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Design and Engineering Aspects of Free-standing Masts and Wingmasts

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ABSTRACT

Since their appearance in the mid-1970's, free-standing masts and wingmasts have become available on a number of production and custom sailboat designs. Being made primarily of composite materials, their engineering and development is complex and subject to much trial and error. Yet practically nothing of this development has been written down outside of what the various manufacturers have recorded for themselves.

This paper outlines the basic criteria for free-standing mast and wingmast engineering and design, discusses current rig arrangements, materials and manufacturing methods, and gives two design examples. Weaknesses in the engineering process are also delineated, and guidelines are given for future research which could eliminate those weaknesses.

NOMENCLATURE

A_s	Sail area
A_{cs}	Area of mast cross-section
A_{le}	Area of cross-section of wingmast leading edge
A_{te}	Area of cross-section of wingmast trailing edge
a	Half wingmast chord length to outside surface
a_1	Half wingmast chord length to inside surface
B	Beam
b	Half wingmast section width to outside surface
b_1	Half wingmast section width to inside surface
C	Mast circumference, wingmast section chord length
CHA	Geometric center of underwater profile
CSA	Geometric center of sail area
D	Diameter of mast cross-section
Δ	Displacement
E	Mainsail base
F_{mh}	Equivalent force at masthead
F_s	Shear force in mast
F_{wb}	Equivalent force at wishbone boom
GM	Metacentric height
I_{cs}	Moment of inertia of the mast cross-section about its centroid
I_{ycs}	Moment of inertia of wingmast cross-section about the y-y axis
I_{yle}	Moment of inertia of wingmast leading edge about the y-y axis
I_{yte}	Moment of inertia of wingmast trailing edge about the y-y axis
L_{oa}	Length over all
L_{wl}	Length on waterline
M_b	Mast bending moment
M_h	Heeling moment
M_r	Righting moment
M_y	Moment of wingmast section area about y-y axis
N	Number of carbon fiber strips
P	Mainsail height
R	Radius of mast cross-section
R_b	Reaction force at base of mast
R_d	Reaction force at deck
SM_{cs}	Section modulus of mast cross-section about its centroid
T	Draft

t	Wall thickness of mast cross-section	
t'	Thickness of carbon fiber strip	
V_{aw}	Speed of apparent wind	
V_h	Boat hull speed	
V_{tw}	Speed of true wind	
w	Width of carbon fiber strip	
$\overline{X}_{cs}$	Distance of centroid of wingmast cross-section area from y-y axis	
$\overline{X}_{le}$	Distance of wingmast section centroid of leading edge shape from y-y axis	
$\overline{X}_{te}$	Distance of wingmast section centroid of trailing edge shape from y-y axis	
ϕ	Heeling angle	
σ	Design bending stress	

Subscripts

aw	apparent wind
b	bending, base
cs	cross-section
d	deck
h	hull, heeling
i	to inside surface
le	leading edge
mh	masthead
oa	overall
r	righting
s	sail, shear
te	trailing edge
tw	true wind
wb	wishbone boom
wl	waterline
y	to y-y axis

BACKGROUND

Free-standing sailboat masts date at least as far back as the Chinese junk rig, a design so old there is no record of its initial use (ref. 1). Structural limitations prevented the junk rig's use on boats larger than about 100 feet long, but it was easily handled on all points of sailing by a small crew.

Elsewhere in the world, various types of stayed rigs predominated, reaching a peak of development with the clipper ships of the 19th century. Steam power put an end to sail in commercial shipping, but recreational sailing came to life, continuing the development of stayed rigs up to the sophistication of present-day ocean racers. In fact, conventional stayed rigs are so well established in the yacht racing industry today that free-standing masts and wingmasts are implicitly prohibited from most races by the racing rules.

Reference 2, rule 802.6A states: "To qualify for measurement under this rule, a yacht must be fitted with a bona-fide forestay." Reference 3, rule 07.01.05 states, "All yachts, including those not having a mast head rig, shall be equipped with a permanent backstay extending to the mast head...." Other features inherent to free-standing masts and wingmasts are also prohibited or penalized, as follows:

Wrap-around luffs & rotating masts	Prohibited by IOR rule 802
Spars built of anything but wood, aluminum alloys, steel alloys, and glass fibre reinforced plastics	Penalized by IOR rule 802.4
Spars built of anything but wood, fiberglass, aluminum alloys, brass, bronze, or steel (including stainless steel alloys)	Prohibited by MHS rule 03.03.01
Battens exceeding 4 or 5 in number of specified lengths	Penalized by IOR rules 845.5 & 848

Why are free-standing masts and wingmasts restricted in this manner? It may be that the aerodynamic efficiency and superiority of these rigs on all points of sail has to be more widely shown and accepted than at present. Also, if the rules were changed overnight, owners, designers, boatbuilders and equipment manufacturers would face an expensive, chaotic situation trying to take advantage of the new allowance for as yet unproven benefits. One thing is certain: if free-standing masts and wingmasts were allowed under the racing rules, development of these types of rigs would be faster and more sophisticated than it is today.

As it is, the restrictions imposed by the racing community have forced the development of free-standing masts and wingmasts into the cruising sailboat market where money and the driving incentive of competition are much more scarce. Nevertheless, there are today in the United States at least seven manufacturers building fifteen different production sailboats with free-standing mast and wingmast rigs. A growing number of custom designs are also available with these types of rigs. Where once racing sailors were the pioneers of sailboat technology, now cruising sailors can claim that role, thanks to the racers' own rules. This situation is very rare in the history of yachting.

For cruising sailors, safety, simplicity, and ease of handling have been the primary incentives for building boats with free-standing masts and wingmasts. Obviously, compared to conventional stayed rigs, free-standing rigs comprise much less equipment that requires maintenance or can give trouble at sea.

Aerodynamic efficiency, though it has played a secondary role, is no less important. References 4, 5, and 6 all discuss assorted model test programs which attest to the improved lift/drag characteristics of streamlined sail sections as found in free-standing mast and wingmast rigs. Reference 7 discusses similar advantages found during full-scale testing and racing of C-class catamarans as well as the reasonable correlation of the test results with computerized performance predictions. For free-standing mast and wingmast rigs to become more widely accepted, their aerodynamic superiority will have to be displayed to a greater degree than it is at present. This can best be done by test sailing identical boats against each other, each fitted with a different type of rig, and by greater participation of boats equipped with free-standing masts and wingmasts in the few organized races open to them.

No further discussion is required on the practicalities of free-standing masts and wingmasts, or on their aerodynamics. What remains is to discuss the masts themselves: what are the different types available, what are they made of, how are they engineered and built, and where is more research required to improve their engineering and manufacture?

BOAT	DESIGNER	BUILDER	RIG TYPE	MAST TYPE FREE-STANDING (FSM) WINGMAST (WM)	MAST MAT'L ALUM. (AL) FIBERGLASS (FG) CARBON FIBER (C.F) WOOD-EPOXY (WE)	BOOM TYPE CONVENTIONAL (C) WISHBONE (W)	SAIL TYPE 1-PLY (1-P) 2-PLY (2-P) FULLY BATTENED (FB)	COMMENTS
PRODUCTION								
F-21	G. HOYT	FREEDOM YACHTS	CAT	WM	CF	C	1-P, FB	FREEDOM YACHTS ORI-
F-25	G. HOYT	″	″	WM	CF	C	I-P, FB	GINALLY HAD 2-PLY SAILS
F-28	K. BURGESS	″	CAT-KETCH	FSM	CF	C	1-P, FB	W/ WISHBONE BOOMS.
F-33	J. PARIS	″	″	FSM	CF	C	1-P, FB	PRESENT ARR'G'T WAS
F-39	R. HOLLAND	″	SCHOONER	FSM	CF	C	1-P, FB	EFFECTIVE 1982
F-40	H. HERRESCHOFF	″	CAT-KETCH	FSM	CF	C	1-P, FB	
F-44	G. HOYT	″	″	FSM	CF	C	1-P, FB	
F-70	G. HOYT	″	″	FSM	CF	W	2-P	
NONSUCH 26	M. ELLIS	HINTERHOELLER	CAT	FSM	AL	W		2 WOOD-EPOXY MASTS HAVE
NONSUCH 30	M. ELLIS	″	″	FSM	AL	W		BEEN MADE FOR NONSUCH 30
TANTON 43	Y. TANTON	OFFSHORE YACHTS	CAT-KETCH	FSM	CF	C	1-P	
WINGS 33	FREEWING GROUP	FREEWING	″	FSM	AL, CF	W	1-P, 2-P	
PEARSON 23C	PEARSON GROUP	PEARSON	CAT	FSM	AL	C	1-P	
WHISTLER 32	C. PAINE	ABLE	CAT-KETCH	FSM	CF	W	2-P	
AMA 45	R. NEWICK	AMA	SLOOP	WM	WE	C	1-P, FB	PRODUCTION TO BEGIN
CUSTOM								
SPONBERG 38	E. SPONBERG	UNDECIDED	SCHOONER	WM	WE	C	1-P, FB	
A.J. LUCIANTONIO	H. HERRESCHOFF		CAT-KETCH	WM	WE	C	1-P, FB	
ROGUE WAVE	R. NEWICK	GOUGEON	SLOOP	WM	WE	C	1-P, FB	
LADY PEPPERELL	HUNTER GROUP	HUNTER	CAT-KETCH	FSM	FG	C	2-P	MASTS ROTATE

