SPONBERG YACHT DESIGN INC.

THE DESIGN RATIOS
A Naval Architect's Dozen (or thereabouts)

A primer on some basic principles of naval architecture for small craft.

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By

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PREFACE

In January 2010, a gentleman in Austria ("Capt. Vimes") visiting the Internet forum BoatDesign.net asked a question about the Center of Flotation and its implications on boat design and performance. I happened to be the first responder to the question, I answered it as simply as I could and gave some insight into what it is used for in boat design and what it means. After a very kind thank you from Mr.Vimes and a few responses and compliments later, another gentleman from Australia ("Landlubber") asked if I could explain <u>all</u> the other naval architecture design ratios in the same way seeing as my answer to the first question was the best description he had ever read.

And so it began. I took up the challenge, and for the next three months I responded with explanations of the various design ratios, publishing one a week. I admit to having an ulterior motive. Many years ago, a client of mine informed me of his invention of what he called the S Number (S#), which is a way to rate the performance of all boats on a scale of 1 to 10 using the Sail Area/Displacement ratio (SA/D) and the Displacement/Length ratio (DLR). He had published an article about it in a regional sailing magazine back in 1988. I found over the years that the S# worked pretty well, and I started using it in my responses to potential clients. A time eventually came where I thought I should publish the concept of the S# again, giving due credit to its originator. I pitched an article about S# to one of the major sailing magazines whom I had written for before, but they declined.

Not too long after that, this opportunity came on BoatDesign.net to discuss some of the basic naval architecture design ratios that are used primarily in recreational craft design, and certainly SA/D and DLR would be part of that discussion. Putting the design ratios in a logical order, I would have a natural progression leading up to S#.

And so it went. The chapters included here are the original complete texts of my "class discussions" on the design ratios, including S#, and repeated in the order in which they appeared on BoatDesign.net. I include the pictures and attachments, plus a few others. I do not include the various questions afterwards—if you want that, you can go to BoatDesign.Net to see the entire thread. See the Table of Contents for the post number position of any topic in the thread. Here is the link to the start:

http://www.boatdesign.net/forums/boat-design/center-flotation-calculation-implications-30857.html

I hope that for those of you who are new to small craft design these pages offer some clear understanding of the concepts involved. Enjoy the reading. And thanks to "Capt. Vimes" and "Landlubber" for their interest and encouragement.

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22 March 2010

CENTER OF FLOTATION

This is where the thread started. "Capt. Vimes" from Austria posed the following question (corrected for spelling and punctuation):

Hi!

I am just trying to understand all the different ratios and calculations regarding a yacht design and I am a little puzzled with this 'center of flotation' and its implication on the balance, stability and performance of a monohull sailing yacht...

In Larsson & Eliasson 'Principles of Yacht Design', this CF is not dealt with at all - it is mentioned but not what it actually means for the design... The calculation of its position is completely neglected... But since this book describes the principles by designing an example yacht (40 ft monohull fin keeled sloop) and all the different parameters are listed, I at least realized that while the LCB is 3,5% LWL aft amidships, the LCF is 6-6,5% LWL aft amidships depending on the load...

What I understand is the principle explanation of the CF:

> *CENTER OF FLOTATION (CF): The CF is the center of the waterline area and is the pivot point about which the boat changes trim, much like the pivot in the center of a teeter totter. On normal sailing hulls the CF is somewhat abaft the CB and, like the CB, is expressed as a percentage of the LWL or a distance from either the bow end of the LWL or from amidships. Of course, as the boat changes trim, due to added weights at one end or the other, the LWL shape changes, so the CF will move slightly.*

How is balance/stability compromised or enhanced if the CF is moved further aft or closer to the CB? What are the performance implications?

Thank's for any help enabling a noob to comprehend this complexity about the one thing we all love so much - woma.... darn - I mean boats....

And here is my reply:

Capt. Vimes,

Your quote is correct, the CF is the center of the flotation waterplane area. Figure 1 shows a picture of the CF. Another way to think of it is, you know that by Archimedes principle a floating body displaces a volume of the liquid whose weight is equal to that of the body itself. So you have a boat floating in the sea, and now imagine that you bring a weight on board that is enough to sink the boat 1 centimeter. The weight that you have brought on is equal in weight to a volume of seawater that is the area of the boat's waterplane times 1 cm thick. The center of that volume of water is located at the CF. Now, imagine, if you will, that when you set that added weight down on the deck, you placed it directly and vertically over the CF. The trim and heel of the boat would not change, but the boat would sink straight down that 1 cm. The weight acts down through its own center of gravity and the added volume of seawater acts in exactly the opposite direction through the CF and through the CG of the weight you added. However, if you set the weight down on the deck at some other location other than over the CF, the boat would trim and heel such that now, the whole submerged volume of the boat at the deeper draft will be equal to the total weight of the boat plus the added weight, and that the center of buoyancy (CB) will be directly under the final center of gravity (CG).

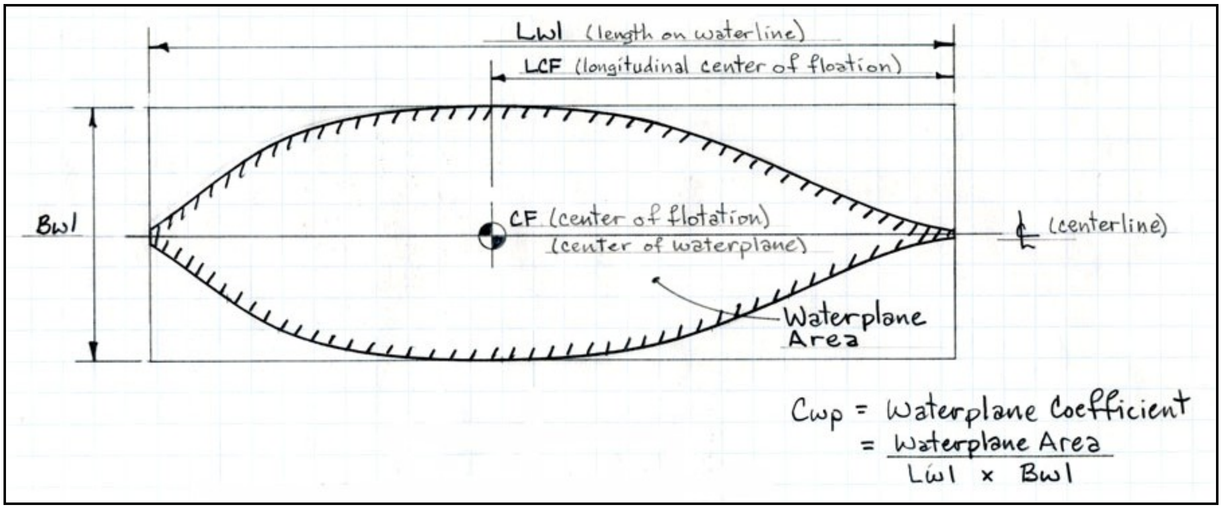

Figure 1. The Center of Flotation and Waterplane Coefficient (see next chapter).

Now, the weight that you brought on board does not have to be that to have a similar effect. Forces acting on the boat have the same effect, such as sailing forces from the rig. These forces do not act at the CF, but elsewhere. They push the boat over and down like an eccentrically placed weight. In order to perform properly, the boat has to balance against those outside forces, and the only way to do that is by virtue of its own hull shape and weight, and by the effectiveness of the keel and rudder. The hull portion--its shape and weight--is a huge factor. If not well shaped, then the boat may heel or trim at odd or weird angles that will affect how much lift comes from the hull itself or the appendages. A boat an unusual angles of trim and heel will generate more a lot more drag that if it is closer to upright. This, of course, affects performance. Generally, the least amount of hydrodynamic drag occurs when a vessel is upright. Drag always increases as the boat trims and heels. This is why you like to minimize heel and trim while underway, and the best way to do that is with a properly shaped boat.

We have very fine examples of this in many round-the-world racers. For example, look at the Vendee Globe fleet (open class designs)--generally these boats are very wide shallow boats and they rocket downwind and off the wind like the blazes. In these conditions, the boat stays more upright than if it is sailing to windward. When sailing to windward, however, these boats are very poor performers, and this is due primarily to their hull shape which is very side aft. As the boats heel over, the CF moves aft quite a far distance. This has the effect of raising the stern, and likewise, pushing down the bow into the sea and away from the wind. That is not the direction you want to go. This can put an adverse angle of attack on the keel. And this is why wide shallow boats are poor performers to windward. The Vendee Globe and other round-the-world racers gravitate to these hull shapes because usually these races are off-the-wind races. The exceptions are the races that have multiple stops, such as the Around Alone (formerly the BOC) and the Portimao Global Ocean Race. When you have to come back into port at multiple stops, you come from pure maritime wind and weather to a mix of maritime and continental weather. Frequently, you have headwinds near the ends of the legs. To get through those headwinds, you need a boat with good windward ability. This is why narrower boats fare better in such races.

So the solution is, in order to have a more balanced boat, you want the CF to move very little as the boat heels over. See Figure 2. Check the position of the LCF at zero degrees heel, and then check it again at 15 or 20 degrees heel. They should be very close in position to each other fore and aft. That way, as the boat heels over, it will have nearly zero tendency to raise the stern and push down the bow. The angle of attack on the keel and rudder are better, and windward ability is very favorable.

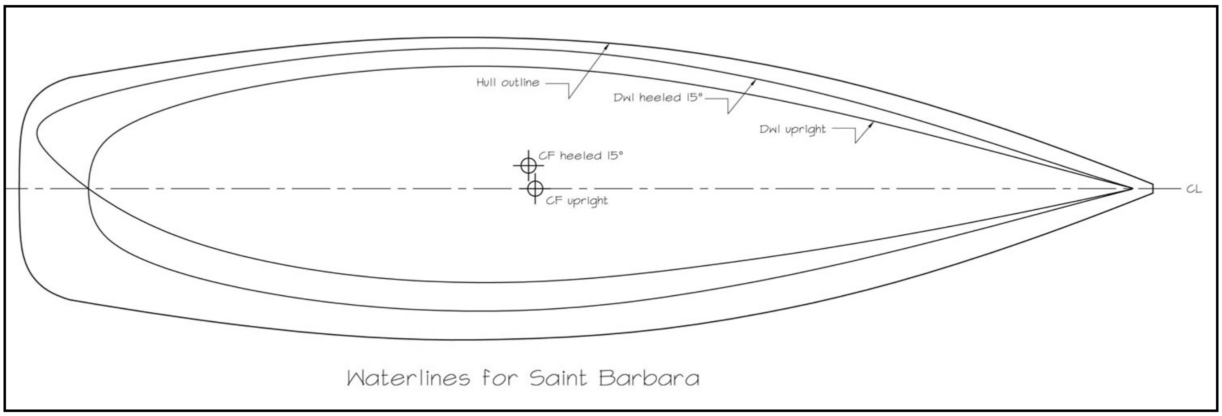

Figure 2. The waterlines for SYDI design *Saint Barbara*, which shows very little fore-aft movement in LCF at 15º of heel. *Saint Barbara* has a very nicely balanced hull.

Take this one step further, like Capt. Nat Herreshoff of Bristol, RI, did in the late 19th century. Many of his designs showed that the CF moved forward as the boat heeled over. This had the effect of raising the bow up and to windward--precisely the direction you want to go--and which enhanced the angle of attack on the appendages and increased lift. The Nat Herreshoff boats were great performers, as everyone knows. Many of his lessons were forgotten in the latter 20th century.

Keeping the CF in a more or less constant position as the boat heels over is easier to do with narrower hull designs. As hulls tend to get wider, CF always moves aft more easily and affects performance in an adverse way.

I think one of the great hoaxes (maybe fallacies is a better word) of modern yacht design is the concept of "powerful stern sections" which became a ubiquitous description of boats in the 1980s, and is still seen today. "Powerful stern sections" implies wider body aft, wider waterplane aft. Certainly, such wider shapes give more room in the cockpit and aft cabins, and they cruise downwind and off the wind OK, but they have a deleterious effect on performance to windward.

I hope that helps. Class dismissed!

BLOCK COEFFICIENT

Today, class, we will take up Block Coefficient, Cb. See Figures 3 and 4. These diagrams came from my notes for a class that I once taught some years ago at the International Yacht Restoration School in Newport, RI, and before that, the boat repair class at the Museum of Yachting, also in Newport.

In the first diagram, we see a perspective view of a traditional sailing yacht sitting in a block of water. The underwater portion of the hull is shaded. If we cut the hull at the waterplane, we have the view in the second diagram, the area of the waterplane. This shows the CF, the subject of last week's discussion, which is the Center of Flotation. The LCF is the Longitudinal distance that the CF is back from the front end of the waterplane. We can measure LCF from any reference point, which we can call Station 0. Traditionally, Station 0 was always the front end of the design waterplane. But with the advent of computers, Station 0 is often taken at the very front extremity of the stem, which is where I usually take it. That way, everything else on the boat is measured as a positive dimension aft of the forwardmost extremity of the boat (excepting bowsprits, of course).

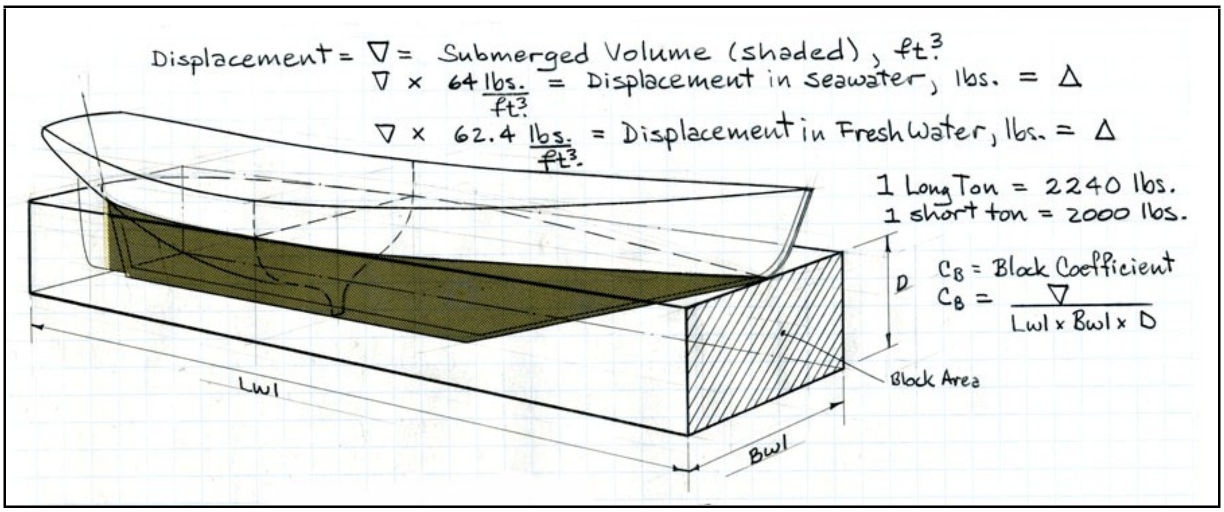

Figure 3. Block Coefficient.

We can see in the second diagram how to calculate the Coefficient of the Waterplane, Cwp. The waterplane is bounded by a rectangle of the length of the Waterplane, Lwl, and the Beam of the Waterplane, Bwl. The waterplane area is always less than the circumscribing box. The ratio of the actual waterplane area to the box area is the Cwp. Cwp = Actual WP area/(Lwl x Bwl). The value of Cwp is always less than 1.0.

Moving onto Block Coefficient, Cb, we can see it is kind of like Cwp with a third dimension added, Draft, D. It is the ratio of volumes instead of a ratio of areas. In the first diagram, we can see that the shaded portion of the hull, which is that portion that it underwater, is bounded by a box, or block, whose volume is Lwl x Bwl x D. The Block Coefficient is the ratio of the actual submerged volume of the hull to the volume of its bounding box, or block, hence the name. The equation is shown in the diagram: Cb = Actual Submerged Volume/(Lwl x Bwl x D). Block coefficient is usually more important in ship design than sailboat or powerboat design, because ship block coefficients tend to be much larger in ships than in recreational boats.

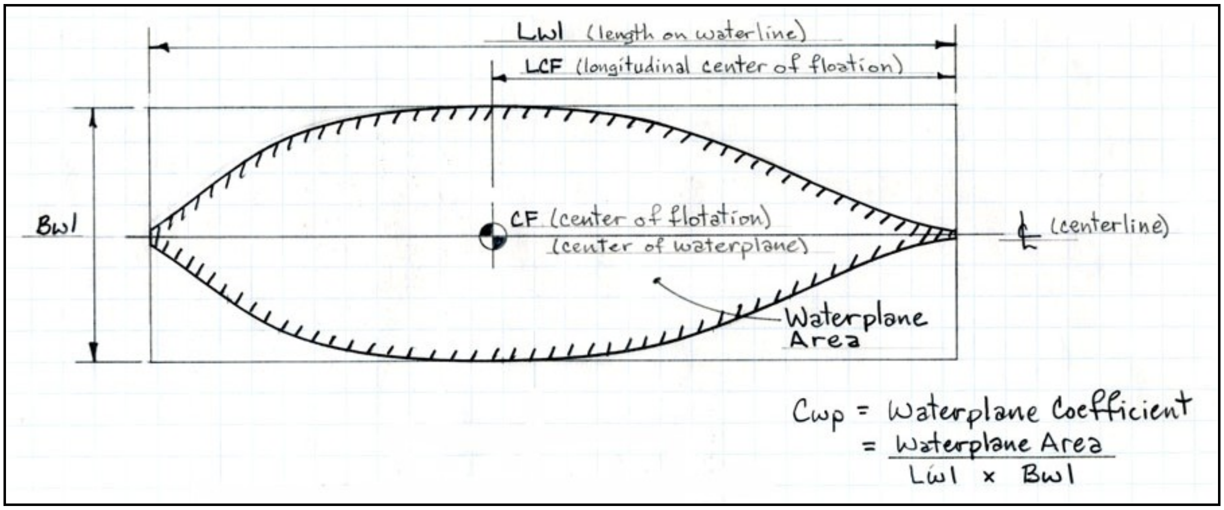

Figure 4. Waterplane Coefficient (same as Figure 1 above).

We can also see that the submerged volume of the hull displaces a volume of liquid that is equal to the weight of the boat. This is Archimedes principle which he discovered back about 220 BC. This is why the weight of a boat is called its "displacement." The density of the water also plays a role. Fresh water is less dense than sea water, so a hull floating is fresh water will sink deeper than when it sits in sea water, in order to make up the volume equal to the weight of the boat.

