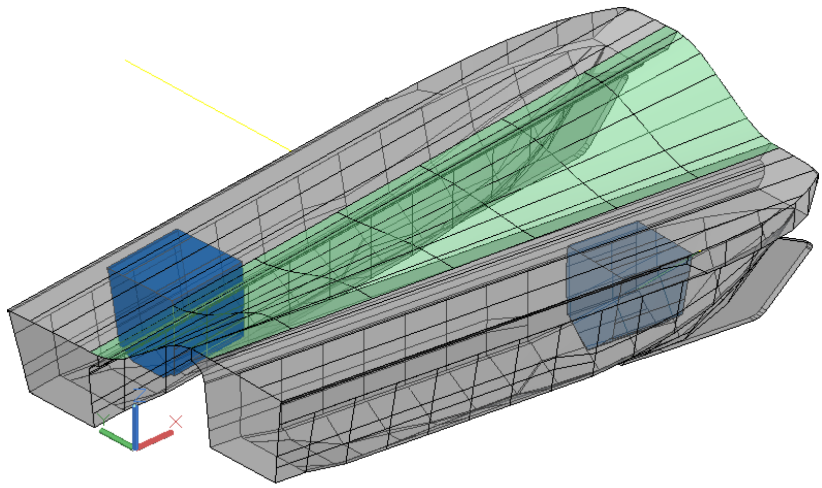
Direct Calculation of CROSS BEAMS conecting Hulls in a CATAMARAN. Wet Deck [1]



[1] These calculations have been incorporated into thelast version of ***SCT*** : small boats scantlings with ISO 12215
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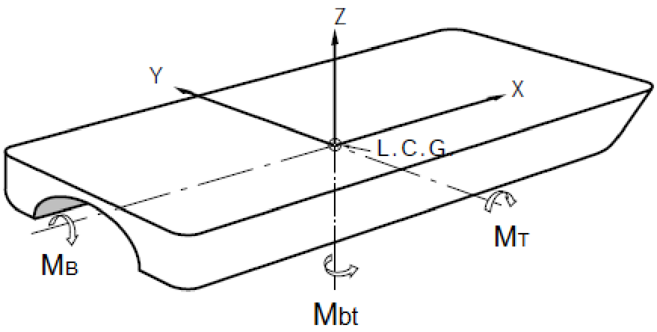
http://657677483.wix.com/tansl#!software

ASSUMPTIONS :

- Hulls are rigid and are joined by the deck, which is flexible
- The upper deck is calculated as always, under normal design pressure for a deck.
- The wet deck is calculated by applying a design pressure equal to the slamming pressure. (See ANNEX I)
- Cross beams are considered embedded in hulls so, besides having "brackets" that materialize this recess, should extend inside the whole width of the hulls.



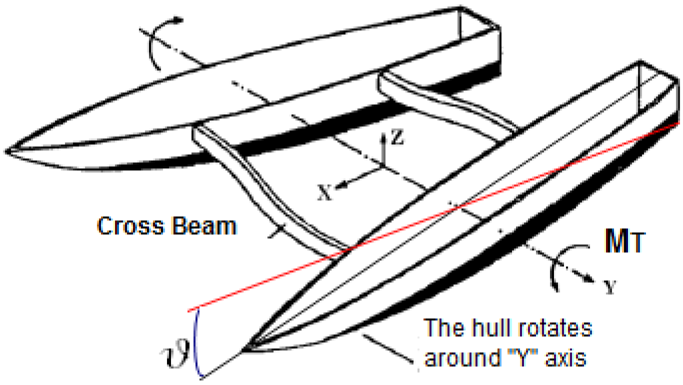
- Hulls can rotate about the three axes of the figure, but being flexible the deck, relative displacements occur between them. These relative movements are those that result in bending moments to be supported by the beams.