TABLE I: CURRENT PRODUCTION & CUSTOM DESIGNS WITH FREE-STANDING MASTS & WINGMASTS

FREE-STANDING MAST AND WINGMAST RIG ARRANGEMENTS

Table I shows most of the current sailboat designs utilizing various types of free-standing mast and wingmast rigs. The Freedom 40 was the first production boat, built in 1975, to resurrect the idea of free-standing masts. Since then, it and the other Freedom yachts have been front-runners in developing new rig features and construction techniques. Freedom yacht masts were originally made of aluminum and were fitted with double-ply wrap-around sails and spruce wishbone booms. These masts were eventually changed to fiberglass, then to a glass/carbon fiber combination, and finally to all carbon fiber in order to reduce weight aloft and increase strength and stiffness. The wishbone booms were changed to aluminum for lighter weight and easier manufacture. In 1982, the wishbone booms and double-ply sails were changed to conventional booms with fully-battened single-ply sails for easier handling. This arrangement seems to be becoming more popular.

The Freedom 21 and 25 are the only production sailboats offering a wingmast rig as standard equipment. When production begins on the 45' trimaran from Ama Inc., it will come with a wingmast as standard equipment.

In the eight years since the first Freedom 40, many other production designs have been developed providing a fairly wide selection of boat size and rig type. If the buyer's ideal combination of hull and rig design cannot be found, a custom boat can be designed and built.

MATERIALS

It was not until the mid-1970's with the advent of advanced composite materials and manufacturing methods, that free-standing masts could be made stronger, stiffer, lighter, and more aerodynamically efficient than conventional stayed rigs. These composite materials are E-glass, S-2 glass, Kevlar 49, and carbon fiber in conjunction with polyester, vinylester and epoxy resins. Wood-epoxy is also used. The reader is assumed to be familiar with the basic physical and mechanical properties of the individual materials; discussion here is restricted primarily to cured laminate properties.

Table II lists the mechanical properties of unidirectional laminates made with the above-mentioned fibers based on a 60-65% fiber content by volume. Figures 1, 2, and 3 show specific strength plotted against specific modulus for tension, compression, and flexure for each laminate. For comparison, the properties of 6061-T6 aluminum are also shown.

Practically all published data for composites is for laminates made with epoxy resin. The data come primarily from the aircraft and aerospace industries where epoxy resin is used almost exclusively. In the boatbuilding industry, epoxy laminates are much less common because of their expense, health hazards, and difficult workability in hand-layup operations. Mechanical properties of laminates using vinylester or polyester resin will be somewhat less than

PROP. \ MAT'L	E-GLASS	S-2 GLASS	KEVLAR 49	CARBON FIBER	ALUMINUM 6061-T6
TENSILE STRENGTH $\times 10^3$ PSI	180	298	220	220	45
YIELD STRENGTH $\times 10^3$ PSI	—	—	—	—	40
TENSILE MODULUS $\times 10^6$ PSI	6.0	7.5	12.5	20.0	10.0
COMP. STRENGTH $\times 10^3$ PSI	107	118	43	177	
COMP. MODULUS $\times 10^6$ PSI	7.0	7.7	12.0	20.0	
FLEX. STRENGTH $\times 10^3$ PSI	218	197	97	287	
FLEX. MODULUS $\times 10^6$ PSI	6.6	7.0	12.0	19.3	
DENSITY LBS/IN^3	0.072	0.069	0.050	0.057	0.098

TABLE II: MECHANICAL PROPERTIES OF UNIDIRECTIONAL EPOXY LAMINATES 60-65% FIBER CONTENT BY VOLUME

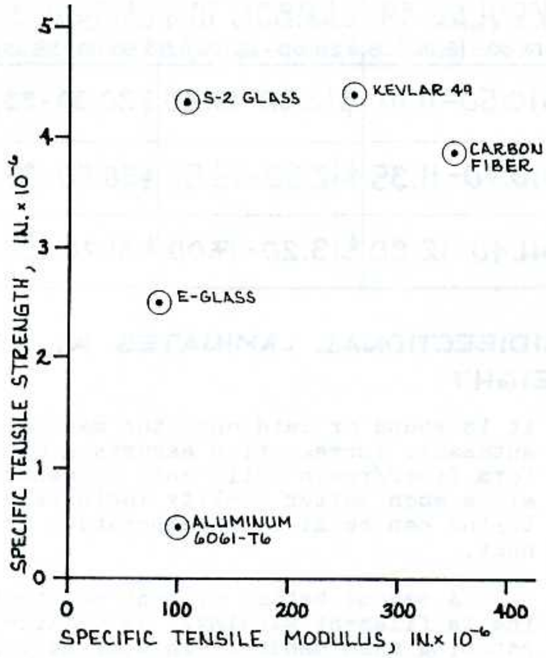

FIG. 1

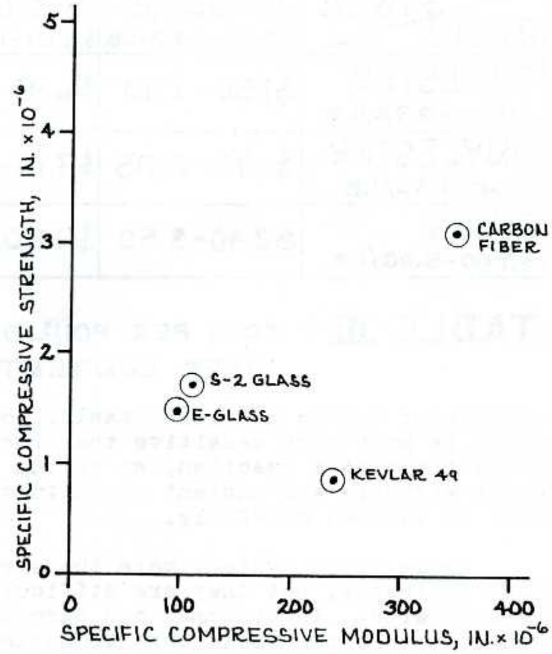

FIG. 2

epoxy laminates, but the relative standing between all laminates using these other resins will remain the same.

Figures 1, 2, and 3, compiled from the data in Table II, show very clearly the relative standing of each laminate. For example, S-2 glass in tension is nearly twice as strong for a similar stiffness and weight as E-glass, but the two are nearly the same in compression and flexure. Kevlar 49 is as strong as S-2 glass in tension, but weaker than both S-2 glass and E-glass in compression and flexure. In all cases, however, Kevlar 49 is about twice as stiff as E-glass and S-2 glass, being directly between glass and carbon fiber. It is readily apparent why carbon fiber is ideally suited for free-standing masts and wingmasts. For its weight, it is the strongest and stiffest material available. Aluminum, as shown, is the least strong and stiff for its weight.

The resin for the laminate plays an important role, not so much in strength and stiffness, but for general laminate quality and ease in manufacturing. Generally, the better grade resins should be used with the better grade fibers to maintain uniformity of mechanical properties. In free-standing mast construction, polyester resin generally is not a good choice. Due to its high styrene content, polyester shrinks considerably on cure and this can cause cracking within the laminate. Such cracking diminishes the strength of the structure considerably.