We can also see that we can measure weights in Long Tons, LT, or Short Tons, ST. One LT = 2,240 pounds, and One ST = 2,000 pounds. I used to know the reason why LT became prevalent in naval architecture, but I have long forgotten it. We can also talk of Metric Tonnes for you metric users. One cubic meter of fresh water weighs 1,000 kgf which equals one Metric Tonne, MT. One MT = 2,204 pounds for those of you interested in the conversion. Note the different spelling of "tons" for imperial units, and "tonnes" for metric units. This is a further clue to detect what another person may be talking about.

Finally, the ratios of Cwp and Cb, and all other coefficients all have the same values in any consistent measurement system. Usually, these coefficients have values less than 1.0. If they don't come out that way, then you have either done something wrong in your calculation, or, the hull is really unusual.

Next time we will take up Midship Area Coefficient, Cmc, and Prismatic Coefficient, Cp.

Class dismissed.

MIDSHIP AREA COEFFICIENT & PRISMATIC COEFFICIENT

Today we take up two coefficients: Midship Area Coefficient, which we'll label Cmc, and Prismatic Coefficient, labeled Cp. See the diagrams attached, Figures 5, 6, and 7.

Cmc is the ratio of the area of largest midship section of the submerged portion of the hull to the area of the box that bounds it. The dimensions of the box are the Beam at the Waterline at the largest section, Bwl, and the Draft at that section, Do. The largest section area may not be at the exact mid-length of the hull, or at the maximum beam or draft of the hull. See the first figure below. The easiest way to determine the maximum size midship section size is to plot the Sectional Area Curve, and the maximum area will be at the peak of the curve. Unfortunately, I don't have an example of my own sectional area curve to show, but you can see one in *Principles of Yacht Design* (Larsson & Eliasson, 3rd ed.) Figure 4.4, pg. 35.

At the fore and aft location of the peak of the sectional area curve, measure the Bwl and the Do. The box bounding area for the largest midship section, therefore, is Bwl x Do. Midship Section Coefficient, Cmc, then is the actual midship section area divided by the bounding box area, as shown in the diagram below. The concept and the equation are shown in the second figure below.

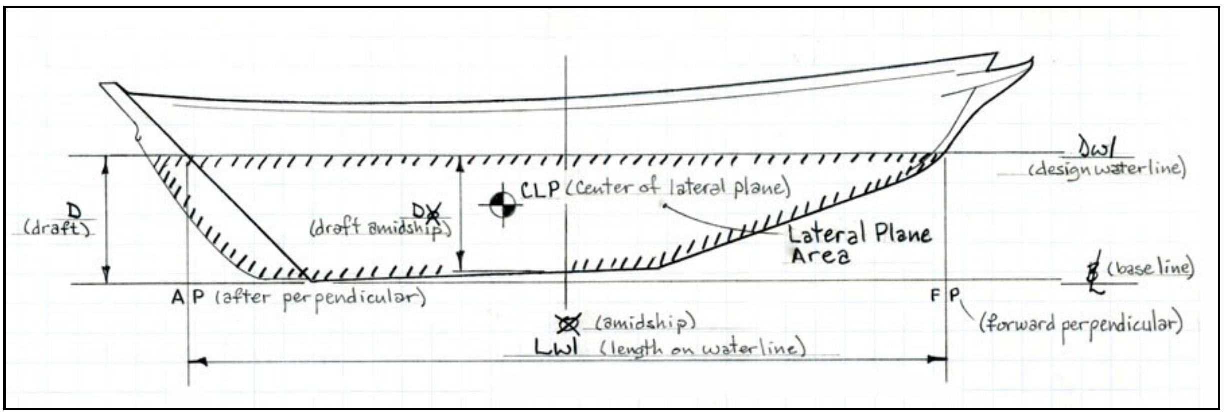

Figure 5. Profile area of a hull and the location of the midship section.

Cmc is used as a gauge to judge the fineness of the midship section. It is useful for comparison between different designs, or to judge how a design is being developed. Say, for example, that you are designing a new hull, and you want the displacement to be within a certain range. On your first pass at developing the lines, you see that the displacement is too large. Maybe the turn of the bilge is a little too sharp. So then you develop a second set of lines and find that the displacement is too low; maybe now the turn of the bilge is not sharp enough. You can compare the shapes of the midship sections, because they are directly related to displacement, and compare the Cmc ratios of each. Analyzing these features will lead you to where you want the final Cmc to be (somewhere in between), and the geometry of the sectional area will show you where to make adjustments in the shape (again, somewhere in between). Cmc is used as an analysis tool, therefore, for developing a new design or comparing two different designs.

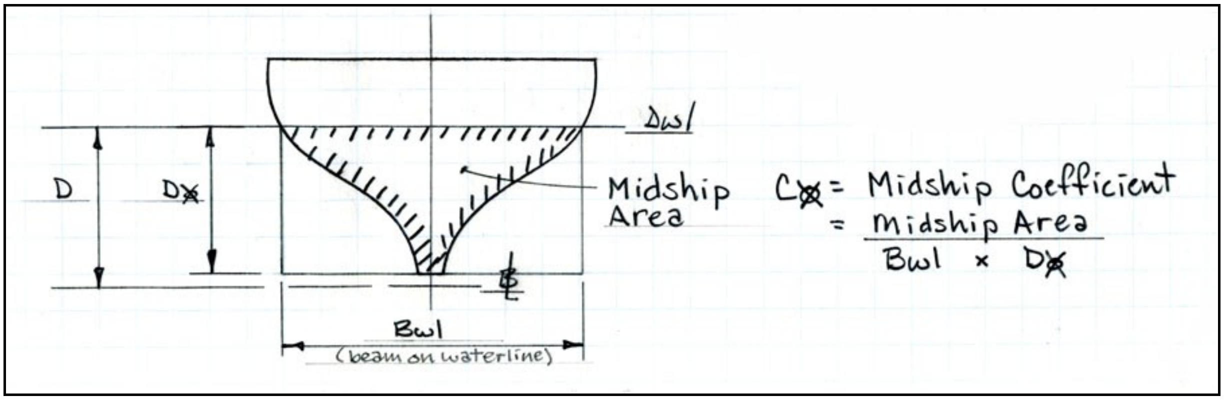

Figure 6. Midship Area Coefficient.

This leads us to Prismatic Coefficient, Cp. Cp is like Cmc, except that we take the calculation one step further with a third dimension--length. Cp is a comparison of volumes, not areas. Cp is the ratio of the volume of displacement of the hull divided by the volume of a prism which is the maximum section area multiplied by the Length on the Waterline, Lwl. See the third figure below. The figure shows the equation. Obviously, the prism volume is always larger than the actual displaced volume, so Cp is always going to be less than 1.0, by definition.

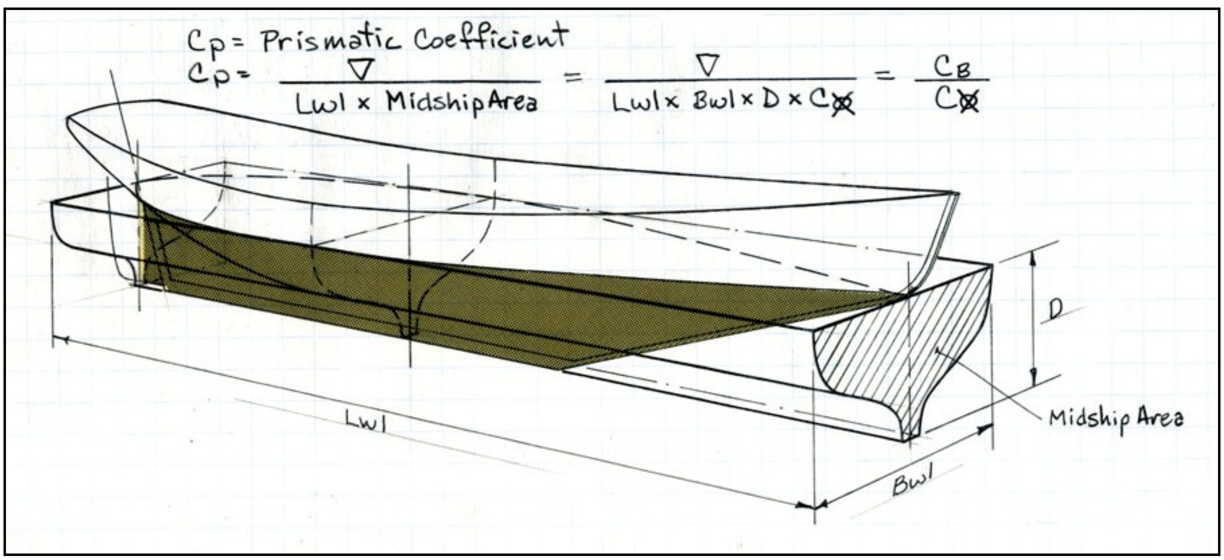

Figure 7. Prismatic Coefficient.

Looking at the equation, we see an interesting thing happen. In the denominator, we see Lwl x midship area. We know from the Cmc above, that if we turn its equation around, Midship Area = Bwl x Do x Cmc. So if we substitute these parameters into the Cp equation, we get that Cp = Vol/(Lwl x Bwl x Do x Cmc). But we know the part Vol/(Lwl x Bwl x Do) is equal to Block Coefficient, Cb. At least it will equal it perfectly if Do is also the maximum draft, D. If Do is not the maximum draft, then the calculation will be a little off. But basically, this all reduces to the fact that Cp is the ratio between Cb and Cmc, as shown in the figure. In words, this is: “Prismatic coefficient is the Block Coefficient divided by the Midship Area Coefficient.”

It turns out, after some 150 years or so of analysis, that performance is closely related to Cp. That is, there is an optimum range of Cp for various speeds of the boat traveling through the water. You can see a table of speed/length ratios versus optimum Cp in *Skene's Elements of Yacht Design* (by Francis Kinney, 5th ed.) pg.284, which I repeat below:

Speed/Length ratio	**Cp**
1.0	0.52
1.1	0.54
1.2	0.58
1.3	0.62
1.4	0.64
1.5	0.66
1.6	0.68
1.7	0.69
1.8	0.69
1.9	0.70
2.0	0.70

Larsson/Eliasson shows a similar range in their book on page 83, Fig. 5.22, in which they plot optimum Cp against Froude Number. Froude Number is very similar to Speed/Length ratio, and if you convert Froude Number to Speed/Length ratio, you will find that Larsson/Eliasson's curve is a bit lower than Skene's curve tabulated above. As is true with many things, therefore, there is some wishy-washiness in the guidelines. Nothing is hard and fast.

Nevertheless, what this tells you is that most displacement boats travel most of the time at Speed/Length ratios of at least 1.0 and slightly above, so you need enough volume to support the hull at those speeds. If volume is either too much or too low—that is if Cp is too big or too small—your hull drag is going to go up. Either the boat is going to have to push too much water out of the way (Cp too big) or it is going to sink into its own waves (Cp too small).

Usually, in sailboat design, the keel and it's draft is left out of the calculation of volume. This is because, as in the Cb calculation, the keel tends to make Cp less sensitive. So we ignore the keel for calculation of Cp. In powerboat design, we do not do this. If we are designing a trawler or lobster boat, for example, we keep the keel in the calculation because it is a major portion of the hull.

Interestingly, I typically design my sailboats with a Cp of about 0.60. I did the same with my Moloka'i Strait motoryachts. This is just below hull speed, Speed/Length ratio = 1.34. (We can take that up in another post, if you wish). You can also see that approaching planing speeds (Speed/Length ratio => 2.50), Cp reaches 0.70. This goes along with very long and narrow hulls—that is, being still in displacement mode at S/L = 2.0, you need a high Cp. This is why catamarans and trimarans (which have long narrow hulls) have very high Cp ratios.

That's a lot of material to digest, so I'll leave it there and wait for questions.

SPEED-LENGTH RATIO AND A/B RATIO

This week we are going to cover two totally unrelated ratios, both of which are relatively short topics: A/B ratio and Speed-Length ratio. The first is very silly and worthless, the other extremely important.

First, A/B ratio—a really a dumb and useless concept. Definition: The A/B ratio is the ratio of two areas, usually in motoryacht and trawler design. The “A” area is the profile area of the whole boat above the waterline, and the “B” area is the profile area of the hull and appendages below the waterline. See Figure 8 below. The ratio of A to B is supposed to be a measure of the boat’s stability and seaworthiness. Nothing could be further from the truth.

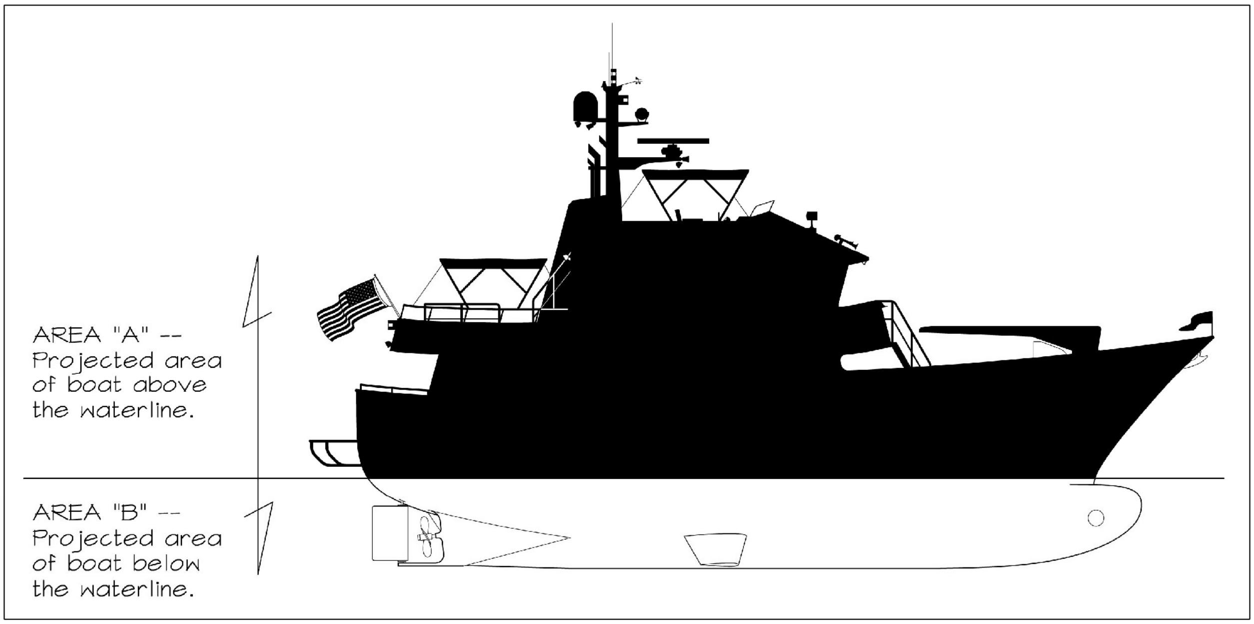

Figure 8. A/B Ratio on the *Moloka’i Strait 72*.

A/B ratio comes from Robert Beebe’s book *Voyaging Under Power*. In that book, Beebe’s total discussion of stability in yacht design centers around the A/B ratio. Metacentric height, the essence of stability, isn’t even mentioned and you can’t find it in the index. Yet, Beebe claims that A/B ratios higher than some unmentioned limit would scare him if the boat were going offshore. He does not define what that limit is. Beebe completely ignores everything else about stability: displacement and center of gravity, submerged volume and center of buoyancy, beam and form stability, free surface effect, righting arm curves, stability tests—everything truly related to stability.

Naval architects, in their formal training, are not taught anything about A/B ratio. You can have two boats, each with the same A/B ratio, and they would have totally different stability characteristics due to those factors just mentioned above. So, get it into your heads right now that A/B ratio is totally meaningless!

Now, Speed-length ratio—very important. You have no doubt heard this term, studied it, and have an understanding that the speed-length ratio equal to 1.34 is called “hull speed.” Indeed it is.

Definition: Speed-length ratio is the speed of the vessel in knots divided by the square root of the vessel's waterline length in feet = V/Lwl^0.5. At speed-length ratios less than 1.34, the vessel is in displacement-mode motion—that is, the hull is simply moving the water out of the way as it moves forward. When speed-length ratio is between 1.34 and 2.5, the vessel is in the semi-displacement or semi-planing mode—that is, it is trying to rise up over its own bow wave to get onto plane. Some boats are designed to operate at these speeds. Above speed-length ratio of 2.0 to 2.5, the vessel is planing and relies on dynamic lift to raise and hold it out of the water so that it can skim along the surface of the sea.

That's the general definition, and there are exceptions to these characteristics. Where does speed-length ratio come from?

Speed-length ratio is a law of physics and nature. The length of a free-running wave on the sea is equal to:

L = 2*Pi*V^2/g

Where:
Pi = 3.14159
V = wave speed in feet/second
g = acceleration of gravity = 32.174 feet/second^2

Therefore, wave length is a function of wave speed and gravity, and that is why sea waves are called gravity waves. If you take the constants to the left side of the equation and put the variables on the right side and convert it to get rid of the speed squared, you have:

(1/2*pi)^0.5 = 0.39894 = V/(g*L)^0.5

V/(g*L)^0.5 is Froude Number, Fn, a dimensionless ratio. It was invented by William Froude, a British naval architect back in the 1870s, who developed the system of measuring and analyzing ship resistance in towing tanks that we use to this day. His contribution was that ship resistance was made up primarily of frictional resistance, form resistance, and wave-making resistance. If you towed a model of the ship that was geometrically similar (same shape only smaller) to the one you wanted to build, you could reduce the drag to dimensionless coefficients that would apply either to the model or to the ship. The coefficient of frictional resistance varied with Reynolds number, another dimensionless ratio. The coefficient of form resistance was the same for both model and ship. And the coefficient of wave-making resistance varied with Froude number.

Through experimentation it was found that when Lwl, the length of the waterline on the ship, equaled L, the length of a free-running wave, ship resistance went up dramatically. This made sense—the length of the wave was as long as the ship, and if the ship tried to go any faster, it would have to create a wave longer than itself, and this requires a tremendous amount of added energy—i.e. more power. The ship would have to start climbing up the back of the wave that it was creating.

Well, if we convert Froude number such that speed is in knots and we put the acceleration of gravity over on the right side of the equation above, we get the following:

V/Lwl^0.5 * (6076 feet/nautical mile)/(3,600 seconds per hour) = g^0.5 * 0.39894

Or

V/Lwl^0.5 = g^0.5*0.39894*3600/6076 = 1.34

And because of this conversion, speed-length ratio is not dimensionless.

So, when speed in knots divided by Lwl^0.5 = 1.34, the length of the ship's wave will be as long as the ship's waterline length, and we can expect resistance to go up dramatically. The boat hits a barrier of resistance—we have "hull speed."