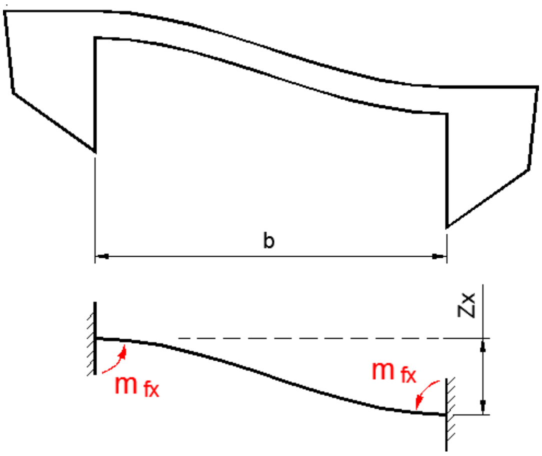
PROPOSAL FOR CALCULATION

SSC Rules by Lloyd's Register : *the torsional stress is to be determined by direct calculation methods using the twin hull torsional connecting moments M_B and M_T*

Torsional moment M_T

produces a relative rotation ϑ of hulls about Y-Y axis.

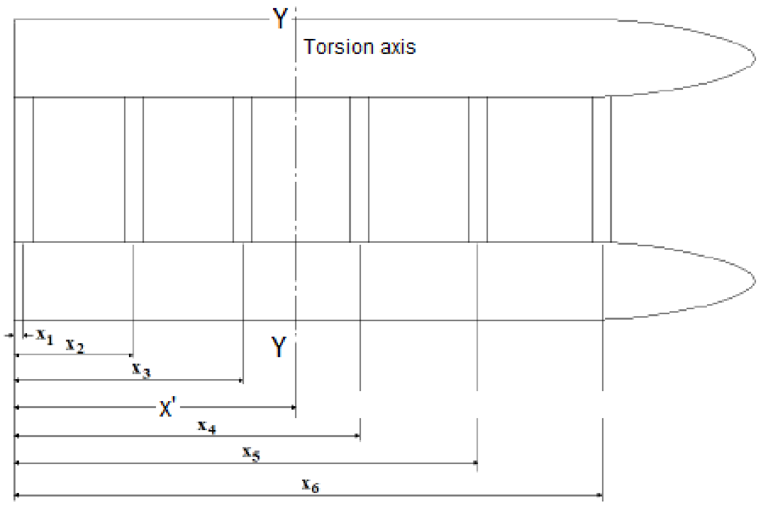




As a result of this rotation, the ends of each beam are no longer in the same plane, a Zx displacement occurs generating a bending moment in the cross beam.

The relative displacement, different for each beam, is directly proportional to the distance of the beam to the torsion axis.

And results in a different bending moment mfx for each beam.



Knowing the overall torque M_T we can calculate Θ, the relative torsion angle between the two hulls :

$$\theta = \frac{M_T}{\sum_1^n \frac{12EI(x_i - x')^2}{b^3} + \sum_1^n \frac{GJ}{b}}$$

See Annex II

where :

b : length of the cross beam
G : material's shear modulus
J : St.Venant torsional constant or twisting module.

Thin wall simple closed section : In this case the shear flow is approximately constant along the thickness of the section wall. Calling A the area enclosed by the middle curve that defines the section, L_Γ its perimeter and e the wall thickness, the torsion modulus is given by the Bredt's formula:

$$J = \frac{4A^2}{\int_\Gamma \frac{ds}{e}} = \frac{4eA^2}{L_\Gamma}$$

And knowing the torsional angle we can obtain the following data for a cross beam at a distance of (Xi - x ') from the torsion shaft:

Bending moment $$m_t = \frac{6\,E\,I\,(Xi - x')\,\Theta}{b^2}$$

E = modulus of elasticity of beam's material
I = second moment of beam transversal section
b = beam length

Bending stress $$\sigma_t = \frac{m_t}{SM}$$ where SM It is the modulus of the beam (second moment of area)

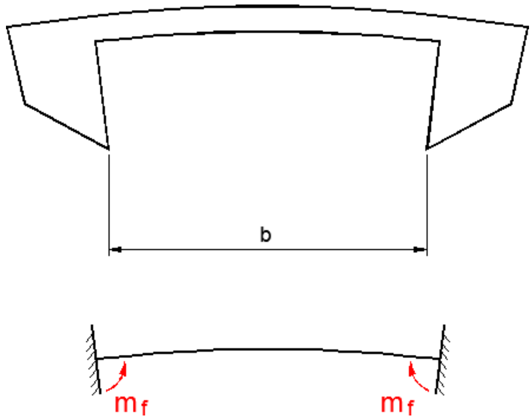
torsional shear stress for a closed profile $$\tau_t = \frac{m_t}{2\,A\,t}$$

where :