Vinylester resin also has a high styrene content, but its curing reaction is quite different and the laminate

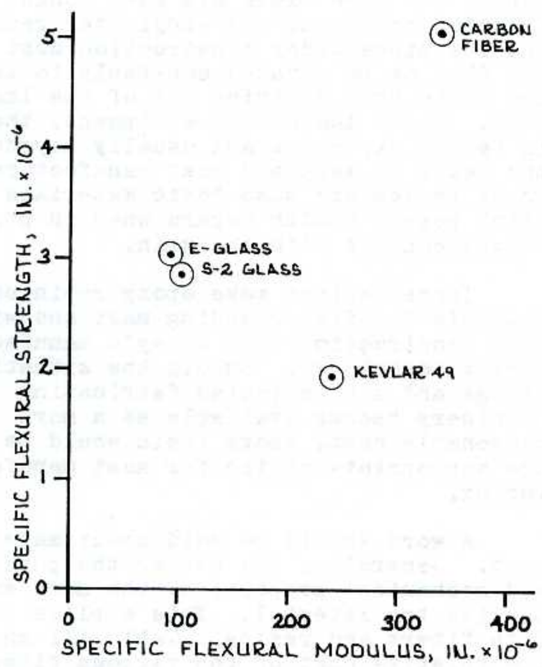

FIG. 3

shrinks very little on cure. Internal cracking, therefore, is not a problem. In addition, vinylester resin has much better adhesion to the fiber which increases the interlaminar shear strength and makes a much more solid laminate.

Vinylester resin utilizes the same curing agents as polyester resin (methyl ethyl ketone peroxide (MEKP) and cobalt naphthenate (conap) or benzoyl peroxide (BPO) and dimethylaniline (DMA)) so that purchasing and handling materials is as easy and familiar as with polyester. The

RESIN \ FIBER	E-GLASS $2.00-2.50/LB	S-2 GLASS $11.00-12.00/LB	KEVLAR 49 $17.00-18.00/LB	CARBON TOW $20.00-25.00/LB	CARBON FABRIC $50.00-55.00/LB
POLYESTER $0.70-0.75/LB	$1.50-1.80	$6.90-7.50	$10.50-11.10	$12.30-15.30	$30.30-33.30
VINYLESTER $1.30-1.40/LB	$1.70-2.05	$7.10-7.75	$10.70-11.35	$12.50-15.55	$30.50-33.55
EPOXY $3.00-5.00/LB	$2.40-3.50	$7.80-9.20	$11.40-12.80	$13.20-17.00	$31.20-35.00

TABLE III: COST PER POUND OF UNIDIRECTIONAL LAMINATES AT 60% FIBER CONTENT BY WEIGHT

vinylester curing reaction itself, however, is much more sensitive than the polyester curing reaction, so curing agent mixtures and ambient conditions must be watched carefully.

Epoxy resin systems make the strongest laminates, but they are difficult to work with. In aircraft and aerospace applications, the resin cure is assisted by heat, pressure and automated fabricating machinery. For room temperature cures, the cure times are much longer than for polyester and vinylester resins, and the piece under construction must lie flat or be rotated constanly to keep the resin from draining out of the laminate. Epoxy laminating equipment, then, is fairly expensive and usually beyond the means of mast and boat manufacturers. Epoxy resins are also toxic materials which pose a health hazard when in prolonged contact with the skin.

These factors make epoxy resin unsuitable for free-standing mast and wingmast construction with today's manufacturing techniques. Should the situation change and sophisticated fabricating machinery become available at a more reasonable cost, epoxy resin would be the appropriate choice for mast manufacturing.

A word should be said about material cost. Generally, the better the physical and mechanical properties, the more expensive the material. This applies to both fibers and resins. Table III shows the relative cost of the various fibers and resins as well as the various laminate combinations at 60% fiber content by weight.

MANUFACTURING METHODS (ref. 8)

Basically, there are three general methods of manufacturing masts with composite materials. The first involves hand-layup of material over a male mandrel. The inner layer of hoop material is wound on, then the axial material is laid on, and then the outer layer of hoop is wound on. Resin wet-out can be by hand, but a better method is by automatic impregnation which applies a measured amount of resin to the material as it is wound or laid onto the mandrel. Automatic impregnation assures a uniform fiber/resin ratio and, in general, a much better quality laminate. Curing can be at room temperature or by heat.

A second method of mast manufacturing is filament winding. It utilizes a rotating male mandrel in combination with a traversing fiber delivery head. A variation of this is a traversing, rotating male mandrel and a stationary fiber delivery head. All the fiber is automatically impregnated and wet-wound onto the mandrel. Various fibers can be used at various orientations from near 0^o to 90^o to the mast axis. Again, curing can be at room temperature, but it is more commonly done by heat. This is the most sophisticated method of manufacture, a clean process, done entirely by machine, and one that lends itself easily to computer control.

The third method of mast manufacture is also a hand-layup operation utilizing a female mold and an inflatable bladder. The wet-out material (again, preferably done by automatic impregnator) is laid into the two halves of a female mold. A deflated bladder is laid into the mold on top of the laminate and the mold is closed. The bladder is inflated which squeezes the resin through the fiber while it cures at room temperature. A major drawback of this method over the other two is that if anything should go wrong during cure, e.g. a bladder leak or pinched fibers between the flanges of the mold, nothing can be done about it until after the laminate is cured. An entire mast can be lost this way; the whole process is much more risky.

Wood-epoxy is being used successfully to build wingmasts. However, not much data are available for engineering in wood beyond reference 9, and that source restricts itself to wood only and does not include wood-epoxy. Some limited wood-epoxy data is available in reference 10. The strength and stiffness of wood depends, of course, on the species, the moisture content, the location of the wood within the tree, and a host of other environmental factors. When engineering in wood-epoxy, therefore, material strength and stiffness tests must be conducted to verify the design parameters.

Wood-epoxy does have a few very attractive advantages. It can be easily molded into the shapes required for wingmast construction. Once built, the mast can be worked easily with common woodworking tools, whereas glass, Kevlar 49 and carbon fiber are particularly hard on tools. Also, wood-epoxy accepts fastenings very well so that attachments to the mast can be installed easily. Fiber composites do not take fastenings well, and in the case of carbon fiber the fastenings must be insulated from it to prevent galvanic corrosion, particularly in a marine environment. Also, the tooling for a wood-epoxy mast can be made much more easily and cheaply than tooling for a fiber composite mast.

The disadvantages of wood-epoxy are that it makes a relatively heavy structure for a highly stressed mast. Also, fabrication must be done with epoxy resin which, as pointed out earlier, is a hazardous material and must be handled carefully.

ENGINEERING AND DESIGN OF FREE-STANDING MASTS

The engineering calculations required for designing a free-standing mast entail, basically, standard cantilever beam theory. The difficult part is to match the required strength of the mast to the capabilities of the material used. The standard cantilever beam equations as found in reference <u>11</u> will apply completely only if the material of the mast is homogenious and isotropic, like metal. In addition, if we talk about a particular alloy of aluminum, for example, its physical and mechanical properties are essentially constant regardless of where it is manufactured since the alloy must comply with various published manufacturing standards. For free-standing masts made of aluminum, therefore, the engineering is fairly simple.

Composite materials pose an infinitely more complex problem. By their nature, composites are non-isotropic. Their physical and mechanical properties depend on the choice of fiber, the type and quality of the fabric, the orientation of the fabric in relation to the loading, the choice and quality of the resin, the weight ratio of resin to the fiber, the curing agents used, the method of fabrication, and whether the cure was at room temperature or under conditions of heat, pressure, or both.

Moreover, the quality and strength of the various materials making up the composite vary from supplier to supplier. A polyester resin from one manufacturer may be significantly different from another manufacturer; resins with a variety of properties are available. A 10 oz./sq.yd. fiberglass cloth from one weaver may have significant advantages over those from another if he has taken the effort to keep the glass strands straight and flat and the weave uniform. Even from a single supplier there can be variances. One carbon fiber manufacturer, for example, has been conscientious enough in improving his manufacturing process that the tensile strength of his carbon fiber has increased over 36% in about 2½ years. This, of course, can work in reverse if a manufacturer fails to maintain the quality of his product.