And, when we are using Froude Number in model tank analysis, of course the L that we use is Lwl of the ship. We can round the decimal fraction above up a hair from 0.39894 to 0.4, and we have equivalence: That is, Fn = 0.4 is equivalent to speed-length ratio of 1.34. In boat design we use speed-length ratio, in model testing we use Froude Number because it is dimensionless. Naval architects like to use dimensionless numbers.

Now, we mentioned before that long narrow hullforms like multihull hullforms tend to not obey this speed-length ratio limit. That's due just to the nature of long narrow bodies generally having less tendency to make waves. Therefore, they can easily go faster than hull speed. This is why speed-length ratio = 1.34 is not a hard and fast rule or law. It is just a really good guideline that is based on physics.

As you are aware, all hydrodynamic resistance is dependent on vessel speed. If a vessel isn't moving, it does not have any resistance. We can get an idea of relative speed where resistance changes by paying attention to speed-length ratio or Froude Number, and this is why speed-length ratio is so important.

Questions?

DISPLACEMENT/LENGTH RATIO

In class today we cover Displacement-Length Ratio, DLR in short-hand notation. This is a commonly used ratio for comparing designs and estimating speed. Here is the definition:

DLR = Displacement/(Lwl/100)^3

Where Displacement is in long tons and Lwl is in feet. A long ton is 2,240 pounds.

DLR is kind of like Block Coefficient, Cb, in that it is a comparison of volumes, really. Since naval architects usually like dimensionless numbers, Displacement should be in expressed cubic feet instead of long tons. Then you would have a dimensionless ratio of cubic feet divided by cubic feet.

Larsson/Eliasson's *Principles of Yacht Design*, uses the inverse of this concept in Length/Displacement ratio, LDR. This is Length in meters divided by the cube root of volume of displacement in cubic meters, a true dimensionless ratio.

LDR = Lwl/Vol^0.333

I am not as familiar with this form as you folks in the metric world may be, and I don't use it in my work. I use DLR. I want to give some history here, so I am going to stick with imperial units and DLR. In the end, you may use either one, depending on your methods of design.

DLR was invented by Admiral David W. Taylor, the father of modern model testing in the United States, and first published in 1910 in his book *The Speed and Power of Ships—A Manual of Marine Propulsion*. Taylor found that when towing models in a towing tank and following Froude's Law of Comparison, the resistance of the model was proportional to displacement. Specifically, he stated: "At corresponding speeds for similar models, resistances which follow Froude's Law are proportional to displacement, and hence pounds per ton are constant."

Zowie! That is actually very, very important. There are very few things that are constant in this world, particularly in naval architecture, so a statement like this is quite profound. For similar hull forms, *at the same Froude Number (or as we saw in the last lesson, at the same speed-length ratio)* resistance per ton of displacement is the same. I stress the terms in italics because the resistance per ton is constant only at that speed. The resistance per ton will be different for different speeds. More on that in a minute.

So, if I test a model that weighs 100 lbs and has a certain resistance, Rm, at a speed length ratio of 1.0, then I can pro-rate that resistance up according to displacement and arrive at a resistance for a larger boat or ship with the same geometrically similar hullform at that same speed-length ratio. To be explicit: resistance per ton = r. Resistance of the model is Rm; Displacement of the model is Dm. Resistance of the ship is Rs; Displacement of the ship is Ds. Therefore:

For the model: Rm = r*Dm

We know Dm because we can weigh the model. (And in fact, that is probably why Taylor used weight rather than volume. The weight of the model is constant during model testing and easily measured on a scale. Submerged volume, on the other hand, varies with the movement of the model through the water as it rises, sinks and trims, and it has to be calculated which back then was more tedious to do than now with our computers.) Also, we know Rm because we measure it by towing the model in a towing tank. So we solve the equation for r:

r = Rm/Dm

We do this for different speeds, and we can plot r versus Froude Number, Fn, or Speed-Length ratio, SLR. Knowing r, and also designing a ship with a displacement Ds, we want to know the resistance at the same speeds. Therefore, for any given speed:

Rs = r*Ds

That is generally the way it works. I do stress that model testing gets a lot more sophisticated and complicated than that when you start breaking total resistance down into its various components. But generally, this displacement rule works.

We know that as speed goes up, the resistance-per-ton, r, goes up exponentially, following a cubic relationship with Fn and SLR. See Fig. 9.1 on page 175 in *Principles of Yacht Design* by Larsson/Eliasson. Also, *Skene's Elements of Yacht Design* (5^{th} edition by Kinney) has a similar graph in Figure 1, on page 85.

So, where does this leave us with Displacement-Length Ratio, DLR? If we are studying a group of boat designs that we like and which are similar to one we want to design (comparing dimensions and characteristics of a population of boats is called a parametric analysis), we can compare their DLRs and deduce some general things about their performance. (I stress the word "general" here. We can deduce trends, not necessarily specific values.) We know that for the same installed power, or sail area and wind speed, heavier boats will be slower, or, that is, boats with high DLRs will be slower than those with low DLRs. Conversely, boats with low DLRs will have more lively performance than boats with high DLRs.

Another way to look at it is, say you have a preliminary design with a certain DLR. It will have a certain r value, resistance per ton of displacement at any given speed. (Or you can say, it will have a certain curve of r over a speed range.) But if you stretch the hull out longer, keeping displacement the same, DLR goes down, and r goes down. The resistance per ton is less, therefore the total resistance will be less, yet the hull is still the same weight. Note that if length increases, wetted surface also increases, so one might expect that frictional resistance goes up, and speed might suffer. Well, that is a common notion, but, the effect on reducing r (that is, reducing the *total* resistance) is greater than the increase in frictional resistance, in general. This primarily plays on the resistance due to form (one of the triad of friction, form and wave-making resistance).

Be aware that increasing length while holding displacement the same also reduces prismatic coefficient, Cp. You would want to be sure that Cp does not fall outside the desirable range. If it does, you may have to change the shape of the hull to maintain Cp in an acceptable range.

Ted Brewer, in his book *Ted Brewer Explains Sailboat Design*, in the first edition (1985), page 9, gives classifications of sailing yacht types based on DLR. These or similar classifications have been stated by other designers over the years, and this is a convenient summary, repeated here:

Boat Type………………………………………**DLR**
Light racing multihull…………………………..40-50
Ultra-light ocean racing boat…..……………..60-100
Very light ocean racing boat………………..100-150
Light ocean racing boat…..………………......150-200
Light cruising auxiliary boat…..……………..200-250
Average cruising auxiliary boat…………….250-300
Moderately heavy cruising auxiliary boat….300-350
Heavy cruising auxiliary boat……………….350-400+

In my career, it has been interesting to see the trend of boat designs getting ever lighter with reducing values of DLR. When I started yacht design in the 1970s, the typical good cruising sailboat had a DLR in excess of 300. Today, that's changed. My sailing yacht designs going back about 15 years are all under 200, for example:

Project Amazon, 1995-6, Open Class 60, offshore racing: DLR = 69.5
Bagatelle, 1998-9, ultra-light ocean racing: DLR = 50.8, later 88.0 with heavier keel
Saint Barbara, 2002, light Great Lakes racing/cruising: DLR = 119.2
Globetrotter 45/Eagle, 2004-5, light auxiliary cruising: DLR = 192.7
Globetrotter 66 (currently in design), light ocean cruising: DLR = 140

All of these designs, save the last, are on my website if you would like additional particulars. The *Globetrotter 66* is an aluminum cat-schooner I am designing for a client in Southern California, who intends to take his family cruising to the far reaches of the planet. The masts will be carbon fiber free-standing rotating wingmasts.

I should point out something else about DLR. Notice that Length is divided by 100 before it is cubed. The reason for doing that is it gives DLR a more understandable and reasonable range of values, generally between about 50 and 400. If we did not divide Length by 100, the DLR would be a tiny, tiny number with about three zeros right after the decimal point before reaching the significant digits. Dividing Length by 100 just makes DLR a little easier to understand.

In another example, let's say we are designing a boat with a certain DLR, and we want to know what size engine to install. We can look at different designs with similar DLRs and see what size engines they have installed. We can be pretty well assured that if we pick a similar size engine, we will achieve performance similar to those other boats. DLR gives us some degree of confidence in making the design decision.

We can get into a deeper discussion about speed and displacement, but we'll save that for another time.

A final note about long tons, one Long Ton = 2,240 pounds. Where does that come from? A short ton is 2,000 pounds. Why the difference?

Aside from wanting to use dimensionless numbers, naval architects also like to use simple numbers, and if you can get rid of decimal digits to the right of the decimal point, so much the better. Sea water weighs 64 pounds per cubic foot. Fresh water, on the other hand, weighs 62.4 pounds per cubic foot. In America during the 19th century, a lot of shipping traffic grew and developed on the Great Lakes (fresh water), almost as much as there was on the sea along the coasts. And coincidentally at this time, naval architecture was going through tremendously rapid development and scientific expansion. Simplifications in ship design were in order wherever they could be found.

As you know, we often convert displacement weight to its corresponding volume of sea water or fresh water. The going standard terminology for a ton, what became known as the "short ton," was 2,000 pounds. So, for the conversion, divide 2,000 pounds by the density of the water:

Sea water: 2,000 lbs/64 lbs/cu.ft. = 31.25 cubic feet (a messy number—it has decimal digits)

Fresh water: 2,000 lbs/62.4 lbs/cu.ft. = 32.05128205 cubic feet (a messier number)

So the naval architects of the time decided to change the definition of a ton that would be easier to use and to convert to volumes of sea water and fresh water. They finally settled on the "long ton". Here is what happened:

Sea water: 2,240 lbs/64lbs/cu.ft. = 35 cubic feet (a very clean and simple number)

Fresh water: 2,240 lbs/62.4 lbs/cu.ft. = 35.8974359 cubic feet (messy, but close enough)

Naval architects will also round numbers up and down if it suits them, and this looks like a good candidate. For fresh water, you can round this up slightly to 36 cubic feet per ton to get a clean number without too much error.

So now, long tons of 2,240 pounds could be easily converted to cubic foot volumes of sea water or fresh water by the using simple conversion numbers:

Displacement (long tons) x 35 cubic feet/long ton = volume in cubic feet of sea water

Displacement (long tons) x 36 cubic feet/long ton = volume in cubic feet of fresh water

And since 2,240 lbs. is larger than the short ton of 2,000 lbs., that is why we have the term "long tons."

Well, that's a lot of material for today.

Next week, we take up Sail Area/Displacement ratio. Questions?

SAIL AREA/DISPLACEMENT RATIO & SAIL AREA/WETTED SURFACE RATIO

I was going to restrict this topic to just Sail Area/Displacement ratio, but it is closely related to Sail Area/Wetted Surface ratio, and so a single discussion on both is in order.

The next topic in our quest for better understanding of naval architecture with regard to sailing yachts rests on two ratios related to sail area. The first and more important one is Sail Area/Displacement ratio (SA/D in short-hand notation), and its little sister is Sail Area/Wetted Surface ratio (SA/WS). The reason I regard one higher than the other is because SA/D is easily calculated from published data, and it gives a better indication of power versus weight—a true power-to-weight ratio—at normal sailing speeds when drag due to displacement—the full total of friction, form, and wave making drag—is significant.

SA/WS ratio is a power-to-drag ratio that gives an indication of the power available to a sailboat for light air sailing when friction drag is the primary drag component and wave making drag is minimal. Few sources publish the wetted surface of boats, so generally, we cannot compare our numbers to other designs, and that limits its usefulness. But we do have some guidelines to follow.

Both ratios are truly dimensionless, and we'll start with SA/D ratio. Here is the formula:

SA/D = Sail Area/(volume of displacement)^0.667

In words, SA/D = the sail area divided by the 2/3rds root of the volume of displacement. We have units of square feet divided by units of square feet. It also works directly in metric units being units of square meters divided by units of square meters—so it is universal in any consistent system of units. Typically, you use the upwind sail area and the design displacement for calculating SA/D ratio.

We calculate volume of displacement, of course, by dividing the weight of displacement by the density of seawater. In imperial units, divide weight in pounds by 64 lbs/cu.ft. of seawater (the norm) or by 62.4 lbs/cu.ft. of fresh water. In metric units using Kgf and cubic meters, we know that 1,000 Kgf = 1 metric ton = 1 cubic meter of fresh water. We want to use sea water to easily compare to other designs. So to convert displacement to cubic meters of sea water, divide the weight in metric tons by 1.025 which is the specific gravity of seawater = volume of displacement in cubic meters of sea water.

We raise the volume of displacement to the 2/3rds power to convert it from cubic units to square units. We leave sail area alone.

There are various published ranges of normal SA/D ratio. Interestingly, *Skene's Elements of Yacht Design* ignores this ratio. Ted Brewer's book on Sailboat Design gives the following ranges which are as good a description as any. Generally, the higher the ratio, the more power the sailboat has, and so the faster it will be.

BOAT TYPE……………………………………….SA/D
Motorsailers………………………………..........13 – 14
Slow auxiliary sailboats……………………..........14 – 15
Average offshore cruisers…………………..........15 – 16
Coastal cruisers…………………….....................16 – 17
Racing yachts…………………….........................17 – 19
Ultra light racers, class racers, daysailers.….…20+

Larsson and Eliasson, in *Principles of Yacht Design*, cite a paper by Miller and Kirkman, *Sailing Yacht Design—A New Appreciation of a Fine Art* (SNAME Transactions, Vol. 98, 1990, which is an update of a 1963 similarly titled paper published in SNAME Transactions by Henry and Miller) which offers a graph, Fig. VII-5, showing a typical range of SA/D ratios between 15 to 22 over lengths on the waterline between 20' and 50'.

In another very good design paper by Jay Paris, *Performance Criteria and the Design of Sailing Yachts* (New England Sailing Yacht Symposium, January 24, 1976, SNAME), Figure 7 shows a lower limit of SA/D of 15 – 17.5, and an upper limit of 17 – 20.5 over the same waterline lengths. All these papers, by the way, are available from SNAME.

In my own experience, the Open Class boats of the 1990s era had very high SA/D ratios, upwards of 40-50. *Project Amazon*, my Open Class 60 for the 1998 Around Alone Race, had a design SA/D of 42.1. Others of my designs are:

Bagatelle, 1998-9, ultra-light ocean racing: SA/D = 38.71, later with heavier keel, 27.47
Saint Barbara, 2002, light Great Lakes racing/cruising: SA/D = 22.90
Globetrotter 45/Eagle, 2004-5, light auxiliary cruising: SA/D = 21.38
Globetrotter 66 (currently in design), light ocean cruising: SA/D = 20.84

I have not been keeping track of design trends in the latest Open Classes, the Volvo Round the World Race, or the America's Cup, but my inclination is that all their SA/D ratios are quite high. The ranges given by Ted Brewer above generally still hold for typical sailing yacht designs.

You use the SA/D ratio to make sure you are not over-powering or under-powering your design. Generally, you want to be within these ranges, or if you are outside these ranges, then justify yourself to your customers as to why. Be prepared to explain.

Moving onto SA/WS ratio, the formula is very simple:

SA/WS = Sail Area/Wetted Surface

Again, this is a true dimensionless ratio and is consistent in all measurement systems. The sail area is again the upwind sail area, and the wetted surface is the entire wetted surface including the keel and rudder (all of the boat is dragging through the water, so the entire wetted surface is included). Obviously, the higher the ratio, the more power the boat has in light air.

Ted Brewer doesn't talk about SA/WS, I guess because most of his designs generally have large wetted surface areas. Larsson and Eliasson refer to the Miller-Kirkman Paper. *Skene's Elements of Yacht Design* gives some guidelines which I will repeat here. Paris' paper summarizes the latter two. Basically, Skene's offers Figure 44 (Kinney, 8th edition, page 287) which shows that for keel boats up to 80' Lwl, the lower limit on SA/WS is about 1.9 at 25' Lwl to 2.88 at 80' Lwl. The upper limit for the same waterline lengths ranges from 2.35 to 3.28. Centerboard boats straddle this upper limit.

SA/WS is usually more important in round-the-buoys racing where light air conditions can prevail over a whole course for a whole race. In the current era, we see lots of racing yachts with minimal keel and rudder planforms, and their Cp's are reduced to the minimum, so wetted areas are very small. Such boats like to sail in flat water (i.e. light wind and tiny waves) and so minimizing friction drag is of paramount importance. Such boats, however, are overpowered easily unless there are lots of crew on board to sit on the rail to keep the boat upright. Therefore, crew commitment and organization play a huge roll in winning races. We could get into a philosophical discussion about the impact of crew on racing boats, and when do crew elements and issues overpower design elements—but that is for another time in a pub somewhere over a beer.

SA/WS is of little concern in an offshore cruising yacht. Such yachts have so much displacement just to carry the owner's stuff, and have keels that are large enough to support the weight of the boat on shore in out-of-the-way places, that wetted surface is necessarily high. We do not try to optimize wetted surface on such designs. As a result, we are more concerned with SA/D ratio.

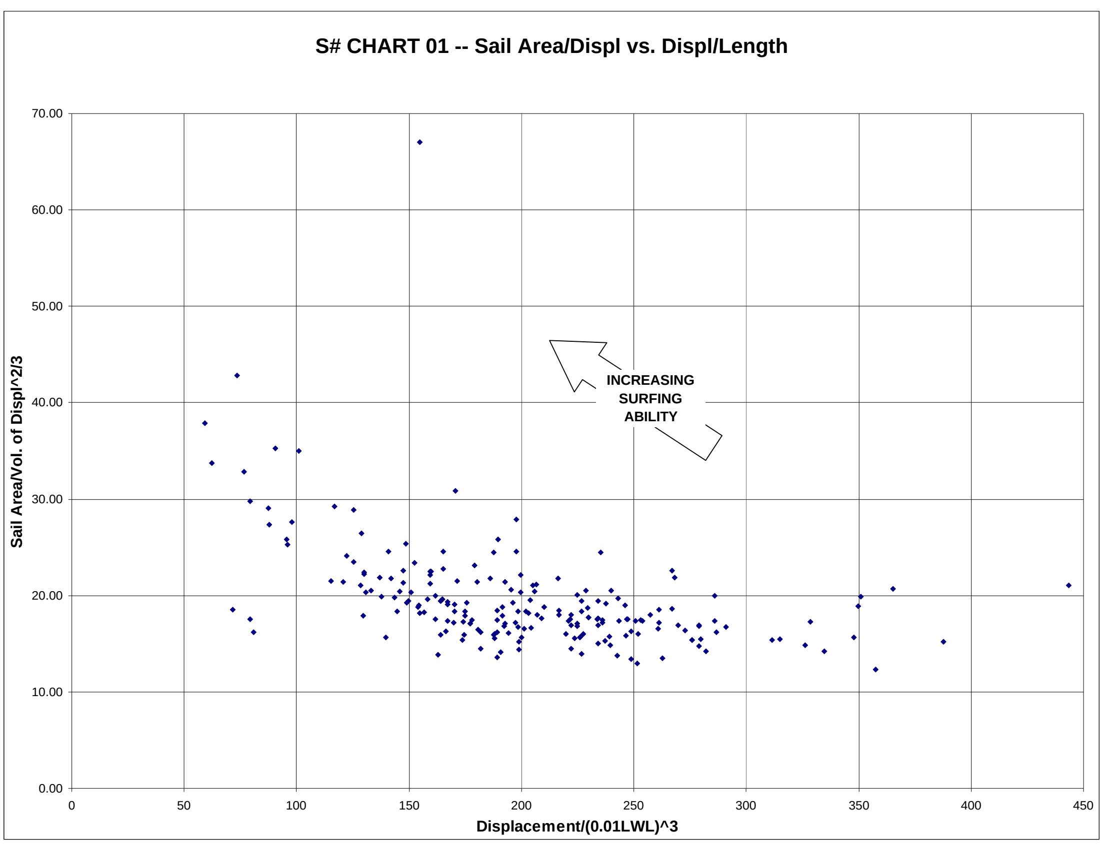

Figure 9. SA/D Ratio versus D/L Ratio.