A = area enclosed by the cross beam profile
t = profile thickness

Torsional moment MB (X-X axis) is supported by all of the transverse beams that, being all equal to each other, each one absorbs a partial bending moment m_f proportional to their number :

$$m_f = \frac{MB}{\text{No. Beams}}$$



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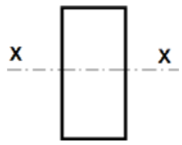
Stresses generated by this bending moment :

bending stress $\quad \sigma_B = \dfrac{m_f}{SM}$ $\quad$ SM is the first moment of the beam area about “X-X”

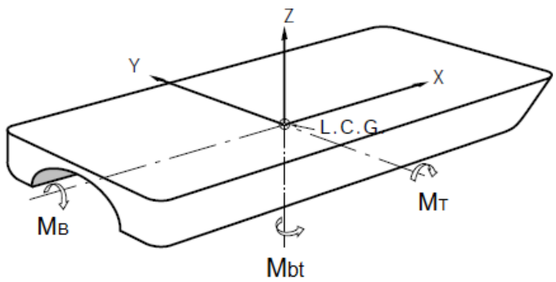


Shear stress $\quad \tau_B = \dfrac{2\,m_f}{A_w\;b}$

where :

A_w = shear area of the cross beam
b = distance between hulls

.The Transverse bending moment Mbt (Z axis)



Mbt as per C3.4.2.3 “HSC” by BV :

$$M_{bt} = \frac{\Delta .b.\ a_t.\ g}{5} \quad \text{kN.m}$$

where :

b = transverse distance, in metres, between the centres of the two hulls
a_t = transverse acceleration

$$a_t = 2,5 \cdot \frac{H_{SL}}{L} \cdot \left(1 + 5 \cdot \left(1 + \frac{V/(\sqrt{L})}{6}\right)^2 \cdot \frac{r}{L}\right)$$

H_{SL} = permissible significant wave height at maximum service speed
r = distance of the point from waterline at mLDC draugh

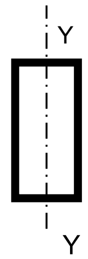
This bending moment is distributed evenly among all the cross beams.

$$m_{bt} = \frac{M_{bt}}{\text{No. Beams}}$$

Stresses generated by this moment are :

bending stress $$\sigma_{bT} = \frac{m_{bt}}{SMy}$$

SMy is the first moment of the beam area about “Y-Y” axis

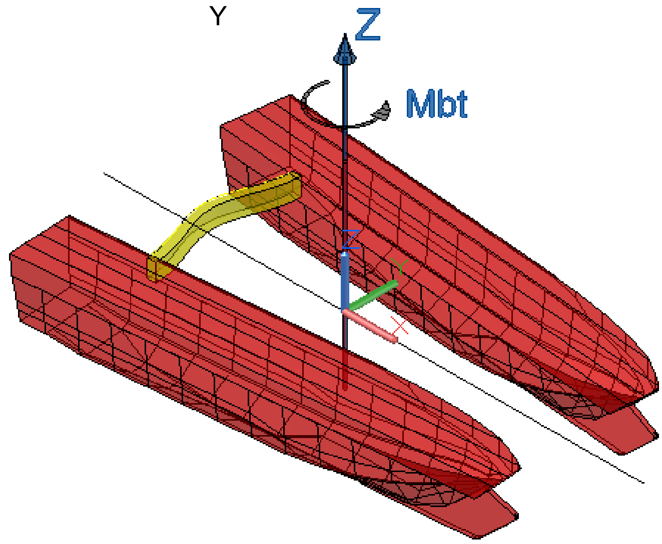


Shear stress $$\tau_{bT} = \frac{2\, m_{bt}}{A_w\ b}$$

where :

A_w = shear area of the cross beam

b = distance between hulls



The superposition principle allows us to calculate the total effort as the sum of the partial stresses

ANNEX I

VALUES FOR MT and MB

According "Special Service Craft" Rules by Lloyd's Register:

$$M_T = G_f \, \Delta \, L_R \, a_v \quad \text{kNm}$$

$$M_B = G_f \, b \, \Delta \, a_v \quad \text{kNm}$$

were :

G_f = service gropu factor.