As a result, engineering composite masts is more difficult because standard cantilever beam theory does not apply completely. Many of the design decisions must be based on test results rather than on empirical equations. Testing of a full-scale mast to destruction is not usually done because of the expense involved, but other non-destructive methods using strain gauges and laboratory test equipment are available and need to be employed if the strength and stiffness of the structure are to be determined.

The engineer must also have a thorough understanding and respect for the manufacturing process being considered and make decisions regarding material types and quantities, layup schedules, and safety factors accordingly. He must also be aware of the type of loading the structure will carry; flexure, tension, compression, shear, impact, and fatigue are all treated differently when developing a composite structure.

In the United States today only a handful of manufacturers are producing composite free-standing masts and wingmasts. They are using some semi- and fully-automated machinery in order to maintain quality, produce in quantity, and still keep costs down. They have also taken some effort to conduct tests on their products and to continually improve them. All of this represents a considerable investment in materials, labor, time and money for which they have earned respected places in the market. Their engineering and manufacturing techniques are not common knowledge, and they would prefer to keep that technology to themselves, at least for the time being.

This paper does not intend to jeopardize this understanding. It is up to the manufacturers themselves to disseminate their technology as and when they choose. The examples worked out in appendices A and B do not dwell on the specifics of any one manufacturer. Consequently, the basis for some of the decisions therein must be taken somewhat on faith, either because the data required for technical backup is proprietary information or because it simply is not available in any other tangible form.

Appendix A is an example of how a carbon fiber free-standing mast is engineered from the information made available from the naval architect, the material suppliers, and the mast manufacturer. Refer to appendix A for the following discussion of the process.

Table A-1 and figure A-1 give the boat and mast particulars. Table A-2 shows the carbon fiber and resin properties which are usually of secondary importance until manufacturing and testing take place. Table A-3 shows the cured laminate properties which are of prime importance at the design stage as the data are used directly in the design process.

The wind loading on the sails is determined by the simplified wind pressure method of reference 12, and then the resulting heeling moment is balanced against the boat's righting moment. The boat's maximum righting moment becomes the maximum design bending moment of the masts.

In a way, designing the mast by how the boat will ride in the water, by its righting moment, is a little backward. The classic approach is to derive the lift and drag characteristics of the sail plan directly, work them into actual loads on the masts, and from those loads draw the shear and bending moment diagrams for the masts. There are several reasons why this cannot be done (at least, not today).

First, no one has done a sufficiently thorough analysis of free-standing masts with various sail combinations and sail/mast attachments to give lift and drag coefficients useful to a general mast design process. References 4, 5, 6, and 7 are about the only studies closely directed at the problem, but they contain little information that can be practically applied to mast engineering.

Second, most aerodynamic studies that do give useful lift/drag data almost always correlate it to the yacht's speed made good to windward. Little consideration is given to how the wind loads are carried by the sailcloth onto the masts and then into the boat to make the boat move.

Third, sail fabrics, like composites, are non-isotropic structures. They seem to act like rigid cambered plates creating lift in moving air, but the loads must be carried by the individual strands in the fabric to points of support on the sail, namely the head, luff, tack, foot and clew. Additionallly, the wind loading on the sails varies according to height above the water (the wind gradient), leading edge shape, sail shape, and a number of other factors. How are all these variables to be calculated into loads on the mast and how are the loads distributed?

The sum of this knowledge is not available at this time. Therefore, the expedient way to design free-standing masts is to look at the response of the yacht in the seaway. How much does it heel and what are the corresponding heeling and righting moments? These are the values to which the mast is designed.

The heeling moment at the design condition is considered to be the maximum working load for the masts, and it should be equal to 1/3 or 1/4 of the boat's maximum righting moment. There are two reasons for this: 1) it gives a safety factor of 3 or 4 against unknown loads that could occur at sea or because of inadequate boat handling, and 2) permanent loss of strength in composites generally occurs after loading has reached about 50% of the structures ultimate strength.

To explain reason 2 further, irreparable damage is done to a laminate as soon as it is loaded, no matter how small the load. The initial damage is minute cracking within the cured resin, but this cracking generally does not reduce the structure's ultimate strength. As the load increases, additional cracking occurs at the resin/fiber interface. At high enough loads, these cracks begin to diminish the structure's strength. Near the ultimate load, actual fibers begin to break and advanced structural failure begins. Acoustic emission testing can be used to determine fairly closely the point where permanent loss of strength occurs for any particular laminate and structure. More complete discussion of this phenomenon is beyond the scope of this paper, but it should be sufficient to point out that the permanent loss of strength does occur at about 50% loading, and the maximum working load should be kept well below this level.

Once the bending moment curve is determined, the corresponding reactions and shear loads can be calculated, as shown.

With the loading known, the total required amount of carbon fiber can be calculated for every point along the mast. Equations (3) through (11) show how this is derived based on the principal equation:

$$\text{section modulus} = \frac{\text{bending moment}}{\text{design stress}}$$

The bending moment is known for every point along the mast, and the derivation of the design stress is also shown.

Equation (11) shows that the required section modulus is dependent only on the mast bending moment, the mast diameter, and the number of strips of carbon fiber, N, the only unknown. The determination of N is shown in figure A-6, and figure A-7 shows how the laminate schedule is made.

In calculating the amount of carbon fiber required, we have not yet considered mast deflection. Deflection could be calculated at every point along the mast by twice integrating the working load bending moment curve. The result, however, would be a masthead deflection far in excess of what actually occurs at sea. This is because the actual bending moment is much less than the working load bending moment, following a curve **similar** to that shown by the broken line in figure A-3. As pointed out previously, the actual bending moment distribution is unknown because the actual sail loading is unknown.

Generally, when the masts are built to the above design, their stiffness will be more than adequate, hardly bending at all. Sails are cut to the mast curve by trial and error until they conform satisfactorily to the shape of the mast bend under a variety of sailing conditions. Being able to accurately predict mast loading and mast bend in the design process would save the sailmaker a lot of work; this is another reason for conducting research to determine actual mast loading.

In addition to determining the amount of carbon fiber, the inside and outside layers of hoop material must also be calculated. In this design example, the sails are assumed to have a wrap-around luff so no torque is applied to the mast. Therefore, the hoop material should be oriented at nearly 90° to the mast axis, that is, in the circumferential direction. If the sail were mounted on a sail track, and thereby tended to twist the mast as well as bend it, the layers of hoop material should be oriented at ±45° to the mast axis. The material should always be balanced in both directions within each layer of hoop so that torsional strength is balanced within each layer of hoop.

In the design example, we have assumed we are designing with a commonly available carbon fiber unidirectional tape. The method of construction depends on the builder. This laminate schedule could be used for female mold techniques and male mandrel techniques. For a filament winding process, a unidirectional tape would not be used but a similar derivation of the laminate schedule could be done. Instead of N representing the number of strips of carbon fiber tape, N would be the number of passes the filament winding delivery head makes over the mandrel.

SPECIAL REQUIREMENTS OF WINGMASTS

A wingmast is a special type of freestanding mast. We will consider the small-section type used in conjunction with a sail, not the whole-wing type as found on some C-class catamarans and other research craft. The wingmast has a symmetrical airfoil section shape and it rotates. The advantages of this type of mast are:

- The smallest amount of frontal area is always presented to the wind, resultin lower form drag.
- The airfoil section shape allows the airflow to remain attached to the sail.
- The mast can always be trimmed into the wind at the optimum angle of attack which will give the best lift/drag characteristics.

Two types of wingmasts are currently being built in this country, keel-stepped and deck-stepped. The deck-stepped mast is not really free-standing as it has a pin support at the base and it requires stays to hold it up. It is used primarily on multihulls where the additional weight of a keel-stepped mast is undesirable in what is a very lightweight boat. Being stayed, a deck-stepped wingmast is a more complex structure to analyze because it undergoes compression simultaneously with bending. Deck-stepped wingmasts, therefore, are beyond the scope of this paper. We will confine ourselves to keel-stepped wingmasts which are unstayed.

The engineering to determine the laminate for a wingmast is exactly the same as for a round mast, in principle. The calculations of moment of inertia and section modulus are more complex due to the shape of the section. Refer to appendix B for the following discussion.

A wingmast rig for the same boat as in appendix A is shown in figure B-1. Note that the boom arrangement is conventional (not a wishbone type). Because of this, the maximum bending moment is assumed to occur between the deck and the gooseneck. The bending moment diagram is shown in figure B-2.