So, we know that SA/D is a power-to-weight ratio—the higher this ratio, the faster the boat will be. And from our last lesson, we know that Displacement-Length Ratio, DLR, is an indicator of drag—the higher this ratio, the slower the boat would be. We can learn some interesting things if we plot the SA/D ratios versus the DLR ratios for any given population of yachts, such as in Figure 9 above.

The data for this plot is something that I have collected over the years, and you can see that the data points form a crescent, if you will, that peaks at the upper left (fast speeds) and again at the lower right (slow speeds). The potential to surf increases going up and to the left. You can match your own designs with a similar plot. This graph is very similar to Figure 8 in the Paris paper, and again, it represents the norm for a lot of production boats—these are typical. If your boat design is outside this range, then it is either because you are trying to design something new and different, or it's because you have made a bad mistake somewhere and need to check your work.

This discussion now leads us to the next question: Is there a way to rate sailboat performance on a scale of 1 to 10? Indeed there is—it is called the S Number, which I refer to in short-hand as S#. S# is not new; I first learned about it in 1988. But next week will be the first time, as far as I am aware, that it will have a worldwide release. And you'll read about it first, right here. Stay tuned.

Questions?

ON A SCALE OF ONE TO TEN—THE "S" NUMBER (S#)

Wouldn't it be nice to rate the performance of all sailboats on a scale of one to ten?

Here's the problem—we have different ways to rate a sailboat's potential performance in the form of design ratios, handicap rules and ratings, and level ratings. In fact, rating systems have been around for centuries, dating back to England and the realm of Queen Elizabeth I—over 400 years. And in all that time, sailors and designers have continuously argued over what makes a boat go fast, and what should be measured and rated in order to allow disparate designs to compete on equal terms. VPP programs and CFD codes have tried to make performance ever more definable, but these tools require sophisticated programs and specialized people to run them. An alternative solution is to race in one-design boats, but, unfortunately, not everyone wants the same boat. On top of that, not everyone wants to race. Still, we want to be able to judge performance—we *always* want to know about performance.

A similar problem crops up with advertising hype—this or that boat is a racer/cruiser, cruiser/racer, racing machine, or simply just a dog that can't get out of its own way. Who defines these things, and how is anyone supposed to make sense out of it all?

A rating number from 1 to 10 might simplify things for the average sailor and designer. What can we do with the information we already have without resorting to a consultant—some way that anyone can rate any boat on a scale from 1 to 10? Has anyone done this? Yes, someone has.

Back in the mid-1980s, I designed a "Boat-in-a-Box" sailboat—that is, a boat that could ship inside a standard 40' shipping container—for a client in Texas, Mr. A. Peter Brooks. At the time, he and I both were also consulting for Cat Ketch Yachts Inc., the builder of the Herreshoff and Sparhawk cat ketches. Brooks was a retired business consultant and author, and he did some writing and marketing for the company. I designed all the carbon fiber free-standing masts for the boats. Brooks invented the idea for what he called the "S" Number (S#)—a single number between 1 and 10 which could rate the performance of all sailboat designs. This idea was published in *Telltales*, a southern Texas boating magazine, in April, 1988. I have never seen anything like it, before or since.

The concept is rather simple and is based on the Displacement/Length ratio (DLR) and Sail Area/Displacement ratio (SA/D), both of which we have discussed in the last few weeks. We know that DLR relates to drag—heavier displacement for a given length results in more drag, and boats with high DLRs are slower than boats with low DLRs. We also know that SA/D relates to power—more sail area for a given displacement results in more speed, and boats with high SA/Ds are faster than boats with low SA/Ds. We have also plotted SA/D versus DLR in a chart, and have seen how the spread of data points relates to boat performance. We use these same ratios—SA/D and DLR—to calculate S#, so we don't need any new computer program to achieve our goal—just one new equation.

The equation for S# is an exponential and logarithmic function using DLR and SA/D as the primary variables. We already know how to calculate DLR and SA/D, and I am going to remove the slash "/" from SA/D so that it is less confusing in the S# equation—we'll use the term "SAD." Although the S# equation looks complex, it can be easily programmed into a calculator or a spreadsheet. Here it is:

S# = 3.972 x 10^[-DLR/526 + 0.691 x (log(SAD)-1)^0.8]

Brooks developted this equation with the assistance of Dr. Fred Young, at the time Dean of the College of Engineering at Lamar University in Beaumont, Texas. I spoke with Dr. Young by telephone some years ago just to make sure I understood the equation correctly, and he was very helpful.

Brooks collected a list of boat designs and their particulars from various published sources and calculated their S#s. Then he classified the boats according to the following categories:

- Lead Sled: S# = 1 to 2
- Cruiser: S# = 2 to 3
- Racer-Cruiser: S# = 3 to 5
- Racing Machine: S# = 5 to 10

The reasons for the ever-broadening scale of category names is simply a function of the logarithmic scale embodied in S#. This appears to be an asymptotic function. You can never reach the number 10, and you can never reach the number 1, both of which are the asymptotes.

Now we have a way to definitively categorize boats, not a wishy-washy, vague notion; we got a unique number for each and every design! I attach the spreadsheet that I used to calculate the SAD and DLR numbers in that chart I posted last week (S# Chart 01, Figure 9, in the previous chapter). Included in that spreadsheet is the S# calculation (pink column), and next to that is the category name for each boat. I sorted the data according to descending values of S# so that you can see how the categories play out. Also included in the spreadsheet is S# Chart 01 of SAD vs DLR for these boats. There are other charts there, too, which you can study or modify at your leisure.

The magazine sources for these sailboat designs are listed at the top of the data table and in the left-most column with the date of publication in the second column. They are all published data from advertisements and design reviews that I have collected over the years. As I review the magazines, I continually add data to this table. The original *Telltales* data that Brooks used and published in 1988 is included. I find it discouraging that in recent years the magazines have been slacking in publishing worthwhile design data on boat designs. Sometimes, it is difficult to get even the most basic of information—some small piece is frequently missing, and you don't necessarily find it on the manufacturers websites. But we gather what we can.

Now here is where my contribution to S# comes in. Overlaying this chart of SAD vs. DLR, I have calculated and plotted the traces of constant S# so that you can see how they subdivide the boat population. I attach this chart as well, S# Chart 02, Figure 10 below. In the data table, I highlighted in yellow a few of my designs, and then have labelled them in the chart, just to give you some context.

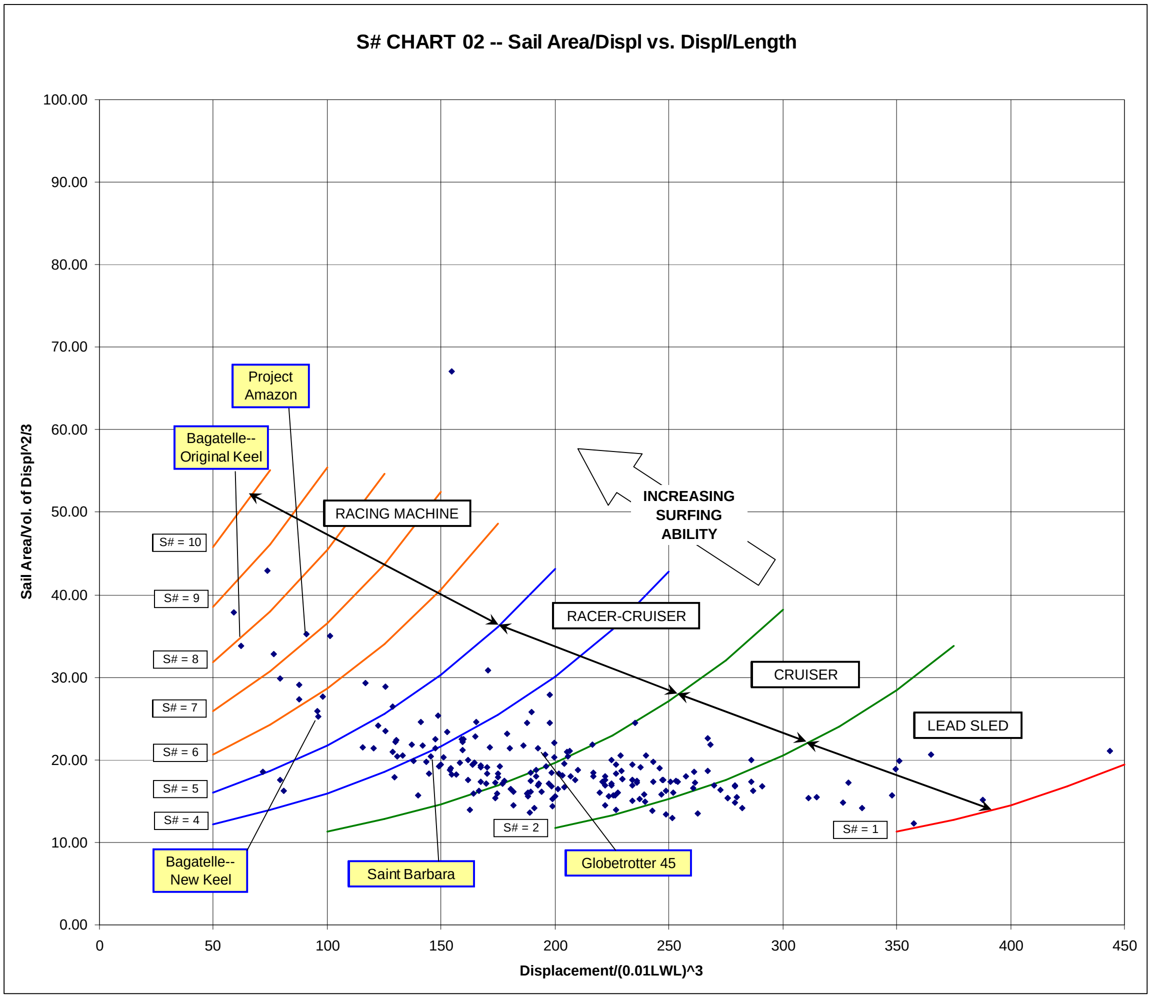

Figure 10. Same as Figure 9 but with contours of S# and the four different categories of boat type overlaid. Some SYDI designs, as mentioned in the text, are also labeled.

So what do we see? S# Chart 02 can be interpreted as follows:

A boat that is very lightweight and has lots of sail area will have a low DLR and a high SAD. It has a high power-to-weight ratio, and so it will be very fast. Its S# will be between 5.0 and 10.0. It will be a "Racing Machine."

On the other hand, a very heavy boat that has a small sail area will have a high DLR and a low SAD. It has a low power-to-weight ratio, and it is not going to be a very good performer. Therefore, its S# will be between 1.0 and 2.0. It will be a "Lead Sled."

For S# values between 2.0 and 3.0, the boat will have a decent amount of volume to carry people and goods but won't necessarily be a real hot-shot sailer. We can place these boats in the "Cruiser" range.

For S# values between 3.0 to 5.0, the boat will be in the middle ground between "Cruisers" and "Racing Machines", so we can call them "Racer-Cruisers" (or "Cruiser-Racers" if you prefer.)

Therefore, the net result of the S# is a clear delineation of sailboat performance using a convenient scale from 1.0 to 10.0, and by this we give definitive meaning to typical descriptive names. In fact, Brooks claimed that the S# is a fairly reliable predictor of PHRF or IMS rating. For two boats of the same length, the one with a higher S# will be faster, will take less time to sail around a course, and therefore will have a lower rating. However, in the same article, this footnote appears: *"Both Dr. Young and the author stress that the 'S' number is not a handicapping or rating system, but a guide to probable boat performance vs. other boats of comparable size."* I personally agree with that opinion.

Something else that is quite interesting is shown in S# Chart 03, Figure 11, also included in the spreadsheet. I had the notion to divide SAD by DLR and plot that against S# and got a surprising result. All of the data forms a unique cluster in a very well-defined curve. These are two independent functions plotted against each other. Rarely in science do we see such a profound correlation of data. I am not absolutely sure of the ramifications of this, and maybe I am reading into it more than I should, but I would have expected a broader spread of data points in this chart. The relationship of the SAD/DLR ratio to S# is extremely solid as indicated by the cluster of points along its trend line. The equation for the trend line shown at the top of the chart is another way to approximate S# in a simpler cubic equation. Throughout the lower categories, S# follows the trend line almost exactly, and it is only in the Racing Machine category where there is some scatter away from the trend line. If we plot S# versus some simple dimension or factor such as LOA or Displacment, we see no discernable relationship to S# at all. But S# vs. SAD/DLR gives us a very unique view of sailing performance.

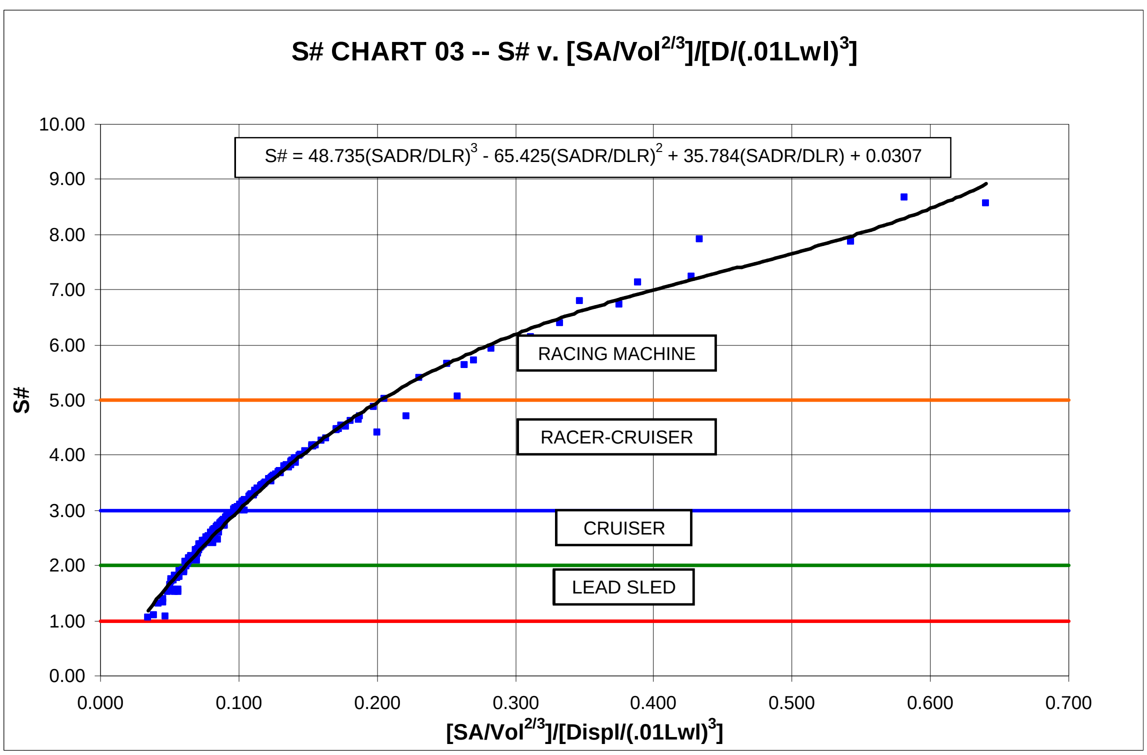

Figure 11. S# versus the quotient of SA/D Ratio to D/L Ratio, nearly an identity.

I am not mathematician enough to explain why this works as we see it. I have, however, on occasion, presented plots like S# Charts 02 and 03 to clients to review the performance they have in their current boats or are trying to achieve in a new design. It seems to give them a clearer understanding of what their current boat does or their new boat is going to do. S# is a way to be a little more scientific in layman's terms. This gives us a better tool to clearly showcase performance without having to go to the model tank, measure resistance factors, plot results, correlate them to full scale, and then do VPPs on top of all that. A picture says a thousand words, and this seems to do a pretty good job.

You will also see in the data table and in the charts a calculation and plot of Ted Brewer's Motion Comfort Ratio (MCR) plotted against S#. I will explain that in next week's topic.

In the meantime, you are all free to use this database and spreadsheet as you please. You may add to it as you do your own designs or review the designs for others. You may change it around and expand it however you want. You may do other analyses and manipulate the data at your will. Send it to your friends, fellow designers and fellow sailors. Pass it around. Discuss it. Use it. The S# is for the public domain, and I hope it adds to our better understanding of sailboat performance. Time will tell.

Questions?

MOTION COMFORT RATIO

At the end of last week's discussion of S#, I mentioned Motion Comfort Ratio which was included in the S# spreadsheet. Motion Comfort Ratio (MCR) is the invention of yacht designer Ted Brewer, which he describes as follows in his book *Ted Brewer Explains Sailboat Design*:

> *This ratio is one that your author dreamed up for an article in* Cruising World *magazine. The article was tongue-in-cheek but the comfort ratio has been accepted by many as a measure of motion comfort, and indeed, it does provide a reasonable comparison.*

"Tongue-in-cheek"??? Well, if he thought it a bit whimsical, Ted nevertheless had some sense of science in creating MCR. He does not show us too much about MCR in his book other than to state the equation and how it works. However, the *Cruising World* article appeared in the September 1990 issue, and MCR was mentioned in a sidebar article called "Looking At The Numbers," written by Danny Greene, one of *Cruising World's* editors. I attach a pdf file of the sidebar article, and MCR is described on the last page.

Here is the equation:

MCR = Displacement/(0.65*(0.7LWL + 0.3LOA)*BEAM^1.333)

Displacement is in pounds and the lengths for LWL, LOA, and BEAM are in feet. This is not a dimensionless number—we have units of pounds per feet^2.333 (add exponents of like units when they are multiplied together.)