We'll take (G4)

$G_f = 2$ for M_B

$G_f = 1$ for M_T

Δ = displacement, in tonnes

L_R = Rule length, in metres

b = transverse distance, in metres, between the centre of the two hulls

a_v = the vertical acceleration <u>at the LCG</u>

The vertical acceleration in the <u>displacement mod</u>e for all craft is to be taken as:

$$a_v = 0{,}2\, g + 34/L_{WL}$$

in <u>planning mode</u> :

$$a_v = (fa * L_{WL} / \Delta) * (Bm * H_{1/3} + 0.084 * Bm^2) * (5 - 0.1 * \vartheta_D) * \Gamma^2 / 1000$$

where :

fa = 4,5 for catamarans and multi-hull craft with partially submerged hulls

ϑ_D = dead rise, in degrees

Bm = hullbeam. Multi-hull will be taken as the sum of the breadts of the single hulls.

$H_{1/3}$ = Minimum significant wave height

Service Group	Minimum significant wave height in metres	value adopted for $H_{1/3}$
1	0.6	4.0
2	1.0	
2A	1.5	
3	2.0	
4	4.0	
5	4.0	
6	4.0	

$$\Gamma = \frac{V}{\sqrt{L_{WL}}}$$

V = speed at displacement Δ, in knots

L_{WL} = waterplane length, in metres

Section 6
Cross-deck structure for multi-hull craft

6.2 Impact pressure

6.2.1 The impact pressure, P_{pc}, acting on the underside of the cross deck ('wet deck') is to be taken as:

$$P_{pc} = \nabla_{pc} K_{pc} V_R V \left(1 - \frac{G_A}{H_{03}}\right) \text{kN/m}^2$$

where

K_{pc} = longitudinal distribution factor

= 1,0 between the aft end of the L_{WL} and 0,75L_{WL}

= 2,0 at the L_{WL} from the aft end of L_{WL}, intermediate values to be determined by linear interpolation

∇_{pc} = cross-deck Impact Factor

= 1/6 for protected structures, as defined in Pt 5, Ch 2, 2.1 Parameters to be used for the determination of load and design criteria

= 1/3 for unprotected structures, as defined in Pt 5, Ch 2, 2.1 Parameters to be used for the determination of load and design criteria

G_A = air gap, as defined in Pt 5, Ch 2, 2.1 Parameters to be used for the determination of load and design criteria

H_{03} = surviving waveheight, as defined in Pt 5, Ch 2, 2.1 Parameters to be used for the determination of load and design criteria $H_{03} = 1.29\ H_{1/3}$

V = allowable speed, as defined in Pt 5, Ch 2, 2.1 Parameters to be used for the determination of load and design criteria

V_R is the relative vertical speed of the craft at impact, in knots. If this value is unknown, then the following equation is to be used:

$$V_R = \frac{8H_{1/3}}{\sqrt{L_{WL}}} + 2\ knots$$

6.2.2 The impact pressure, P_{pc}, should not be taken less than zero.

ANNEX II

THEORETICAL JUSTIFICATION

The strain energy of a deformed cross beam is given by the expression:

$$U = \int_0^b \frac{m_{fi}^2}{2EI}dy + \int_0^b \frac{m_t^2}{2GJ}dy$$

where the bending moment is given by : mfi = mf0i (1-2y/b), and mt is the torsional momento given by $\dfrac{GJ\theta}{b}$.