The first step is to calculate the section modulus for an airfoil section which is equivalent to the round section at the deck. A few points must be considered when selecting an airfoil section.

First, it is desirable to have the chord length as short as possible to keep the weight of the mast down and to have the smallest amount of area exposed to the wind when the boat is moored. Wingmasts alone will drive the boat, and this effect needs to be minimized when the boat is moored. A short chord length also will make the transition from the airfoil section to the round section through the deck as gentle as possible. Too sharp an angle at point A in figure B-1 will create a hard spot. As it bends, the mast tends to buckle the circular section at this point. (An alternative is to increase the diameter of the mast at the deck to be the same as the chord length, thereby eliminating the hard spot.)

Second, the latest designs indicate that the section width should be 1/2 to 1/3 the chord length, and the maximum section width should be located at about 0.5 C. Thinner mast sections would probably have better lift/drag characteristics, but they would also be more sensitive to angle of attack and therefore require constant trimming to prevent stalling.

Third, the leading edge should be elliptical as this gives a desirable aerodynamic shape, and, almost as important, it makes calculation of the section modulus relatively easy. The after end of the section can be a gentle curve terminating in a trailing edge no thicker than an appropriate sail track.

Fourth, every section along the mast should be geometrically similar to the base section, not only for aerodynamics, but, again, for ease in calculating section modulus. The top section chord length should be no less than 75% of the bottom section chord length. Any shorter, though perhaps aerodynamically desirable renders the mast too flexible and sometimes too difficult to build, particularly on small boats.

Figures B-3 through B-6 show a probable section shape drawn within the guidelines above. We have chosen a 2:1 airfoil section for better athwartships stiffness characteristics. The calculations for moment of inertia and section modulus are shown for the general case as well as the 2:1 case.

When building a wingmast we are not restricted to an existing male mandrel, as with round masts. Wingmasts are most easily built in female molds, and we are free to choose whatever basic parameters seem appropriate.

Figure B-7 is similar to figure A-6 in appendix A where N, the number of strips of carbon fiber, is calculated in the same manner by equation (32). Figure B-8 shows the laminate schedule. Although it would be a difficult process, wingmasts could be filament wound, and N in such a case could again represent the number of passes of the fiber delivery head. The hoop material is designed in the same manner as for round masts.

The laminate for the transverse web is determined next. A web is required because the hollow airfoil section itself is not very stiff in the athwartship direction, yet wingmasts do have a tendency to bend sideways. This is because the sail usually has some twist in it and does not always pull in line with the airfoil section's major axis. Some recent masts built without a web have proven to be too flexible. The builders have rectified this by installing spreaders and diamond wires on the mast to stiffen it (which seems to defeat the purpose of a clean, no-wire mast), or they have experimented with other ways to make the laminate stiffer with limited success.

The easiest way to maintain section shape is to add a web at the neutral axis; this holds the sides of the mast in place. This web can also be made of carbon fiber and need not be very thick as the loads in it are low. A thickness the same as the carbon fiber skins is recommended. The layup for the web in the design example is shown in figure B-9.

The final section of appendix B is the weight calculation. Hardware would be extra. Note that the round mast and the wingmast are about the same weight. The wingmast will actually be somewhat heavier as the web must be tabbed into place with extra glass. This extra weight aloft should be offset by the improved aerodynamics of the wingmast, but no one knows for sure if this is true because few tests have been done to verify the performance of these types of rigs. It is important, however, to keep weight aloft to a minimum, consistent with strength, stiffness and safety.

SECTION SHAPE BUCKLING

In appendices A and B it was assumed that the laminates had carbon fiber running in the axial direction and E-glass running in the hoop direction. This arrangement reflects current building practice. It was also pointed out that lack of hoop stiffness and strength contribute to mast failures because the section shape buckles or collapses. Section shape buckling is related to bending, so a stiff material is required to maintain section shape. E-glass and even S-2 glass are probably not the materials to use; they are not stiff enough.

Carbon fiber would appear to be the best choice for hoop material, but there are some practical considerations which prevent this. Carbon fiber is conductive. If a carbon fiber laminate is sanded or machined, carbon fiber dust can short out nearby electrical equipment and cause expensive maintenance problems. Also, if exposed to other materials, particularly metals, carbon fiber can cause galvanic corrosion problems because it is very low on the electromotive potential scale as shown in Table IV.

POTENTIAL VOLTS	MATERIAL	
+2.4	MAGNESIUM	ANODE, LEAST NOBLE, CORRODED METAL
	MAGNESIUM ALLOYS	
+1.7	BERYLLIUM	
	ZINC	
	ALUM.1100, GALV. STEEL	CURRENT FLOW ↓
	CADMIUM	
	ALUMINUM 2024-T4	
+0.44	STEEL or IRON	
	CAST IRON	
	CHROMIUM IRON (ACTIVE)	
+0.25	NI-RESIST	
	304 STAINLESS (ACTIVE)	
	316 STAINLESS (ACTIVE)	
	HASTELLOY "C"	
+0.13	LEAD TIN SOLDERS	
	LEAD, TIN	
	NICKEL (ACTIVE)	
	INCONEL	
	HASTELLOY "B"	
-0.34	BRASSES, COPPER, BRONZES	
	COPPER-NICKEL ALLOYS	
	MONEL	
	SILVER SOLDER	
	NICKEL (PASSIVE)	
	INCONEL (PASSIVE)	
	CHROMIUM IRON (PASSIVE)	
	304 STAINLESS (PASSIVE)	
	316 STAINLESS (PASSIVE)	
-0.8	SILVER	
	TITANIUM	
-1.7	GRAPHITE, GOLD	CATHODE, MOST NOBLE, PROTECTED METAL
	PLATINUM, CARBON COMPOSITES	

TABLE IV: THE GALVANIC SERIES, REF. 13

Therefore, a hoop material other than glass or carbon fiber is needed to protect the axial carbon fiber from the elements, both during manufacture and in the finished product. A viable alternative might be Kevlar 49, which is both lighter and stiffer than glass and which has very good impact and abrasion-resistant qualities. An even better alternative might be a Kevlar/carbon hybrid if the materials could be arranged in the laminate such that the Kevlar masked and protected the carbon.

Another factor contributing to section shape buckling is wall thickness, which relates more to the geometry of the section than to material stiffness. Because free-standing masts are built with such high-strength materials, little material is required to carry the loads. The mast section becomes a thin-walled tube, and, as such, suffers from local buckling effects. In appendices A and B this was accounted for by reducing the design stress to one half of the compressive strength of the material. This meant using twice the amount of material in the section as was really needed, at twice the weight and twice the cost.

Section shape buckling in composite thin-walled tubes has not been studied very extensively, so it is not known if hoop strength and wall thickness are at all related or if they are independent of each other. Certainly wall thickness is added every time a layer of hoop material is added, but is the advantage gained proportional to the increased weight? If it is not, some other way of increasing wall thickness without increasing the amount of the axial fiber or the hoop fiber must be found. One possibility is the use of core materials in the wall of the mast to build up wall thickness to prevent section shape buckling.

The core materials available to do this are balsa, Klege-cell, Airex, Nomex, and Coremat. It is not known which of these cores is the most desirable but nearly all of them are available as thin as 3/16". Coremat is available as thin as 1 mm and might be the most ideally suited if very thin layers can be distributed throughout the laminate. All of this is an area of research yet to be explored.

FUTURE RESEARCH

Table V shows the general cycle of design for free-standing masts, together with input on where more research is required to refine the engineering. New free-standing mast designs are usually built by trial and error without full data and research to back up some of the design choices. Examples of this have already been noted.

Much of the needed research can be derived from parametric studies in a wind tunnel, collected into a volume of design parameters with the corresponding aerodynamic effects. This would provide useable data for practical mast and rig design. For instance, for a given hull shape under certain sailing conditions, would a cat-ketch rig or a schooner rig be better? Or, what would be the best leading edge shape, trailing edge shape, chord length, and section width for any particular mast design? More definition and guidelines are required in these areas.

Some of the research on strength and stiffness of materials, material selection, fiber orientation, and mast section shape could be done with models. Some of the materials used in advanced composites are very expensive, and destructive testing of full-scale masts is cost-prohibitive. Moreover, the test equipment has to be fairly massive and strong to carry out testing to the full-scale design loads. By scaling down the material weights and laminate thicknesses, models of mast sections can be tested fairly easily and give meaningful results.