In the *Cruising World* article, Brewer divides comfort zones into three parts [*Commoditas est omnis divisa in partes tres.*—"All comfort is divided into three parts.", to coin a phrase, with apologies to Julius Caesar. I can't help it; my sister is a Latin teacher.] These are called Lesser, Average, and Greater comfort. Not clever or original, but they get the point across. It is actually easiest to simply quote Ted Brewer's own description from his book, which is pretty clear:

> *The comfort ratio is based on the fact that motion comfort depends on the rapidity of the motion; the faster the motion, the more upsetting it is to our human gyroscopes. Given a certain force, such as a wave, the speed of motion depends upon the weight of the object (the boat) and the amount of surface that is acted upon (the LWL area). Greater weight, or lesser area, means a lower motion, thus more comfort.*
>
> *Beam enters into it also as wider beam will generate a faster reaction, particularly in beam seas. In effect, the comfort ratio measures the displacement of the vessel against its waterline area, adds a factor for beam, and thus is a means to compare motion comfort for boats of various sizes and types.*

> *One finds that smaller yachts, having a higher beam/length ratio, are lower on the comfort scale. Also, older designs get higher marks for comfort as they are from the era of heavy displacement and narrower beam. Comfort ratios will range from 5.4 for a Lightning class daysailing sloop to the high 60s for a heavy vessel such as a Colin Archer pilot boat. The moderate and successful ocean cruiser, such as the Whitby 42 [a Ted Brewer design—EWS] and Bob Perry's Valiant 40, will be in the low to middle 30s.*

And that is the sum total of what Ted says about MCR.

The chart in *Cruising World* magazine is a little more revealing, displaying a chart of MCR versus Length. I am guessing that Ted came up with these divisions with some sense of actual experience, because there is no other way in analyzing MCR that one could sense where the divisions should be based on the equation alone. Ted himself does not even have this chart in his book, nor does he state the definitions of the three parts, and he does not even list MCR in the index, so I wonder how seriously he takes this. Maybe not very much. But there it is, he gave it to us, so let's take a closer look.

In the spreadsheet, look at MCR Chart 01, Figure 12 below. This is exactly the same chart as in the *Cruising World* article, except that I have carried both length and MCR down to the origin: Length = 0 and MCR = 0. We see that the division lines between Greater, Average, and Lesser are all straight lines. That means that the MCR limits are always a set ratio in relation to length. Ah, but which length—LOA or LWL? Both Ted and Danny Greene speak in generic terms of LOA: "Whitby 42," "Valiant 40", "light displacement 50-footer", "heavy displacement 30-footer".

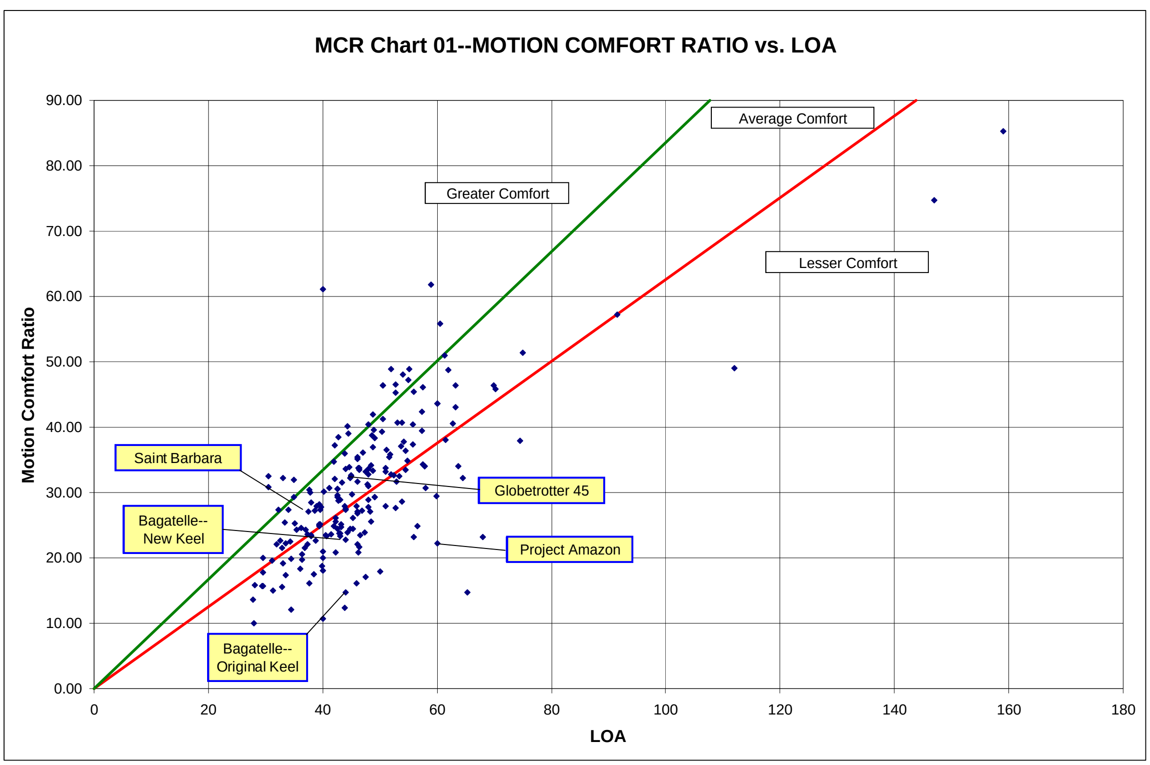

Figure 12. Motion Comfort Ratio, MCR, versus LOA with categories of comfort defined by Ted Brewer.

So I take “length” in the *Cruising World* chart to be LOA. That is how we think of boats in regular terms, by their LOA. If this is so, then in my chart I found that the division line between Greater and Average is always MCR => 0.835 of LOA, and the division line between Average and Lesser is always MCR <= 0.626 of LOA. So this gives us a useful tool—with any population of boat designs, we can easily calculate both the MCR and what division it is in.

Before I go further, let’s have a closer look at the equation. The numerator is easy, it is simply displacement in pounds. But look at the denominator, we basically have an equation for waterplane area = 0.65 x L x B. The 0.65 is a generic waterplane coefficient, but L is actually two parts, 70% of the length on the waterline plus 30% of the length overall. This gives a length that is just longer than the design flotation waterline, and the reason for doing this is to simulate an “actual” wetted waterline length as the boat moves through the water. The beam is overall beam raised to the 1.333 power to give it a bit more influence in the equation.

But why the power of 1.333? It is probably because we know that a boat’s rolling motion will have higher accelerations if the metacentric height is really large, and lower accelerations if metacentric height is really small. It’s the accelerations that kill you and make you feel uncomfortable. Metacentric height, GM, is proportional to the moment of inertia of the waterplane, which in turn is proportional to length and to beam cubed. Therefore, comfort is inversely proportional to L x B^3. Large beam is going to give higher metacentric height, and therefore lesser comfort. But in his MCR equation, I think Brewer is looking for “influence”, not equality. He wants a resulting MCR number that has the same order of magnitude as length, caused by the influence of beam. He could have used beam cubed, but that would make MCR a pretty small number. And if naval architects like anything, it is reasonable numbers, so Brewer lets beam have some greater influence, but not too much.

Now look at MCR Chart 02, Figure 13. This is a plot of S# versus MCR. The regions for S# are defined by colored horizontal lines, and the regions for MCR are the two diagonal lines across the chart. All regions for both factors are labeled. How did I come up with this chart? I did a sort of the data and plotted only those boats with high S#s and MCR labels of “Lesser” comfort, and drew the Lesser comfort boundary appropriately to the right side of all the data. Then I did the same with the Average boats and set that boundary. Finally, all the Greater comfort boats ended up in the lower right part of the chart. It was simply a brute-force method of charting. I show labels where some of my own designs, as I have mentioned before, appear to fall.

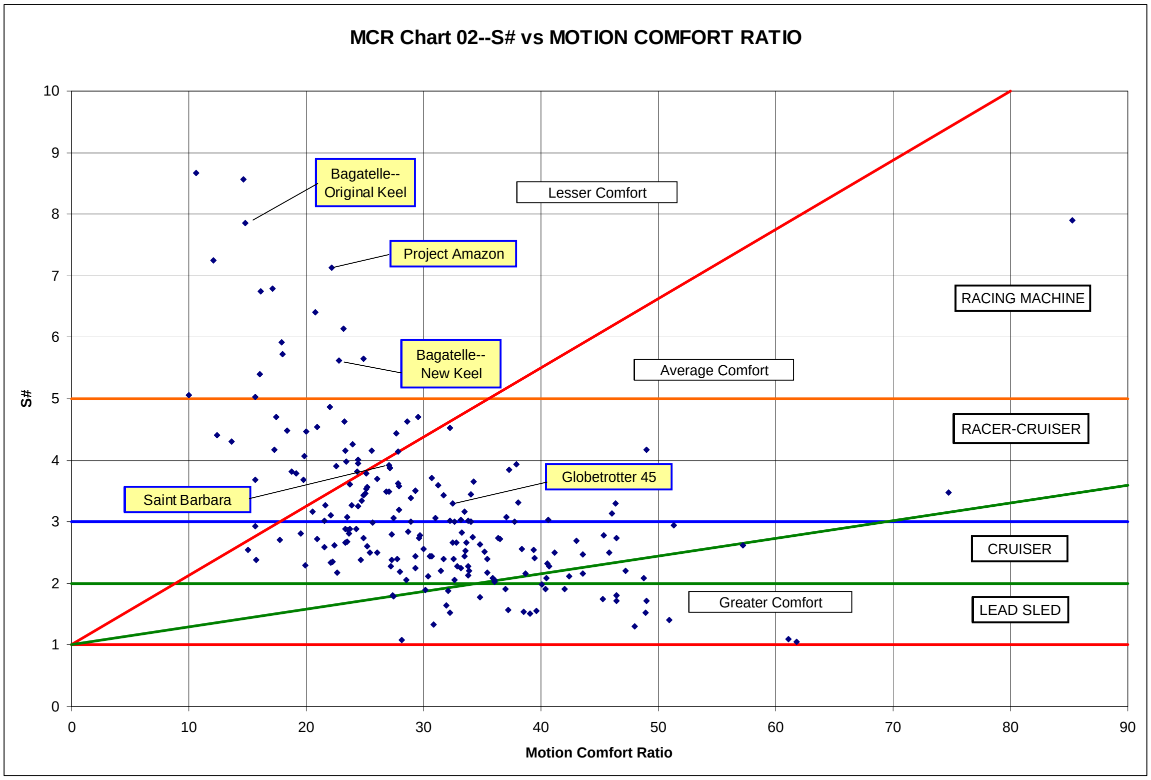

Figure 13. MCR versus S#--a very comprehensive view of sailboat performance.

If you think about it and study the chart, it seems to make sense. Certainly, boats with higher S#s are going to be faster, more lively, and therefore have less comfortable motion. Boats with lower S#s will be slower, less lively, and have more comfortable motion. It is all consistent and makes sense. So here we have in this chart a very interesting and revealing picture of boat performance. I show this to my clients as we discuss their new boat designs, and it reveals how a potential new design will fit in the overall scheme of things and in relation to other boats of known performance. With just the basic design parameters of length, beam, displacement and sail area, you can calculate both S# and MCR, vary the parameters, and see how the proposed boat moves around this chart. This helps the client and designer decide what the overall proportions should be for the performance that is desired. It is quite a useful analytical tool.

Next week, I would like to discuss Dellenbaugh Angle.

Questions?

DELLENBAUGH ANGLE

Up to now, the discussions of the various design ratios all had to do with hull form and displacement. Today, we move a little deeper into naval architecture with a topic that touches on stability—Dellenbaugh Angle for sailing yachts. I suppose that a comprehensive discussion of stability should precede this, but stability is a very broad topic, and it would take several weeks to cover it completely. Instead, I refer you to the basic design texts such as Kinney's *Skene's Elements of Yacht Design* or Larsson/Eliasson's *Principles of Yacht Design* for a thorough review. Probably one of the best texts that I have read on sailing yacht stability is "Chapter IV, Stability," in Pierre Gutelle's *The Design of Sailing Yachts*. Gutelle's book is not the best overall, but the stability chapter is very good. Going forward from here, you should have a good understanding of stability and the concept of metacentric height, GM.

So now, Dellenbaugh Angle. First of all, who was Dellenbaugh? Dellenbaugh was Frederick Samuel Dellenbaugh, Jr., a professor of Electrical Engineering at the Massachusetts Institute of Technology (MIT), beginning as an assistant professor in the early 1920s. He was also the coach of the 1923 men's heavyweight varsity crew team. At that time, MIT also had a prestigious department of Naval Architecture and Marine Engineering. In 1921, Dellenbaugh's master's degree thesis was on analyzing harmonic curves and plots by the use of mechanical machines like planimeters and integrators which we use in naval architecture. He even invented his own electric analyzing machine for this work. Interestingly, the righting arm stability curve of a floating vessel is a harmonic curve, and buried in the thesis (if one were to read it, which I did) you can see where Dellenbaugh Angle eventually came from, some of which I show below.

Frederick Dellenbaugh developed the Dellenbaugh Angle for sailing yacht analysis in the 1930s. How or why the calculation actually came about, I don't know, but it is fairly simple as you'll see below. It is known that Dellenbaugh had some correspondence with Olin Stephens of Sparkman & Stephens (S&S) in the 1930s, which would have been early in Olin's career as a yacht designer. If you Google "Dellenbaugh Angle" on the Internet, all references point back to Francis Kinney's version of *Skene's Elements of Yacht Design* which was first published in 1962, and then later revised in 1973 (the version I have). Francis Kinney worked on and off for S&S from after WWII until the 1970s. So, barring any other facts that may be known by others, I would say that Dellenbaugh Angle came about during this association between Olin Stephens and Frederick Dellenbaugh, Jr.

Just to fill in the picture (because I like the history so much, and it gives some context), Frederick Jr's. father was Frederick S. Dellenbaugh, Sr., who was a renowned artist, photographer, explorer and traveler. He participated in the second Powell expedition on the Colorado River in the 1870s, and in the Harriman maritime expedition of the coast of Alaska in the 1899. Frederick Jr's. son was Warren G. Dellenbaugh, who was a principal in a company called US Yacht which was instrumental in the development of O'Day Yachts in the US, a well-known builder of fiberglass sailboats. Warren's sons, and Frederick Jr's. grandsons, are world-renowned sailing experts Brad and David Dellenbaugh.

Here is the equation for Dellenbaugh Angle (DA):

$$DA = \frac{57.3 \times \text{Sail Area} \times \text{Heeling Arm} \times 1.0}{\text{GM} \times \text{Displacement}}$$

Where:
Sail area is in square feet.
Heeling arm is in feet and is the distance between the center of effort of the sail plan and the center of the lateral plane of the underwater profile of the hull.
GM is the metacentric height in feet.
Displacement is in pounds.

Consistent metric units will give you the same result, with the lengths in meters, area in square meters, and displacement in kilogram force. The number 1.0 (pounds per square foot) in the numerator must change to 4.883 (Kilogram force per square meter).

This is simply the heeling moment divided by the righting moment resulting in an angle of heel when the wind pressure is one pound per square foot (or equivalent metric measure). This may be hard to see, so let's look further. You should recall from your study of stability that:

Heeling Moment = Sail Area x Heeling Arm x Wind Pressure (1 lb/sq.ft. in this case)

And:

Righting Moment (at any angle of heel) = Displacement x Righting Arm

We know that Righting Arm = GZ = GM x Sin θ

So, Righting Moment = Displacement x GM x Sin θ

Refer to DA Chart 01, Figure 14, which shows the full righting arm curve for my Globetrotter 45 design, *Eagle*, which favored very well in the Westlawn Yacht Design competition in 2007 (it placed 7th out of 10 in the official judging, but it won the popular vote by a wide margin). At small angles of heel, you can see that the value of righting arm, GZ (black line), closely approximates its own slope at zero degrees of heel (green line).

The slope of the GZ curve = GZ/θ = GM x Sin θ/θ

Since Sin θ approaches θ as θ approaches zero, the slope of the curve at the origin is the metacentric height. That is, if the righting arm continued to increase at the same rate as at the origin, along the green line, it would be equal to GM at a heel angle of one radian, 57.3º. Look at DA Chart 02, Figure 15, which is a close-up of the origin of Chart 01. Notice that the black GZ line is close to but not exactly on the green slope line. At about 12º heel, the black line departs significantly from the green slope line, and this is because as the boat continues to heel, the shape of the waterplane gets narrower, so its moment of inertia is less, and as a result the metacenter slides down closer to the

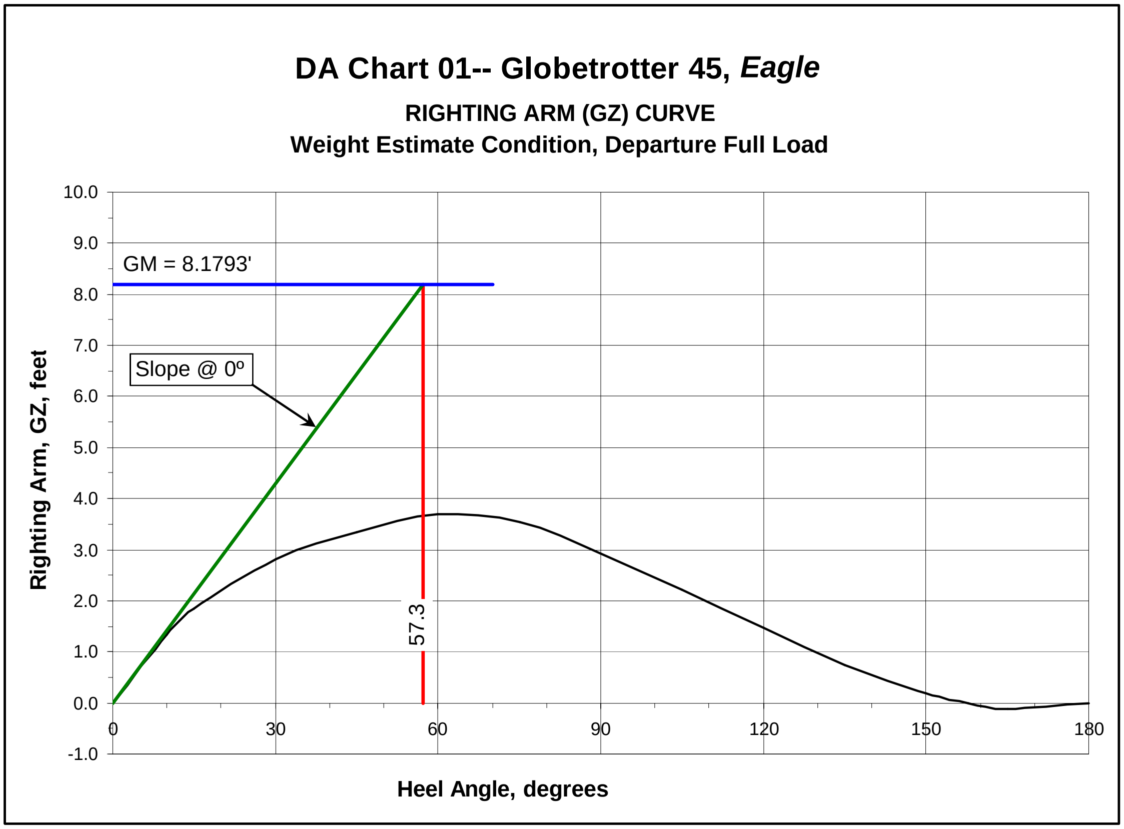

Figure 14. Dellenbaugh Angle for SYDI *Globetrotter 45* design *Eagle*, which shows how DA is defined.