By operating is obtained :

$$U = \int_0^b \frac{m_{fi}^2}{2EI}dy + \int_0^b \frac{m_t^2}{2GJ}dy = \int_0^b \frac{m_{f0i}^2\left(1-\frac{2y}{b}\right)^2}{2EI}dy + \int_0^b \frac{\left(\frac{GJ\theta}{b}\right)^2}{2GJ}dy = \frac{m_{f0i}^2}{2EI}\int_0^b\left(1-\frac{4y}{b}+\frac{4y^2}{b^2}\right)dy +$$

$$+\frac{GJ\theta^2}{2b^2}\int_0^b dy = \frac{m_{f0i}^2}{2EI}\left(y-\frac{2y^2}{b}+\frac{4}{3}\frac{y^3}{b^2}\right)_0^b + \frac{GJ\theta^2}{2b^2}(y)_0^b = \frac{m_{f0i}^2}{2EI}\left(b-\frac{2b^2}{b}+\frac{4}{3}\frac{b^3}{b^2}\right)+\frac{GJ\theta^2}{2b^2}b =$$

$$= \frac{m_{f0i}^2 b}{6EI} + \frac{GJ\theta^2}{2b} = \frac{(6EIz(x_i))^2}{6EIb^3} + \frac{GJ\theta^2}{2b} = \frac{6EIz^2(x_i)}{b^3} + \frac{GJ\theta^2}{2b}$$

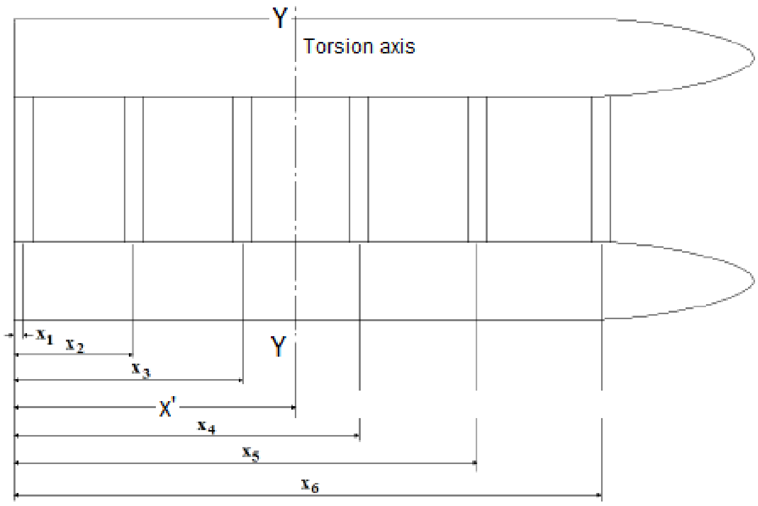
As assumed rigid hull, it holds that Z(x) = (xi – x')Θ for each beam, being (xi – x') the distance between the axis of each beam and the torsion shaft. The strain energy of all beams can be determined by the expression:

$$U_T = \theta^2\left[\sum_1^n \frac{6EI(x_i - x')^2}{b^3} + \sum_1^n \frac{GJ}{2b}\right]$$

Equating this formula to the external work done by the torque applied to the ship, you can find the angle of rotation:

$$\frac{1}{2}M_T\theta = \theta^2\left[\sum_1^n \frac{6EI(x_i - x')^2}{b^3} + \sum_1^n \frac{GJ}{2b}\right] \Rightarrow \boxed{\theta = \frac{M_T}{\sum_1^n \frac{12EI(x_i - x')^2}{b^3} + \sum_1^n \frac{GJ}{b}}}$$

once ϑ angle is known :



For a cross-beam located at a distance *Xi* from the twist axis, we have the following :

$$\text{maximum shear force } = \frac{12\ E\ I(Xi - x')}{b^3}$$

$$\text{shear stress } = \frac{12\ E\ I(Xi - x')}{b^3\ Aw}$$

Aw is the area of beam's transversal section

$$\text{maximum bending moment } = \frac{6\ E\ I(Xi - x')\vartheta}{b^2}$$

$$\text{bending stress } = \frac{6\ E\ I(Xi - x')\vartheta}{Z\ b^2}$$

Z is the section first moment of area

$$\text{maximum torque on the beam : } \quad mt = \frac{G\ J\ \vartheta}{b}$$

$$\text{torsional shear stress tangential for a closed profile } = \frac{mt}{2\ A\ t}$$

t profile thickness, *A* area enclosed by the beam's profile