DESIGN CYCLE	RESEARCH REQUIRED
1) DETERMINE SAILING CONDITION 2) " BOAT, MAST, SAIL PLAN	1) OPTIMIZE SAIL PLANS: SLOOPS, CATS, CAT-KETCES, YAWLS, SCHOONERS. DETERMINE EFFECTS OF HEADSAILS; HOW BIG NEED THEY BE? IS ROACH EFFECTIVE; HOW MUCH IS NECESSARY?
3) DETERMINE WIND LOADS ON SAILS 4) " SAIL LOADS ON MASTS 5) " MAST LOADS ON HULL 6) DRAW LOAD, SHEAR, & BENDING MOMENT DIAGRAMS 7) DETERMINE MATERIAL PROPERTIES	2) HOW ARE AERODYMIC FORCES CAPTURED BY SAIL FABRIC? HOW DO TYPE OF FIBER, WEAVE, FABRIC ORIENTATION, & SAIL CONSTRUCTION AFFECT DISTRIBUTION OF LOADS IN SAIL? HOW ARE SAIL LOADS DISTRIBUTED ON MAST? WHAT ARE LOADS ON MAST: BENDING, TORQUE, COMPRESSION, SHEAR, ETC.?
8) DETERMINE MAST SECTION SHAPE, DIMENSIONS, I, & SM	3) OPTIMIZE WINGMAST SECTION SHAPES FOR IMPROVED AERODYNAMICS. DETERMINE BEST LEADING & TRAILING EDGE SHAPES, CHORD LENGTHS, SECTION THICKNESSES, & MAST TAPER.
9) DETERMINE MATERIAL LAYOUT 10) " METHOD OF CONSTRUCTION	4) IMPROVE CONSTRUCTION TO OBTAIN MORE STRENGTH FROM THE MATERIALS. INVESTIGATE HOOP STRENGTH & STIFFNESS & OTHER POSSIBLE HOOP MATERIALS. DETERMINE OPTIMUM RATIO OF HOOP FIBER TO AXIAL FIBER. INVESTIGATE USE OF CORE MATERIALS TO STIFFEN MAST WALL AGAINST BUCKLING.
11) DESIGN ATTACHMENTS FOR RIGGING, AND SAIL, MAST, & BOOM CONTROL 12) CALCULATE WEIGHTS & COSTS	5) IMPROVE METHODS OF ATTACHING GEAR TO COMPOSITE MASTS. IMPROVE METHODS OF WINGMAST ARTICULATION CONTROL. IMPROVE BEARING ARRANGEMENTS. IMPROVE BOOM & GOOSENECK ARRANGEMENTS. IMPROVE REEFING ARRANGEMENTS.

TABLE VI: FUTURE RESEARCH REQUIRED TO REFINE ENGINEERING & DESIGN

A few areas of research are done best at full scale. A boat could be rigged with strain gauges in the sail fabric and on the masts to gather data on how the rig responds to the sailing conditions. This would determine the loads in the sails, the load distribution along the masts, and the overall strength and stiffness of the masts.

The ultimate goal is a computer program into which one could feed all of the boat design parameters, rig design and layout, etc., together with the wind and sailing conditions, and have it return the mast laminate schedule. Other factors relating to mast performance could also be worked into the program. For example, suppose it was desirable to have the upper half of the mast bend to leeward 5° in a 22 MPH wind. Such a condition would be a design variable, and the program would alter the laminate schedule accordingly.

Some of the above research is probably already underway by private companies and institutions. It will not be compiled easily or quickly, but perhaps the above provides some direction for the ongoing research. It will then be necessary to collect all the data and information into a central library and reduce it to a form suitable for practical mast engineering and design.

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APPENDIX A

ENGINEERING AND DESIGN OF A TAPERED, ROUND-SECTION CARBON FIBER FREE-STANDING MAST

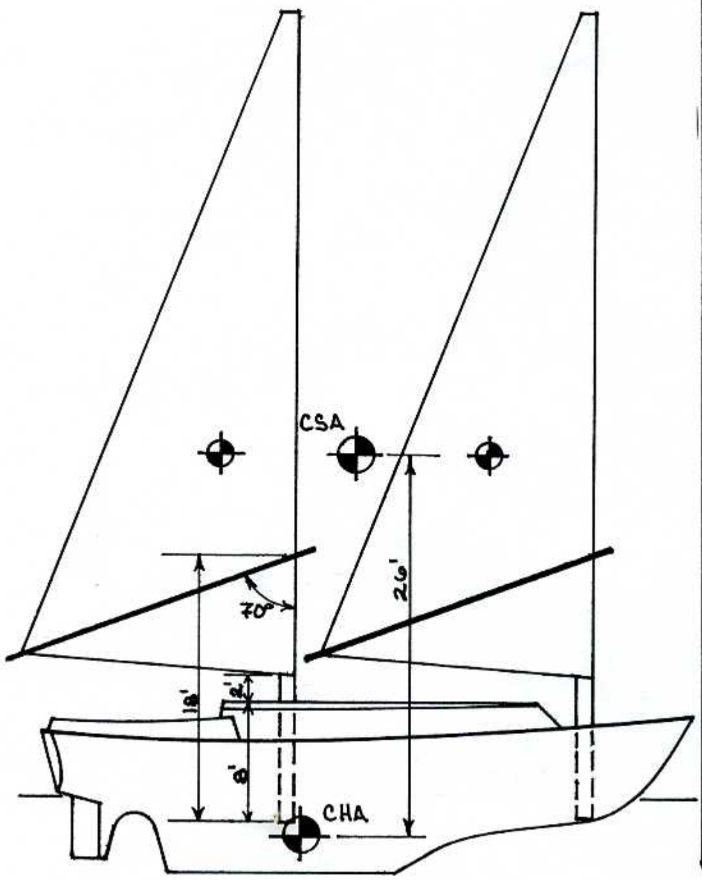


FIG. A-1

BOAT DESIGN PARAMETERS	
BOAT PARTICULARS	
Loa	44'-00"
Lwl	38'-00"
B	11'-00"
T	5'-00"
Δ	23,000 LBS
A_S	810 FT^2
M_r @ 1° HEEL	1,750 FT-LBS
MAST & RIG PARTICULARS	
CAT-KETCH RIG; BOTH MASTS HAVE THE SAME DIMENSIONS	
MAST LENGTH	55'-00"
MAST BASE I.D.	10"
MAST TOP I.D.	5"
MAST SECTION DIAMETER CONSTANT @ 10" FOR BOTTOM 15', THEN TAPER @ 1/8"/FT. FOR 40'.	
MAST O.D. AS APPROPRIATE FOR LAMINATE	
LUFF LENGTH	45'-00"
BURY LENGTH	8'-00"
SAIL AREA/MAST	405 FT^2
P	45'-00"
E	18'-00"
WISHBONE BOOMS 22'-0" LONG AND MAKE ANGLE WITH THE MAST OF 70°, 18'-0" ABOVE THE MAST BASE	
SAILING CONDITION	
THE BOAT SAILS AT HULL SPEED UP TO 22 MPH APPARENT WIND SPEED AT AN APPARENT WIND ANGLE OF 60°. AT HIGHER WIND SPEEDS THE SAILS ARE REEFED.	

TABLE A-1

Calculate the loads on the masts for a 44'0" cat-ketch. Draw the corresponding bending moment diagram. Determine the laminate schedule.

First, determine the true wind speed and the wind pressure loading on the sails. Hull speed is attained at 22 MPH apparent wind.

$$V_h = 1.34\sqrt{L_{wl}} = 1.34\sqrt{38'0"}$$

$$= 8.26 \text{ knots} = 13.94 \text{ ft/sec}$$

$$V_{aw} = 22 \text{ MPH} = 32.27 \text{ ft/sec @ } 60^\circ \text{ to boat's course}$$

Triangle solution, figure A-2, shows that

$$V_{tw} = 28.0 \text{ ft/sec} = 19.1 \text{ MPH}$$

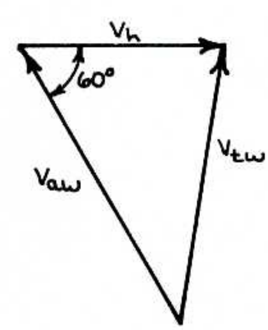


FIG. A-2

RAW MATERIAL PROPERTIES

CARBON FIBER PROPERTIES	
TENSILE STRENGTH	420,000 PSI
TENSILE MODULUS	33,000,000 PSI
ELONGATION	1.10 %
DENSITY	0.063 LBS/IN.^{3}
UNIDIRECTIONAL TAPE	12" WIDE × 0.006" THICK × 0.05 LBS/LIN. FT.