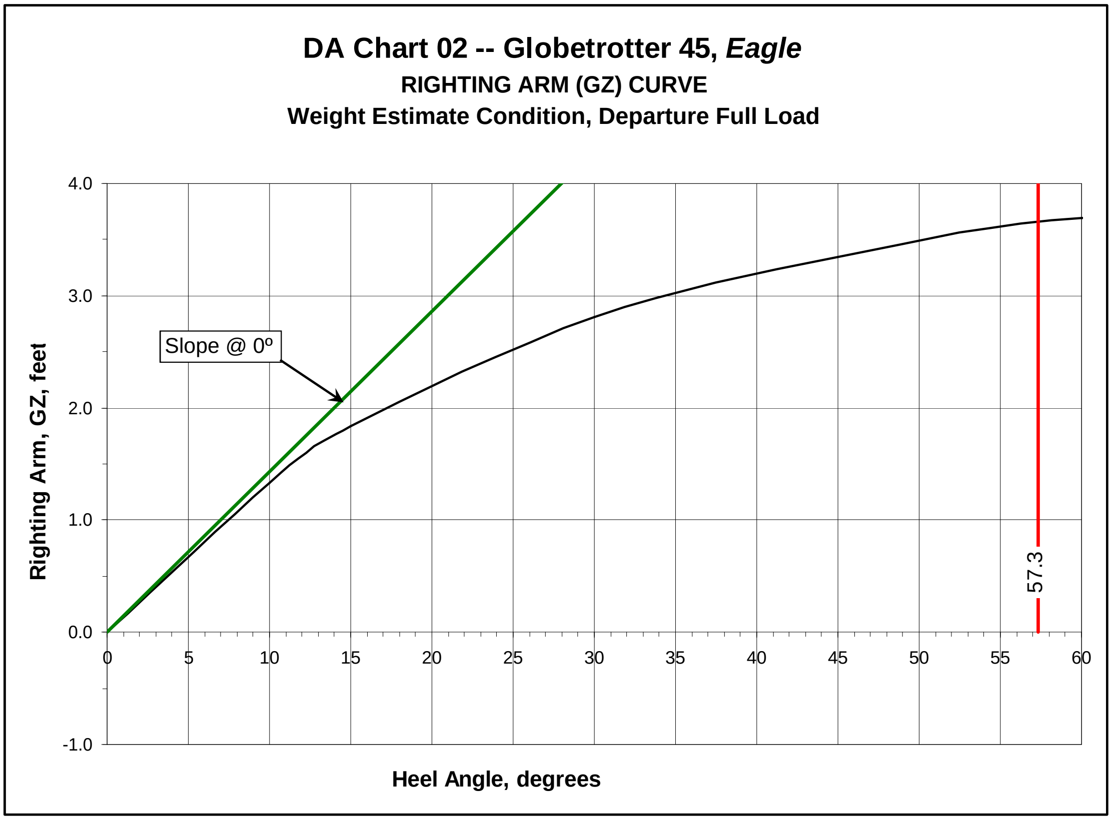

Figure 15. A close-up of the origin of Figure 14 for Dellenbaugh Angle.

waterplane, and so GM is slightly smaller. As GM goes smaller, GZ goes not increase as fast, and so the black GZ curve starts to bend away from its zero-degree slope.

However, at small angles of heel, and without doing a full righting arm curve calculation, we can approximate the righting moment at 1º of heel:

$$\text{Righting Moment at 1º of heel} = \frac{\text{Displacement x GM}}{57.3}$$

If we divide the Heeling Moment at wind pressure of 1.0 pound per square foot by the Righting Moment at 1º of heel, we will get the approximate heel angle at which the righting moment of the boat equalizes the heeling moment. This will be the estimate of heel angle along the green slope line, the Dellenbaugh Angle:

$$DA = \frac{\text{Heeling Moment}}{\text{Righting Moment at 1º}}$$

Substituting for Heeling Moment and Righting Moment at 1º of heel:

$$DA = \frac{\text{Sail Area x Heeling Arm x 1.0}}{\text{GM x Displacement/57.3}}$$

Straightening out the math signs:

$$DA = \frac{\text{57.3 x Sail Area x Heeling Arm x 1.0}}{\text{GM x Displacement}}$$

This is the equation for Dellenbaugh Angle that we started with. Obviously, the smaller the DA, the stiffer the boat is. Stiffer boats will generally sail better—be faster and point higher—than tender boats.

That is, Dellenbaugh Angle is the estimate of how much the boat will heel over in moderate conditions. One pound per square foot of wind pressure is equivalent to a wind speed of about 16 miles per hour, Beaufort Force 4. If you look at *Skene's Elements of Yacht Design*, page 297 (8^{th} edition, Kinney), you'll see a chart of wind pressure versus wind speed in miles per hour, which is simply a plot of what Kinney calls Martin's Formula, P = 0.004 x V^2. The student is encouraged to prove the validity of this equation. The 0.004 is the result of the density of air and the consistency of units using a wind speed in miles per hour against a sail area of one square foot.

Dellenbaugh Angle is quite accurate at small angles of heel, where the green slope line and the black GZ line come very close together. DA is less accurate at higher angles of heel where these lines start to separate.

Dellenbaugh Angle is quite useful for estimating the stability of a boat design early in the design process, before you have created the 3D hullform when you can do a full stability curve calculation. For example, when I was designing the Scandinavian Cruiser 40 in 2008, the client and I were going through various permutations of the sail plan as the overall arrangement of the boat was being worked out. I was working in AutoCad with my intended hull shape, but this was before I had even started on a 3D hull model. I made some estimates of the stability of the boat based on the drawings and weights up to that time, and I used Dellenbaugh Angle calculations to work out the heel angle of various sail combinations and wind speeds. This helped us focus on the desired sizes and arrangement of the sails. See the sample plot below, Figure 16, SC40 Heel Angle.

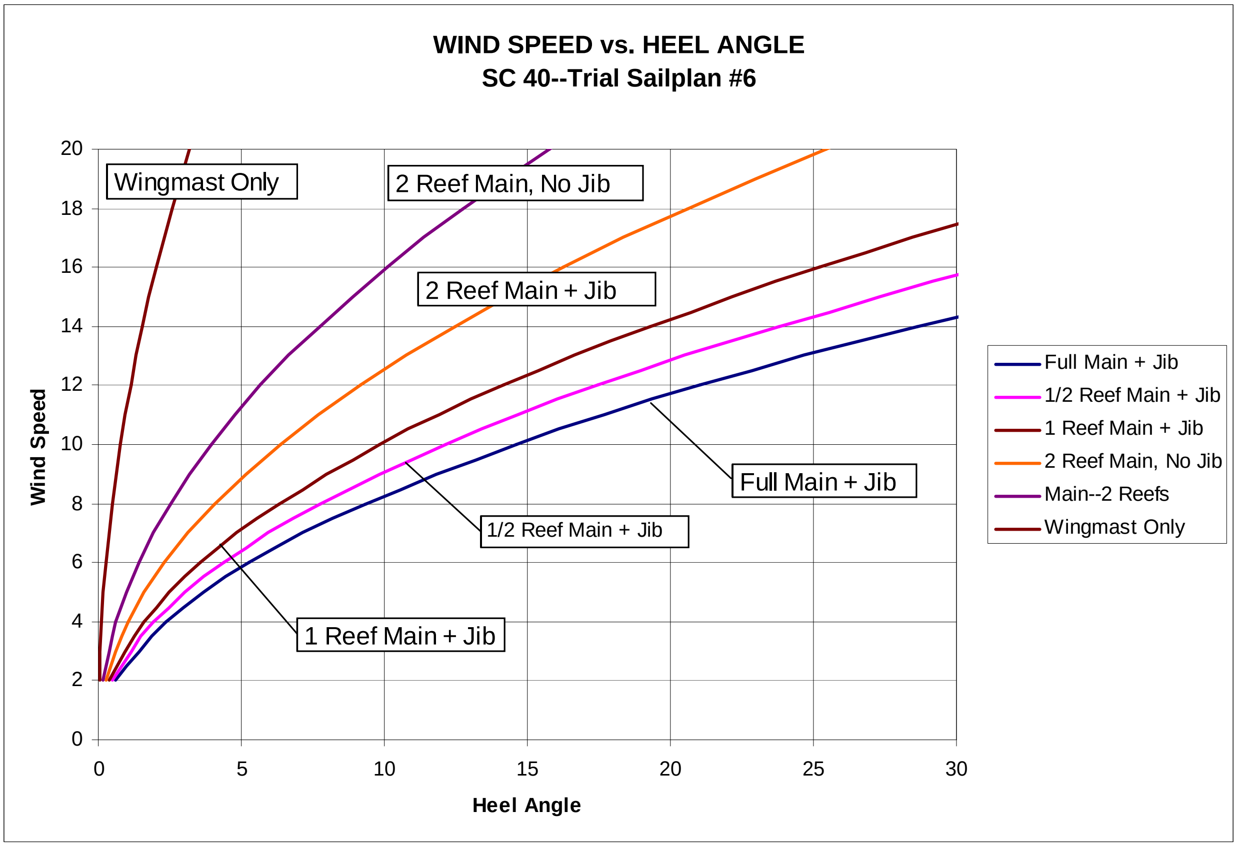

Figure 16. Dellenbaugh angle for various sail combinations of SYDI's design of the *Scandinavian Cruiser 40*, a "Boat-in-a-box" daysailer that will be built in China and shipped in standard 40' shipping containers.

In *Skene's Elements of Yacht Design*, Kinney shows a table and a chart of Dellenbaugh Angles for various types of craft, and he separates these out by length and whether they are centerboarders or keel boats. The chart was first published in Kinney's 1962 edition, which he then updated 10 years later. The caption to the chart states, "Today (1972) new keel sailboats are 25% stiffer than shown on this chart." I dare say that this trend has probably continued in general, that boats with the current crop of keel designs are considerably stiffer than they were back in 1972 (now, 38 years later).

Well, that was a pretty involved topic, and I hope everyone understood it.

Questions?

BRUCE NUMBER

This week we venture into the world of sailing multihulls with a discussion of Bruce Number, which is basically a Sail Area/Displacement ratio. Unfortunately, you don't find this in any of the classical design texts like *Principles of Yacht Design* or *Skene's Elements of Yacht Design*, because those books don't cover multihulls. But Bruce Number is quite important to multihull designers and enthusiasts.

First of all, who was Bruce? Well, you maybe have heard of the Bruce Anchor or perhaps the Bruce Foil? Same guy. Edmond Bruce was British, an active member and prolific writer for the Amateur Yacht Research Society (AYRS) in the United Kingdom which published his seminal book, *Design for Fast Sailing*, publication #82, 1976, written with co-author Henry A. Morss, Jr. In this book is described the Bruce Number:

BN = SA^0.5/Displ^0.333

Where:
SA = Sail Area in square feet
Displ = displacement in pounds

In words, the Bruce Number is the square root of the sail area divided by the cube root of the displacement. It is not dimensionless, and the units are imperial for a reason, feet per pound. Bruce felt that a Bruce Number should approximate the boat's speed ratio compared to the true wind speed. That is, a Bruce Number greater than 1.0 meant that the boat speed could exceed the true wind speed on some points of sail in some conditions. Obviously, this thinking will not hold with metric units. However, if using a metric version of Bruce Number consistently for comparison of different designs, then it can be valid merely to compare the power-to-weight ratios of different designs. That's all that Bruce Number is—a power-to-weight ratio.

I have to admit to being not as completely versed in multihull design as perhaps some of you multihull enthusiasts are, and you may know a lot more about Bruce's work than I do. Those who are more interested in reading about Bruce's multihull research can purchase his book from the AYRS. Another thread on this forum has useful multihull performance information, at this link:

http://www.boatdesign.net/forums/multihulls/multi-speed-length-relationship-22529.html

That is, about the first half of the thread is interesting; the second half descends into bickering among the posters, so you can ignore that.

Another interesting and informative website is Multihull Dynamics Inc., whose link is here:

http://www.multihulldynamics.com/

Bruce Number has a few significant drawbacks that prevent it from being more comprehensive than you may think. In addition to sail area and weight, a sailing multihull derives power from the distance between the hulls (wider hull-to-hull beam =

more stability = more power) and from the length-to-beam ratio (higher length-to-beam ratio = less wave making drag = more power). But these factors do not appear in the Bruce Number equation. So if you are comparing two multihulls of the same length and weight so that they have the same Bruce Number, the boat with the wider spread between the hulls will have more power and potentially faster speed. Likewise, a boat with narrower hulls compared to a sister generally will have less hull drag and, therefore, more speed. Therefore, Bruce Number has to be used judiciously in order to make valid comparisons.

I used Bruce Number recently in a design project on my desk right now. A client wants to build his own 24’ catamaran, and he wants to go really fast. He does not want to race; he just wants the thrill of fast speed. He also wants to build the boat himself so that he can hone his boatbuilding skills. This is a stepping stone to a grander project. The next boat will be a 35’er that he can use for charter sailing and fishing. Finally, he wants to build a 70’er for a charter fishing business in the Caribbean. This process will take some years, but the client is only in his late twenties or early thirties, so he has plenty of time.

Unfortunately, in my research of comparable designs, there just weren’t that many at that size to compare to. Most beachable multihull designs range from 12’ to 20’, and between 23’ and 30’, I found four Stiletto models and two RC models. So I did a parametric analysis on the boat listings that I could find and made various plots. See the Figures 17, 18, 19, and 20. These are:

Sail Area vs. Length
Displacement vs. Length
Sail Area vs. Displacement
Sail Area/Displacement Ratio vs. Bruce Number

I also attach the spreadsheet so that you can see the original data and charts for yourselves.

These data in the spreadsheet are all published figures, and I did not try to verify any of the specifications independently. My goal was simply to find out that if I want to design a 24’ catamaran that can go really fast, what weight and sail area ball park should I be in? From the first three plots, the scatter of the data is all over the place, although the first plot of Sail Area versus Length is the tightest. But once you address weight, consistency goes out the window. However, I know I am going to be in the realm of the Stiletto and RC catamarans, and the other designs at the lower end of the scale help to give me some context as to where the design should be. Interestingly, the Stiletto boats, which are all made using carbon fiber pre-prepregs and Nomex honeycomb—meaning really lightweight construction—are in fact really quite heavy boats and not so lightweight at all.

The final plot, Figure 20, is actually almost an identity—two factors almost exactly the same plotted against each other. These are Sail Area/Displacement ratio, which we talked about before, versus Bruce Number. One would expect a very tight plot like this. In all these plots, I have labeled the exemplar boats (Stiletto and RC) so that I can place my new design in context with them. If boats of the Stiletto and RC weights are possible, then I am confident that my client can build his boat at my intended weight with conventional composite materials.

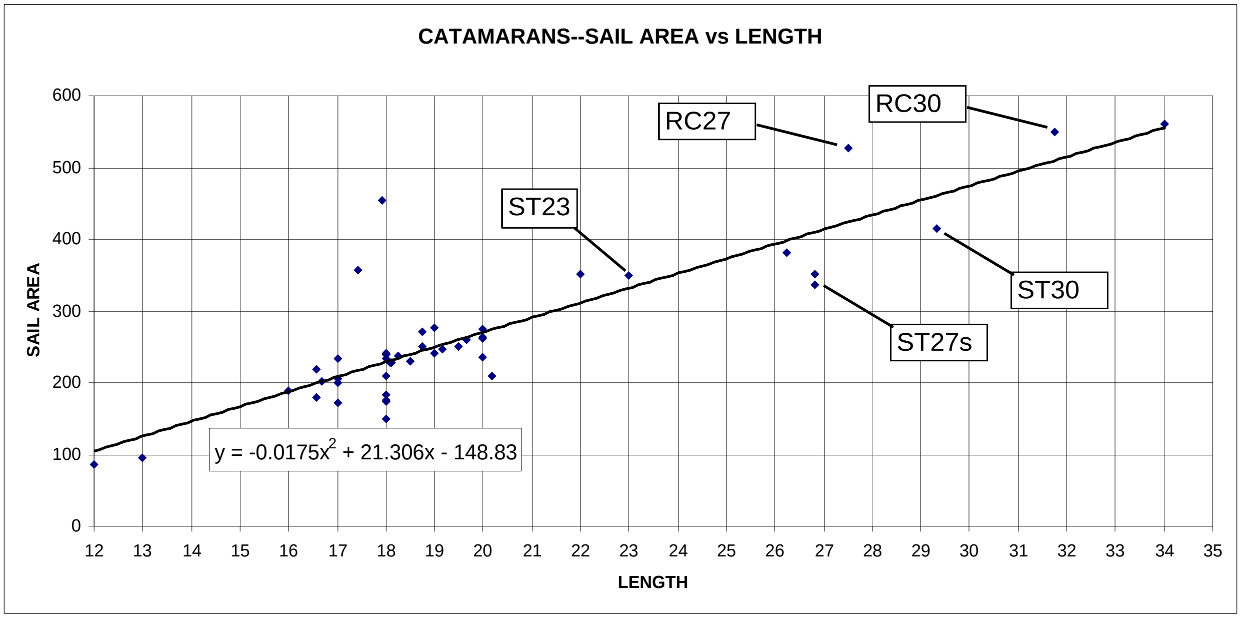

Figure 17. A population of current catamaran designs, Sail Area versus Length. Plots like this are called parametric analysis, that is, an analysis of various design parameters (dimensions).

