/mm^2	G_1	2946	G_2	2157
Poisson's coefficient	—	ν_1	0,405	ν_2	0,114

Table 3: Mechanical properties of both fibers, MAT 300 and ROV 800.

	CSM Chopped strand mat	...	WR/bidirectional 0/90 quoted "BD+"
ε_{ut}	1,35	...	1,55
ε_{uc}	1,70	...	1,40
ε_{uf}	1,88	...	1,84
γ_u	2,00	...	1,70

Table 4: Values in Table C.6 in Appendix C of the ISO 12215-5 Norm.

9.3 Mechanical properties of the composite

Finally, taking into account what we have computed so far in the previous sections, we can compute the mechanical properties of the composite. The properties of both the MAT 300 and the ROV 800 are shown in table 3.

The definite properties of the composite are computed by the formulae (7). Using that expressions, and the datum from table 3, we get

$$w = 1,1 \text{ kg/m}^2; \quad (34)$$

$$t = 1,74 \text{ mm}; \quad (35)$$

$$E = 12122 \text{ N/mm}^2; \quad (36)$$

$$G = 2474 \text{ N/mm}^2; \quad (37)$$

$$\psi = 42,9\%; \quad (38)$$

$$\rho_f = 2,56 \text{ kg/m}^3. \quad (39)$$

For the fracture strains, we should take the smallest value of the two components, therefore we have (see table 4):

$$\varepsilon_{ut} = 1,35\% = 0,0135; \quad (40)$$

$$\varepsilon_{uc} = 1,40\% = 0,0140; \quad (41)$$

$$\varepsilon_{uf} = 1,84\% = 0,0184; \quad (42)$$

$$\gamma_u = 1,70\% = 0,017. \quad (43)$$

The design ultimate stresses for this combined fiber (MAT300-ROV800) are computed, according to Table C.8 of the Norm, by multiplying the corresponding modulus (Young's (36) or Shear (37)), obtained by the previous process, by the ultimate strain (linear (40), (41), (42) or angular (43)), obtained above using the values Table C.6 of the Norm:

$$\sigma_{ut} = 163 \text{ N/mm}^2; \quad (44)$$

$$\sigma_{uc} = 169 \text{ N/mm}^2; \quad (45)$$

$$\sigma_{uf} = 223 \text{ N/mm}^2; \quad (46)$$

$$\tau_u = 42 \text{ N/mm}^2. \quad (47)$$

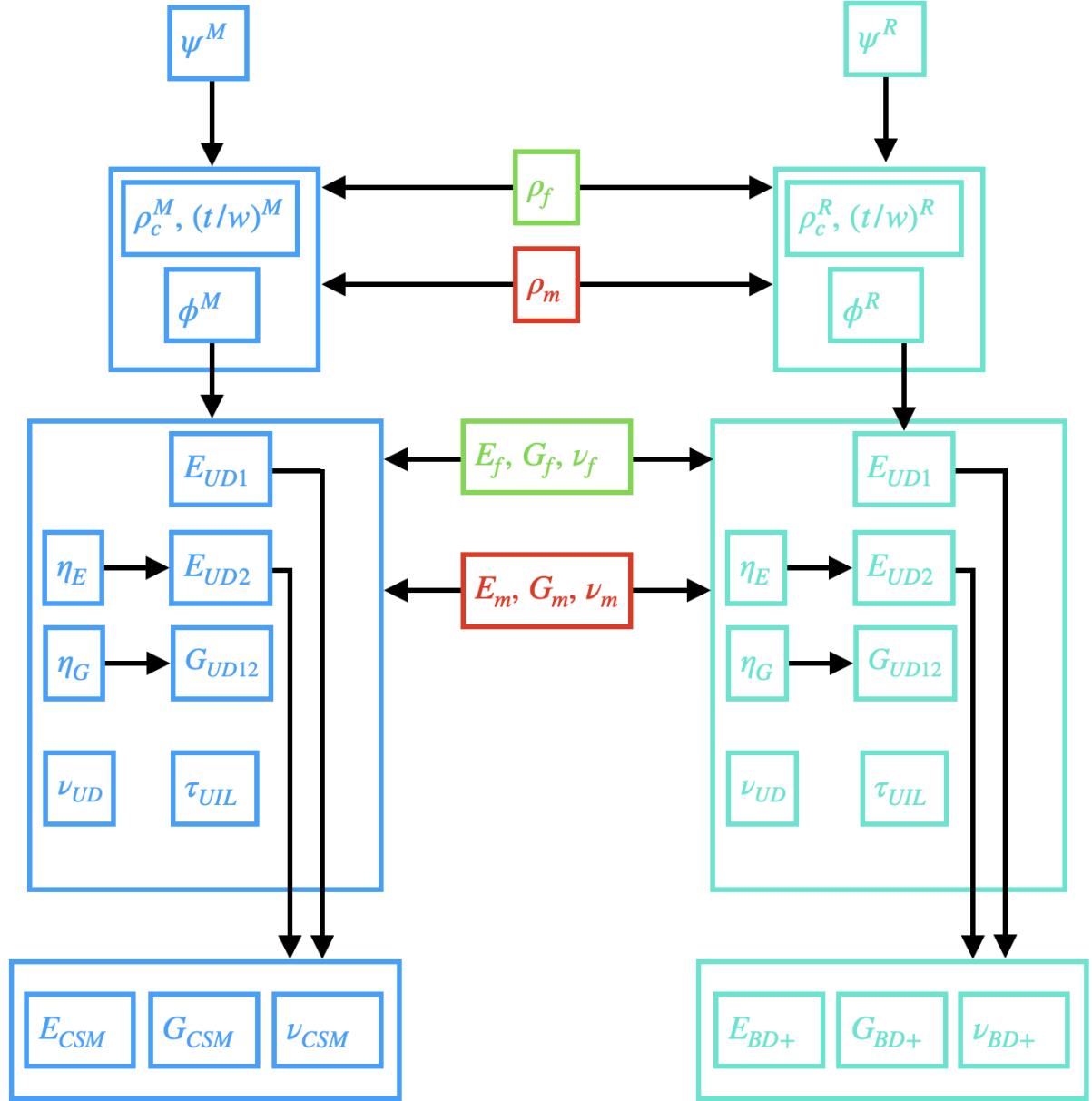


Figure 31: Flux diagram indicating the steps to follow in the calculations performed in this section and showing the dependencies between the different parameters. The super-indexes M and R refer to the MAT and Roving fibers, respectively.