VINYLESTER RESIN PROPERTIES	
UNCURED	
RESIN/STYRENE RATIO	53/47
VISCOSITY	500 CPS @ 77°F
SP. GR.	1.04
WEIGHT PER GALLON	8.70 LBS.
1/8" THICK CURED CASTINGS	
TENSILE STRENGTH	11,600 PSI
TENSILE MODULUS	470,000 PSI
TENSILE ELONGATION	5.00 %
FLEXURAL STRENGTH	19,400 PSI
FLEXURAL MODULUS	450,000 PSI
BARCOL HARDNESS	38
SP. GR.	1.12

TABLE A-2

From ref. 12 pg. 297,

wind pressure = 1.5 lbs/ft^2

Sail loading:

main = 405 ft^2 x 1.5 lbs/ft^2 = 607.5 lbs
fore = 405 ft^2 x 1.5 lbs/ft^2 = 607.5 lbs
total 810 ft^2 = 1,215.0 lbs

M_h = heeling moment

M_h = sail loading x heeling arm (1)

For each mast

M_h = 607.5 lbs x 26 ft = 15,795 ft-lbs

or total

M_h = 1,215 lbs x 26 ft = 31,590 ft-lbs

The righting moment is

$M_r = \Delta \times GM \times \sin\phi$ = 1,750 ft-lbs (2) @ 1° heel

Solve for GM

$$GM = \frac{1,750}{23,000 \times 0.01745} = 4.36 \text{ ft}$$

If the righting moment equals the heeling moment at 31,590 ft-lbs, the angle of heel will be approximately

$$\sin\phi = \frac{31,590}{23,000 \times 4.36} = 0.3150$$

CURED LAMINATE PROPERTIES

THE CARBON FIBER UNIDIRECTIONAL TAPE IS IMPREGNATED AUTOMATICALLY & UNIFORMLY WITH VINYLESTER RESIN TO A FIBER/RESIN WEIGHT RATIO OF 60/40.

TENSILE STRENGTH	240,000 PSI
TENSILE MODULUS	21,000,000 PSI
TENSILE ELONGATION	1.10 %
COMPRESSIVE STRENGTH	170,000 PSI
COMPRESSIVE MODULUS	19,500,000 PSI
COMPRESSIVE ELONGATION	1.00 %
FLEXURAL STRENGTH	285,000 PSI
FLEXURAL MODULUS	19,000,000 PSI
SHORT BEAM SHEAR STRENGTH (INTERLAMINAR)	12,000 PSI
TRANSVERSE:	
TENSILE STRENGTH	8,200 PSI
TENSILE MODULUS	1,300,000 PSI
TENSILE ELONGATION	0.70 %

TABLE A-3

ϕ = 18.4°, or say about 15-20°

The maximum righting moment will probably occur at about 30° heel, when the lee rail starts to submerge.

M_r at 30° = 23,000 x 4.36 x 0.500

M_r = 50,140 ft-lbs

The heeling moment of 15,795 ft-lbs per mast should be considered the maximum working load for the mast. The ultimate load per mast should be 3 or 4 times this value and in the neighborhood of the boat's maximum righting moment.

3 x 15,795 = 47,385 ft-lbs
4 x 15,795 = 63,180 ft-lbs
sum 110,565 ft-lbs

ave M_b = 55,283 ft-lbs

say 55,000 ft-lbs

The design bending moment for each mast, therefore, is to be 55,000 ft-lbs which compares favorably with the boat's maximum righting moment of 50,140 ft-lbs.

The maximum working load will occur either at the wishbone boom attachment or at the deck, depending on sail set, wind conditions, etc. Therefore, the design bending moment should extend along the mast between the deck partners and the wishbone boom attachment, and taper to zero at the ends. The bending moment diagram is shown in figure A-3.

The reactions and shear loads are calculated as follows:

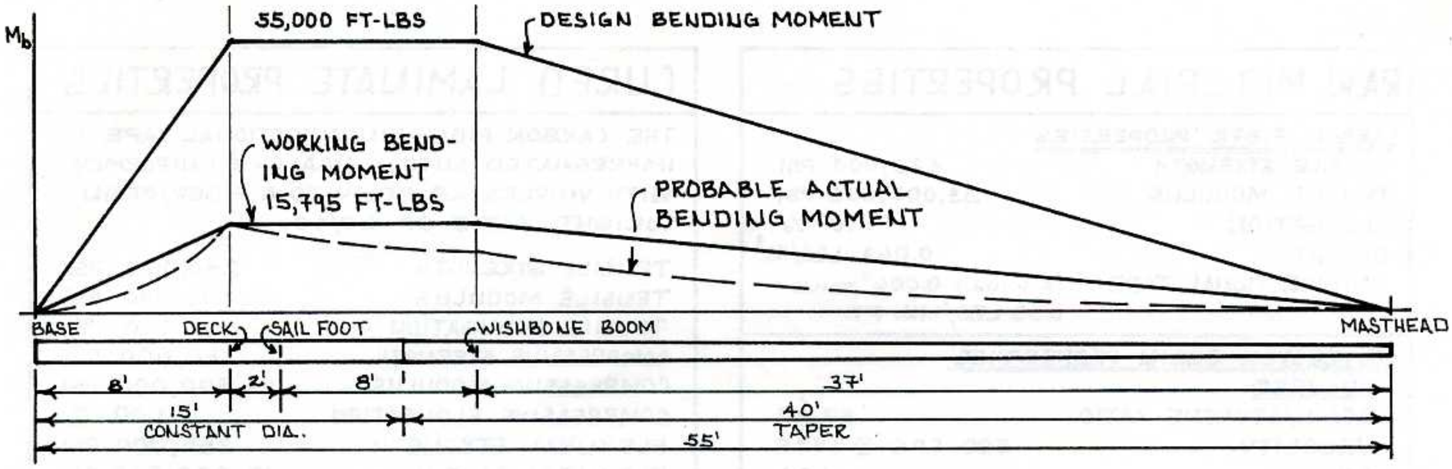


FIG. A-3 BENDING MOMENT DIAGRAM

Moments at WB:

$$F_{mh} = \frac{55{,}000 \text{ ft-lbs}}{37 \text{ ft}} = 1486.5 \text{ lbs}$$

Moments at D:

$$-F_{wb} \times 10 \text{ ft} + F_{mh} \times 47 \text{ ft} = 55{,}000 \text{ ft-lbs}$$

$$F_{wb} = \frac{1486.5 \times 47 - 55{,}000}{10}$$

$$F_{wb} = 1486.5 \text{ lbs}$$

Moments at B:

$$R_d \times 8 \text{ ft} - F_{wb} \times 18 \text{ ft} + F_{mh} \times 55 \text{ ft} = 0$$

$$R_d = \frac{1486.5 \times 55 - 1486.5 \times 18}{8}$$

$$R_d = 6875.1 \text{ lbs}$$

Reactions:

$$R_b - R_d + F_{wb} - F_{mh} = 0$$

$$R_b = 6875.1 - 1486.5 + 1486.5$$

$$R_b = 6875.1 \text{ lbs}$$

Next, calculate the required moments of inertia and section moduli at one-foot increments along the mast. They can be calculated in terms of the number of strips of unidirectional material (carbon fiber tape). Because carbon fiber is such a strong material, not much thickness will be required; therefore, the mast can be assumed to be a thin-walled tube.

$$I_{cs} = \pi R^3 t = \frac{\pi D^3 t}{8} \quad (3)$$

$$SM_{cs} = \frac{I_{cs}}{R} = \frac{\pi R^3 t}{R} = \frac{\pi D^2 t}{4} \quad (4)$$