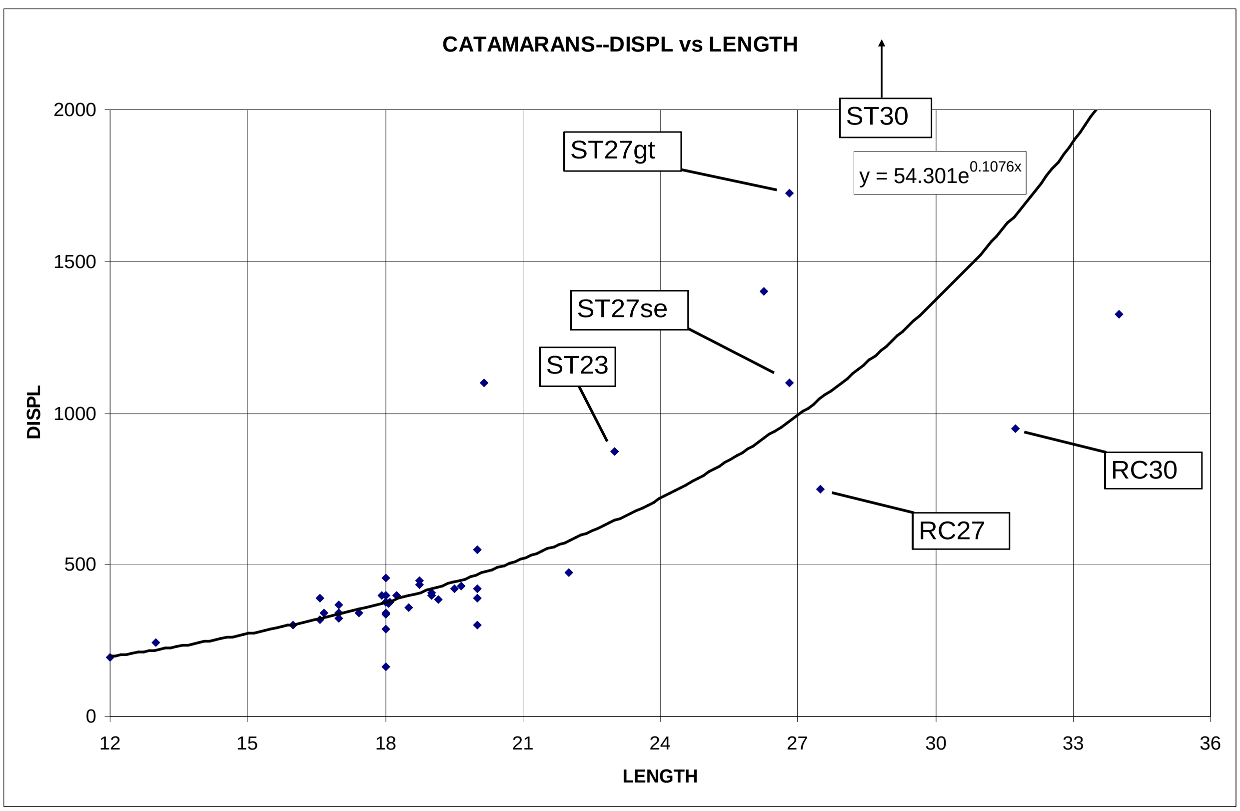

Figure 18. Catamaran study, Displ versus Length.

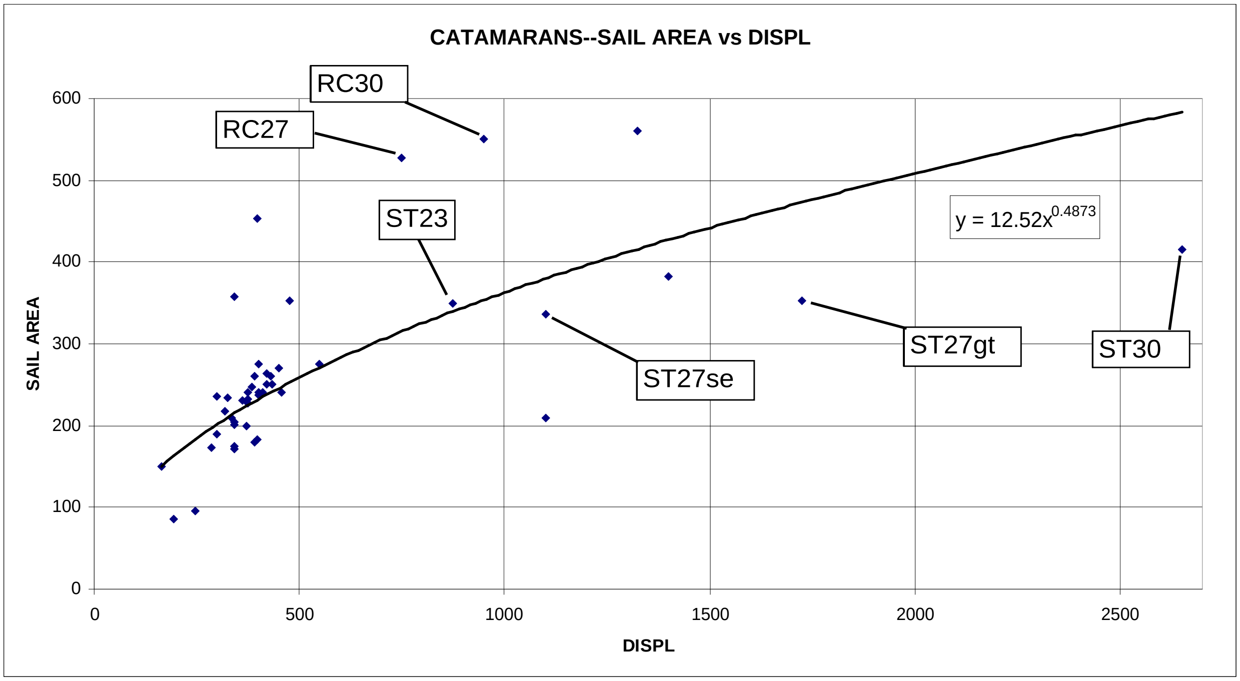

Figure 19. Catamaran study, Sail Area versus Displ.

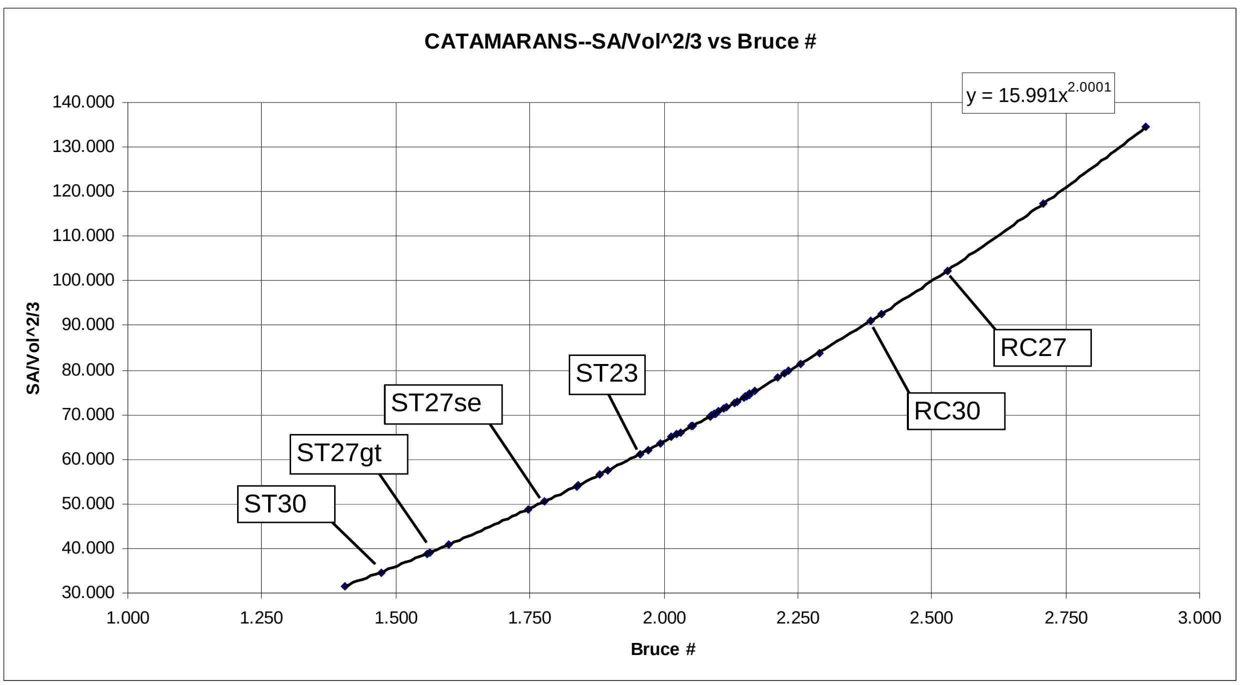

Figure 20. Catamaran study, Sail Area/Displ ratio versus Bruce #, nearly an identity because the numbers are of the same type of parameter and the spread of data is extremely tight.

So, what do I draw from these plots? I decided that I did not want to go too far out on a limb because my client is a novice, and so I chose a Bruce Number of 2.5. This puts me squarely between the RC 30 and the RC 27 by comparison. I know my client is going to be experimenting with this project. If the first boat is not right, he is ready to modify it or build a second or a third one. The big variations that he is equipped to toy with are beam spacing between the hulls and sail area. So Bruce Number leads me to a starting point where I know he can build a reasonably lightweight boat that can go fast. I attach a rendering of the hull that I finally developed. Picture two of these hulls side-by-side in a spread that has yet to be determined.

At one point, my client and I talked about chined hulls and lifting strakes, and I recommended we wait on those concepts until he has been around the block at least once with building a basic design. Once we get some time on the water with what he can build, then we can do systematic changes to improve performance.

In this instance, Bruce Number proved useful because it gave me a snapshot of the current market in small beach-type and trailerable cats. This is a tool to evaluate initial performance so that I can place my design in an area where top performance is possible, without being too dangerous.

And so ends the discussion for today. I welcome any input from other multihull enthusiasts who care to shed some more light on this topic.

We are approaching the end of this discussion thread on the design ratios. I have two topics left, and they involve speed and power calculations for powerboats. These are the Displacement Speed Formula and Crouch's Planing Speed Formula which I will cover in the next (and last) two weeks. I have found these formulas to be particularly useful in my work, and so would like to review them here.

When this series is over, I will collect all the lectures (12 of them) in a master pdf document and post it here so that you can download it as a complete collection.

Questions?

DISPLACEMENT SPEED FORMULA

Taking another 90º turn in naval architecture (from sailing monohulls two weeks ago, to multihulls last week, and now to powerboats and motoryachts), we'll cover the displacement speed formula. You might well ask, "Which displacement speed formula—there are several of them." Right you are, and the one I am referring to comes from Dave Gerr (pronounced like "bear" as in black bear or grizzly bear, not "Geer," as in transmission gears). Dave Gerr, as you may know, is a very well known boat designer and the director of the Westlawn Institute of Marine Technology, the correspondence school in boat design.

A second question you might ask is why are we breaking away from design coefficients and considering speed and power? Good question. First of all, we have just about used up all the good design coefficients, and secondly, as boat designers and naval architects, we have to be aware of the relationships between boat speed and the power required to reach a given speed—that's the whole point of designing, building, and using boats: to travel over the water at some speed. One very well known naval architect once said that "Performance is boat design objective number one." He was right—no matter what else your boat design is supposed to be or to do, it has to move well through the water. Think about it—if a boat does not move well, it's useless, and there isn't any other design feature that will ever compensate for poor performance, really. Note that I do not say that the boat has to be the fastest or to move efficiently, but only that it has to move well. That is, the boat has to achieve a speed that meets its design objective.

Also, it is my belief that any naval architect or boat designer worth his salt could conceivably design any sort of craft because he understands the physical, mechanical, and engineering principles involved. He may not choose to design certain craft, but he could if he wanted to. And ladies, I use the pronoun "he" generically—what I say applies to you as well; you are included. A good naval architect or boat designer will be well versed in performance criteria, whether they apply to powerboats or sailboats.

Unfortunately, the following discussion is not covered very well in either Larsson and Eliasson's *Principles of Yacht Design* or in Kinney's version of *Skene's Elements of Yacht Design*. But fortunately, Dave Gerr published his displacement speed formula in his excellent book, *The Propeller Handbook*, by International Marine in 1989. By the way, I'm the guy that wrote the *WoodenBoat* magazine review of this book, issue #92, wherein I said, "Buy two copies, because the first one will probably wear out fast." I meant it—if you don't have this book, go buy it. Now.

In *The Propeller Handbook*, Chapter 2, "Estimating Speed," on page 10, Gerr states his displacement speed formula:

SL ratio = 10.665/(LB/SHP)^0.333

(and let's call this formula "Version A", for reasons that will become apparent shortly.)

Where:

SL = Speed/Length Ratio = V/Lwl^0.5 (which we covered earlier)
LB = Displacement in pounds (and let's use our earlier designation "Displ")
SHP = Shaft Horsepower at the propeller
V = Boat speed in knots
Lwl = Length on the waterline in feet

Gerr qualifies this equation by saying it is useful for predicting displacement and semi-displacement speeds, and he provides a chart of SL Ratio versus LB/HP with delineations of displacement speeds (up to SL Ratio = 1.4) and semi-displacement speeds (up to SL Ratio = 2.9). He further says that it is assumed that the propeller has an efficiency between 50 and 60 percent, "with 55 percent being a good average." Yes, that is pretty average—almost all conventional propellers center around 55% efficiency.

We know that speed/length ratio is not dimensionless, and in order for this equation to work, the other side of the equal sign must be of the same units as speed/length ratio. That's where the coefficient 10.665 comes in. Part of the coefficient's role is to hold all the conversion factors to make both sides of the equation equal each other in consistent units. But the other thing that is important to this equation is that 10.665 depends a little bit on what kind of boat you are looking at. For example, this value may be the appropriate numerator for lobster-style powerboats, but not so for twin-screw motoryachts, or vice versa. You have to be careful. When comparing two different boats, particularly of different sizes, they should be of the same family or hull style. Use the equation with caution.

For example, when I was designing the Moloka'i Strait motoryachts, which are very heavy displacement, we made very good speed predictions for the MS 65 with the model testing that we did at the Institute for Marine Dynamics in St. John's, Newfoundland, Canada, under the guidance of Oceanic Consulting Corporation. During later variations in the Moloka'i Strait designs at other lengths, from 58' to 85', I was able to make quick predictions of required power using the displacement speed formula rather than go back through more complicated test data and calculations. But to be careful, I recalculated the coefficient.

On the MS 65, displacement was 181,000 lbs, SHP (actually rated BHP) was 440 HP, and Lwl was 56.58'. It is OK to use BHP instead of SHP, so long as you keep your factors consistent, and you have to assume that the drive trains are going to be similar—similar transmissions, shafting, and bearings. I knew that with these values we had indeed achieved hull speed, SL ratio = 1.34. But when I calculated the other side of the equation, the speed length ratio came out greater:

10.665/(Displ/BHP)^0.333 = 10.665/(181,000/440)^0.333 = 1.434

Which was too high—both sides of the equation did not equal each other. So I recalculated the coefficient:

Coeff = 1.34 x (Displ/BHP)^0.333 = 9.966

Part of this change in coefficient accounts for the fact that I am using BHP instead of SHP, and another part is due to the fact that it was my unique hullform. However, using the new coefficient, I could solve for the BHP of the new MS 58 motoryacht which was going to be of similar hull design as her larger sister with displacement of 115,745 lbs and Lwl = 48.75'. Using my "Moloka'i Strait" coefficient of 9.996, I can reliably calculate the BHP that I need in the new design:

SL ratio = 1.34 = 9.966/(Displ/BHP)^0.333

Or, so that we have only one division sign in the equation, we can flip the fraction in parentheses upside down:

SL ratio = 1.34 = 9.966(BHP/Displ)^0.333

Solve the equation for BHP, the only unknown:

BHP = 1.34^3 x Displ/9.966^3 = 2.406 x 115,745 / 989.835 = 281 HP

So, I knew to start looking for engines in the 280-300 HP range, and I was confident that I had a very reliable result.

And the speed was going to be, at hull speed:

V = 1.34 x Lwl^0.5 = 1.34 x (48.75)^0.5 = 9.36 knots

Well, this is all very well and good, but then two years ago, Dave Gerr changed his formula. In the June 2008 issue of *The Masthead*, the design newsletter from Westlawn, Dave published this version:

SL ratio = 2.3 – (((Displ/SHP)^0.333)/8.11)

(and so we'll call this "Version B")

And, solving for speed:

V = Lwl^0.5 x [2.3 – (((Displ/SHP)^0.333)/8.11)]

The variables are the same as before, but the equation is totally different. Dave Gerr says that this version is more accurate at SL ratios below 2.0, and it has the benefit of not requiring a coefficient that can change with hull type.

You can access *The Masthead* newsletter at this link for a complete discussion of Dave's views on displacement speed formula:

http://www.westlawn.edu/news/WestlawnMasthead06_June08.pdf

In fact, *The Masthead* newsletters are open to the public domain and you can browse through and download any of the issues that you wish. Explore the Westlawn website; this is quite a good design resource.

I have not worked with this version of the displacement speed formula yet, as I found Version A fairly easy to deal with and am used to it. You will see that in both versions of the formula, speed is dependent on the square root of vessel length, and the cube root of the quotient of vessel displacement and horsepower. Why is that? It all comes down to the definitions of horsepower and hydrodynamic force.

The horsepower needed to drive a vessel through the water is called "Effective Horsepower", EHP. Power is force driven at a speed, F x V: pounds times feet/second in imperial units. One horsepower is 550 lbs-ft/second. The equation for EHP for vessels is:

EHP = (Rt x V)/325.6

Where:

Rt = Total resistance of the vessel moving through the water, a force, in pounds
V = Vessel speed, in knots
325.6 is the unit of horsepower converted so that we can use knots of speed, which we are familiar with, instead of feet per second, which we do not typically use at this level of design.

325.6 = (550 ft-lbs/sec)/(1.689 ft/sec/knot)

We also know that any hydrodynamic (or aerodynamic) force, such as vessel resistance, can always be expressed as follows:

Force = (CpAV^2)/2

Where:

C is a coefficient, be it lift or drag or whatever
p is the mass density of the fluid involved, be it air or water
A is generally area, but it can be an function of length squared (which is area, by definition)
V is the speed of the thing through the fluid.

You will recall that all aerodynamic and hydrodynamic forces can relate back to this form. We don't care at the moment what the constants or coefficients are; we only care that Force is proportional to speed squared. Vessel resistance is a force—it has the same units, pounds—and so it is proportional to speed squared. We also know from our discussion of displacement-length ratio that resistance is directly proportional to displacement. This is to be expected; they have the same units—pounds. So, we can substitute these facts into the EHP equation above, and while we are at it, let's ignore the constants and make the equation a proportional relationship:

EHP ~ Rt x V

Rt ~ Displ x V^2

EHP ~ Displ x V^2 x V = Displ x V^3

That is, EHP is proportional to displacement times the cube of the speed.

Solve for speed and we have to take a cube root:

V ~ (EHP/Displ)^0.333

But, the term (EHP/Displ)^0.333 occurs in the denominator of Version A of the displacement speed formula, and that is why EHP and Displ are reversed:

V ~ 1/(Displ/EHP)^0.333

Which is saying the same thing.

Corresponding speeds here are very important, and that is why we have speed-length ratio on the left side of the equation. That is the “corresponding speed” as we learned when discussing speed/length ratio and Froude number. For resistances of different models of vessels to be the same, their corresponding speeds—speed-length ratios—have to be the same.

In Version B of the displacement speed formula, Dave Gerr brings the term (Displ/SHP)^0.333 into the numerator. Technically, he probably should have inverted the fraction, but he goes on to change all the other numbers and the form of the equation so that he can get away from having to use a varying coefficient due to changes in hull form. Be that as it may, the formula is still a function of the cube root of a quotient of Displ and Horsepower.

A further note on power. EHP is the effective horsepower necessary to drive the vessel through the water. This is exactly equal to the horsepower delivered by the propeller, that is, the power on the after side of the propeller blades. The power on the forward side of the propeller blades is the Shaft Horsepower at the prop, and the quotient of EHP to SHP is the overall propeller efficiency.

ηp = EHP/SHP

or written another way:

EHP = ηp x SHP

So EHP is directly proportional to SHP, and SHP, of course, is a direct function of break horsepower, BHP. SHP is the rated horsepower of the engine less the losses due to the transmission and the shafting bearings. Since these horsepowers are all directly related, we can interchange them into the displacement speed formulas and compare different vessels so long as we keep all the types of horsepowers consistent. That is, if we have a known SHP from one vessel, and we are trying to use displacement speed formulas to find the powers or speeds for a different vessel, we will be using or finding SHP for the new vessel. Similarly, if we start with BHP of the known vessel, we will be using or finding BHP for the new vessel.

You are encouraged to read more on horsepower and speeds. Review *The Masthead* newsletter in more detail. Pay particular attention to the fact that speed-length ratio—corresponding speeds—is very important, and that the displacement speed formulas apply to vessels only going at displacement speeds, and maybe at a stretch, semi-displacement speeds.

You will also see in *The Masthead* a discussion of Wyman's displacement speed formula. This was first published in the August/September 1998 issue of *Professional Boatbuilder* magazine, issue # 54, by naval architect and professional engineer David Wyman, where you can read the complete description of his formula as well. Wyman's formula also gets away from having to use coefficients varying with hull form, and actually, it is basically identical to what is known as Keith's formula for power. We'll cover that next week along with Crouch's formula in our last session.

That's quite a lot of information.

Questions?

**CROUCH'S PLANING SPEED FORMULA,
KEITH'S FORMULA, AND
WYMAN'S FORMULA**

In our final discussion of this series, we take up one primary performance formula, Crouch's Planing Speed Formula, and two other ones, Keith's Formula and Wyman's Formula which I mentioned at the end last week on the discussion of displacement speed formula. Boats travel either at displacement speeds, or planing speeds, two very different hydrodynamic regimes. There is, of course, the semi-displacement regime which is that middle region between the two, but for the sake of discussion, we will differentiate these two basic regimes of motion.

Larsson and Eliasson's *Principles of Yacht Design*, 3rd edition, discusses high speed craft, but they don't mention anything about Crouch's Formula. However, *Skene's Elements of Yacht Design* does, as does Gerr's *Propeller Handbook*. I have used Crouch's Formula a number of times, and it always seems a little bit different each time. First of all, who was Crouch?

George F. Crouch was a famous American naval architect in the early part of the 20th century. An 1895 graduate of Webb Institute of Naval Architecture, he went to work in industry for nearly a decade, but returned to his alma mater to become professor of math for about ten years, then ultimately became a full professor of naval architecture and resident manager of the college. In 1923, I guess he'd had enough of academia after nearly two decades, and decided to go where the money was. I may not have the order perfectly correct, but we know that in 1924, Crouch was vice president of design for Dodge Watercar, a new boatbuilding venture and brainchild of Horace E. Dodge Jr., son of one of the founding brothers of the Dodge automobile manufacturing company. Crouch held this position for a number of years. Horace wanted to race boats, and he also wanted to manufacture an "everyman's" boat on an assembly-line basis much like his father's automobiles, hence the name "watercar". This was to be in direct competition with the Chris Crafts and the Gar Woods of the time, all of them centered in the Detroit, Michigan, area.

I guess Crouch was still able to do consulting and custom design on the side because his iconic racing boat design *Baby Bootlegger* became the Gold Cup racing champion in 1924, and it repeated its victory again the following year. I say iconic because *Baby Bootlegger* really was a different stroke in boat design with three notable features: An unusual rounded shear; a canoe stern that overhung the aft end of the planing surfaces by some few feet; and an innovative wedge-shaped rudder. Many reproductions of *Baby Bootlegger* have been built; you can buy plans and build one yourself; and you can buy model kits of this famous racer. *Baby Bootlegger* probably overshadowed everything else that Crouch designed, including sailboats later in his career. *Baby Bootlegger* still exists and was meticulously restored about 25-30 years ago. A wonderful article about her appeared in *WoodenBoat* magazine, issue #60, September/October 1984. Crouch also worked for Nevin's Boatyard in New York for a long time, where *Baby Bootlegger* was built, and he died in 1959.

So what is Crouch's Formula, which he surely used to great effect in his power boat designs? Here it is:

Speed, V = C/(Displ/SHP)^0.5

Where:
V = speed in knots
Displ = boat displacement in pounds
SHP = shaft horsepower at the propeller
C = a coefficient depending on boat type.

Kinney in *Skene's Elements of Yacht Design* gives a general range of the value of C as from 180 to 200. This is not too helpful, but Gerr gives a better breakdown in *The Propeller Handbook*, namely:

C	**Type of Boat**
150	Average runabouts, cruisers, passenger vessels
190	High-speed runabouts, very light high-speed cruisers
210	Race boat types
220	Three-point hydroplanes, stepped hydroplanes
230	Racing power catamarans and sea sleds

Personally, I don't let these coefficients color my thinking too much. I know there is a range, but what you are supposed to do is find similar boats of the type that you are analyzing or designing, and back-calculate what the coefficient C is. That is:

C = V x (Displ/SHP)^0.5

Then proceed with that coefficient with the boat that you are analyzing or designing. That is, C is very much an empirical number—it changes all the time. Note that C must have units that allow the quantity 1/(Displ/SHP)^0.5 to end up in knots of speed.

A curious thing here is that the quotient Displ/SHP is taken to the square root power, whereas last week when we talked about displacement speed formula, this same quotient was taken to the cube root power? Why the difference?

To tell you the truth, I don't have a good answer. I find this very curious because last week we saw that a power calculation is the product of Force times Speed:

Power = Force x Speed, with Force in this case being the hydrodynamic drag

HP ~ Rt x V

Rt ~ Displ x V^2

HP ~ Displ x V^2 x V = Displ x V^3

That is, horsepower is proportional to displacement times the cube of the speed, and if we are to solve for speed, we have to take the <u>cube root</u> of HP/Displ. But Crouch's equation, which we can rewrite thus to get rid of one division sign:

V = C x (SHP/Displ)^0.5

uses the <u>square root</u> of this quotient.

I don't know the answer to this quandary. Did Crouch figure that cube root was probably involved, but decided he did not want to go through the labor of calculating cube roots and settled for a square root function??? Rather than deal with cube roots to get an exact result, he maybe relied on the square root of the quotient and a sliding scale of his power coefficient, C, to make the equation work quickly and easily. Back then, he did not have nifty calculators or computers that automatically calculate numbers to any conceivable power or fractional power to the greatest degree of accuracy. But he most likely had a slide rule, and if he had a "K" scale on his slide rule, he could easily calculate cubes and cube roots. That's why I think this is all very curious.

Slide rules date back to the early 17^{th} century (I still have mine from college, and still use it from time to time), and were highly refined by the time Crouch was born. (Did you hear about the constipated mathematician?? He worked it out with a slide rule. HAR, HAR—GROAN! That's an old childhood joke.) The long and the short of it is, I don't know the answer—a square root does not make sense in the physics of the matter. Go back through any hydrodynamic study in the last 50-60 years, and you will always find a force related to speed squared, and power related to speed cubed—there is just no way around it. So I wonder if Crouch's formula simply was a mathematical simplification yet still have horsepower within the formula.

So, does Crouch's formula work? Yes, pretty well. I recall once talking to the vice president of engineering at a major muscle-boat manufacturer who claimed that he successfully brought a claim against his engine supplier for faulty engines when he discovered that he was not getting the power out of the boat that he should have been getting—his race boats weren't as fast as they should have been as calculated using Crouch's formula. His coefficient was supposed to be C = 225 which he had proven time and again on the race course for his style of boats. But when the test results of the latest speed runs on a new boat came back with low speeds, and knowing that the boat came in on weight and on spec so that C was still reliably 225, the only thing that had to be off was the horsepower. The engine manufacturer relented and replaced the engines, and the problem was cured.

For myself, I have used Crouch's formula to characterize the design at hand, or to mimic other designs when I am trying to home in on new power and weight parameters. I'll calculate the C coefficient for known other boats of similar form, and then use that coefficient for the new design at different weight and power. That coefficient usually does not conform too well to Gerr's classification above, and that is why I generally ignore the names—I am only interested in the value of C at hand. Once I am in the ballpark, then the design effort becomes much more specific in choosing the correct engine, reduction gear, and propeller size. For that, I rely on two sources, the first of which is the Bp-d method (pronounced “Bee-Pee-Delta”) of propeller specifications, which is found in *The Propeller Handbook* and is also covered in other major professional naval architectural texts. Then I revert to more detailed calculations using the NavCad software from HydroComp Inc. in New Hampshire. More often than not, Bp-d and NavCad come out in very close agreement. Crouch’s formula is a quick and easy way to get started with the right ballpark of horsepower and weight.

Isn’t there a more universal formula that covers both performance regimes, you may ask? Indeed there is, two of them, in fact. The first of these is Keith’s Formula which, interestingly, is mentioned only in Kinney’s *Skene’s Elements of Yacht Design*, and not anywhere else that I can find. Who was Keith? I haven’t the foggiest idea. I have been researching all over the Internet and asking the most renowned powerboat design experts in the US and the world, and no one can come up with any original information or source for Keith or his formula. Keith is a total enigma. Nevertheless, here is his formula, and I mention this only because it leads us directly into the other formula that is available, Wyman’s Formula:

Keith’s formula states:

Speed, V = Lwl^0.5 x C x ((BHP x 1000)/Displ)^0.333

Where:
V = speed in miles per hour (not knots)
BHP = Break Horsepower
Displ = Displacement in pounds
C = Coefficient that ranges between 1.3 to 1.5

Notice a few interesting things compared to Crouch’s formula. First of all, horsepower is Brake Horsepower (installed horsepower of the engine, before it gets to the prop), and that horsepower is multiplied by 1000. Or, you could also say that the 1,000 number is the displacement divided by 1,000. Either way, this has the effect of reducing the coefficient C down to a very low number. Note also that the quantity that includes the quotient of BHP/Displ is taken to the cube root, as we should expect.

The other interesting thing that we see in this equation is the value of the square root of the length on the waterline on the right side of the equation. If we move it back to the left side of the equation, it goes to the denominator, of course, and that gives us speed/length ratio:

V/Lwl^0.5 = C x ((BHP x 1000)/Displ)^0.333

which results in exactly the form of the equation of the displacement speed formula from last week. So it is the same thing written just a slightly different way (only one division sign).

In Keith’s Formula, Kinney does not give any good description of how the coefficient C varies, only that it does, and that you do the same thing as with Crouch’s formula—back-calculate C from exemplar boats, and use it again for more or less reliable predictions on new designs. That can be a problem if you don't have any exemplar design to refer to, or if their weight, speed, and horsepower data are incomplete.

To solve that, we can turn to Wyman's Formula. Wyman's formula is exactly the same as Keith's Formula, it is just that Wyman has given a fair bit more effort to defining the coefficient C. First of all, who is Wyman?

David B. Wyman is a practicing naval architect and professional engineer currently living in Maine. He trained in naval architecture at the US Merchant Marine Academy, has taught at the Maine Maritime Academy, and has done extensive work for the US Navy. He currently specializes in the design of small submersible vessels. In the August/September 1998 issue of *Professional Boatbuilder* magazine (*PBB*), issue #54, he described his formula and the derivation of his coefficient. This information is repeated in *The Masthead* issue of June 2008 that I cited last week, whose link I repeat here:

http://www.westlawn.edu/news/WestlawnMasthead06_June08.pdf

Wyman's Formula is:

V = Cw (Lwl^0.5) [SHP/(Displ/1000)]^0.333

Where:
V = boat speed in knots
Cw = Wyman's Coefficient
Lwl = length on the waterline
SHP = Shaft Horsepower at the prop
Displ/1000 = Displacement in pounds divided by 1,000

Note that it is displacement divided by 1,000, not SHP multiplied by 1,000. This is exactly the same as Keith's formula, which Wyman admits he started with. This formula also includes the square root of length on the waterline, so it is a "corresponding speed" formula. But Wyman was disturbed that C in Keith's Formula was not well defined, so he started to collect performance data from a wide variety of boats over the years (over 50 different designs) in order to determine an empirical value, equation, or chart for C, which he now calls Cw, the Wyman Coefficient. This he plotted against speed/length ratio, and got a surprisingly consistent straight-line relationship for Cw. A plot of his data appears in the *PBB* article, and a slightly different version with the axes reversed appears in *The Masthead* article. Cw = 0.7 at S/L = 0.0, and Cw = 2.5 at S/L = 10.4, according to the *PBB* plot. You can do straight-line relationships with these limits if you don't have access to the *PBB* article which appears to be the more comprehensive plot of the two.

By the way, you can buy PDF downloads of any issue of *Professional Boatbuilder* magazine for US$3.50. Here is the link:

http://www.woodenboat.com/wbstore/index.php?main_page=index&cPath=73

The complete series of *Professional Boatbuilder* magazines, always updated to the current issue, is available on a flash drive for US$145. Link:

http://www.woodenboatstore.com/Professional-Boatbuilder-The-Complete-Collection/productinfo/199-USB/

And just so that you know, all *Professional Boatbuilder* issues since #95 in June/July 2005, are on *PBB*'s website at: http://www.proboat.com/digital-issues.html. Anyone can access any issue since then, a tremendous resource.

So, what do we do with Wyman's Formula? The same thing that we can do with the displacement speed formula or Crouch's Formula. Wyman's Formula is the most comprehensive of all of them because it applies equally well to displacement boats, semi-displacement boats, and planing boats. Pick the Cw coefficient that corresponds to the speed/length ratio, and solve for the unknown that you are looking for—speed, length, weight, or horsepower. That is, Cw is always determined for you; you don't have to go searching for it, and that is why Wyman's Formula is so useful. It has been determined empirically from existing boats.

Now that you see how this is done, of course, you can back-calculate the coefficient if you want to. Let's say you have your own collection of boat designs, different from Wyman's. You can solve the Wyman Formula for your population of boats and determine your own Wyman Coefficient. This is a way to home in a little more accurately for your own style of design. This is essentially what I have done before with both displacement speed formula, where the 10.665 coefficient was fixed but I changed it to suit my designs, and Crouch's Formula where I again determined by own coefficient. In the end, we see that Crouch's Formula appears to be an unnecessary simplification of what Wyman's Formula now does with ease. So I say, let's forget about displacement speed formula, Crouch's Formula, and Keith's Formula, because Wyman's Formula does it all quite well because it is grounded in physics and is based on what actual boats do.

THE END

Ladies and gentlemen, this is the end of the series on *The Design Ratios*. This has been a fun ride, and I am grateful to all of you for your compliments and comments that have been posted for the benefit of all the readers. In particular, I wish to thank Captain Vimes for his original question (Thank you, Capt.) and to Landlubber who suggested doing a whole series on design ratios (Thank you, LL).

Within the next few days, I will post a PDF document containing all the discussions in the order they were presented, edited to clean up the text a little bit, and with the diagrams and pictures embedded within the text so that they make for a little easier reading. Some documents were posted separately, such as a few spreadsheets and an article or two, and I'll post those again so that they are all available at the same place. Finally, I will post everything on my own website so that there is a second repository for the information.

To all of you who have been following this series, thank you for your interest. This series means nothing if not for readers who read it. I hope that *The Design Ratios* has helped you to understand boat design a little better, and to all the budding designers as well as practicing professionals, I hope that the history and derivations of some of the material have been enlightening.

Good luck to you all; it has been a pleasure to talk with you.

Best regards,

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REFERENCES
(alphabetical order by title)

1. *Design & Detail—Looking At The Numbers*, by Danny Greene, *Cruising World* magazine, September 1990.

2. *Design for Fast Sailing*, by Edmond Bruce & Henry A. Morss, Jr., Amateur Yacht Research Society, publication #82, 1976.

3. *The Design of Sailing Yachts*, by Pierre Gutelle, International Marine Publishing, English language edition, 1984.

4. *On a Scale of One to Ten*, by A. P. Brooks, *Telltales*, April, 1988.

5. *Performance Criteria and the Design of Sailing Yachts*, by Jay Paris, SNAME, New England Sailing Yacht Symposium, January 24, 1976.

6. *Practical Solutions—Wyman's Formula*, by David Wyman, *Professional Boatbuilder* magazine, issue # 54, August/September 1998.

7. *Practical Speed and Powering Calculations*, by Dave Gerr, *The Masthead*, newsletter from Westlawn Institute of Marine Technology, June 2008.

8. *Principles of Yacht Design*, by Lars Larsson & Rolf Eliasson, International Marine/McGraw Hill, 3rd edition, 2007.

9. *The Propeller Handbook*, by Dave Gerr, International Marine Publishing, 1989.

10. *Sailing Yacht Design—An Appreciation of a Fine Art*, by Robert G. Henry & Capt. Richards T. Miller, USN, SNAME Transactions, Vol. 71, 1963.

11. *Sailing Yacht Design—A New Appreciation of a Fine Art*, by Richards T. Miller & Karl L. Kirkman, SNAME Transactions, Vol. 98, 1990

12. *Skene's Elements of Yacht Design*, by Francis S. Kinney, Dodd, Mead & Company, 5th edition, 1973.

13. *The Speed and Power of Ships—A Manual of Marine Propulsion*, by Admiral David W. Taylor, John Wiley & Sons, 1910.

14. *Ted Brewer Explains Sailboat Design*, by Ted Brewer, International Marine Publishing, 1985.

15. *Voyaging Under Power*, by Robert Beebe, Seven Seas Press, 2nd edition, 1984.