For any given section, the area of the

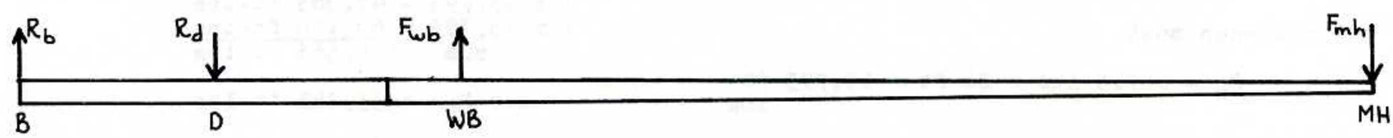


FIG. A-4 REACTION LOAD DIAGRAM

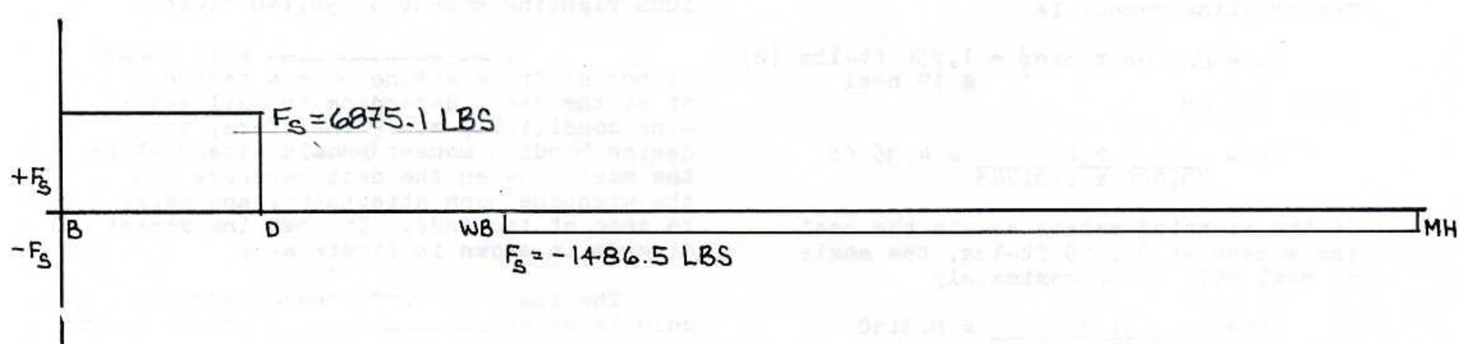


FIG. A-5 SHEAR LOAD DIAGRAM

section will be

$$A_{cs} = Ct = \pi Dt \qquad (5)$$

Also, if the cross-section is comprised of N strips of carbon fiber, 12" wide by 0.006" thick, the cross-sectional area can also be written

$$A_{cs} = Nwt' \qquad (6)$$

$$A_{cs} = N \times 12" \times 0.006" = 0.072N$$

These two areas are always equal

$$\pi Dt = Nwt' = 0.072N \qquad (7)$$

Split the expression for I_{cs} to obtain a πDt expression

$$I_{cs} = \pi Dt \times \frac{D^2}{8}$$

Substitute (7) for πDt

$$I_{cs} = Nwt' \times \frac{D^2}{8}$$

$$I_{cs} = \frac{0.072\ ND^2}{8} = 0.009ND^2 \qquad (8)$$

Do the same for SM_{cs}

$$SM_{cs} = \pi Dt \times \frac{D}{4}$$

$$SM_{cs} = \frac{0.072\ ND}{4} = 0.018ND \qquad (9)$$

Therefore, I_{cs} and SM_{cs} are dependent only on N, the number of strips of unidirectional material (carbon fiber tape), and D, the diameter of the section. π and t fall out of the equations.

It should be noted that equations (8) and (9) are true only for the unidirectional tape selected. Similar expressions must be worked out for other types of fabric.

Current technology has shown that a composite circular-section beam under a bending load will almost always fail on the compression side. Therefore, the compression strength of the unidirectional material becomes the limiting stress, in this case, 170,000 PSI. Further, past experience has also indicated that failure usually involves section shape instability due to the fact that the laminate is non-isotropic and of very thin wall. Strength and stiffness in the hoop direction (i.e. circumferential), which maintains section shape, is very much lower than that in the axial direction. To account for this weakness, a reasonable figure for design stress has been shown to be about half the compression strength, or 85,000 PSI in this case.

For a beam under load,

$$\text{section modulus} = \frac{\text{bending moment ft-lbs}}{\text{stress} \quad \text{lbs/in}^2} \times 12\text{in/ft} \qquad (10)$$

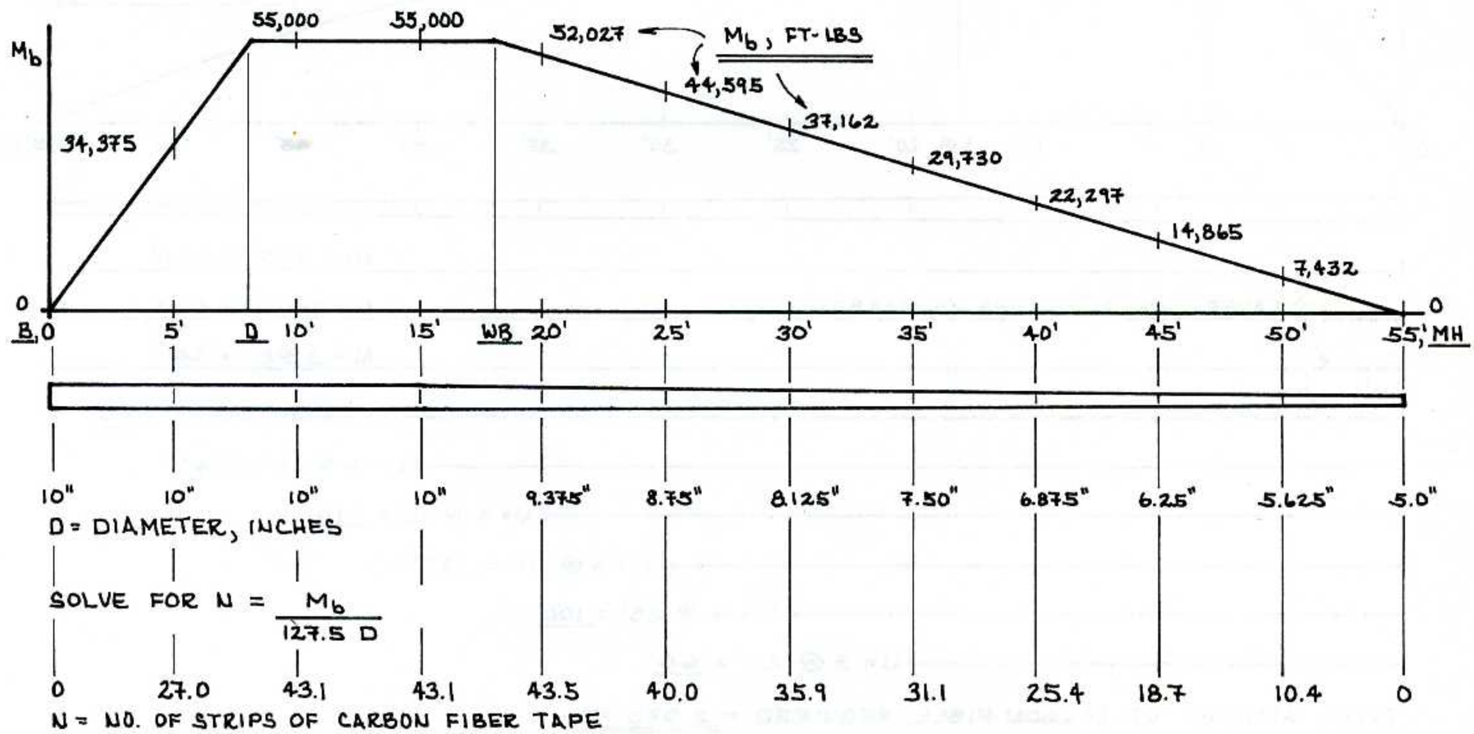


FIG. A-6 DETERMINING N. VALUES SHOWN ARE AT 5' INCREMENTS FOR CLARITY; ACTUAL CALCULATION TO BE DONE AT 1' INCREMENTS.

Setting equations (9) and (10) equal to each other gives

$$SM_{cs} = 0.018ND = \frac{M_b \times 12}{\sigma} \qquad (11)$$

Solving for N

$$N = \frac{M_b \times 12}{\sigma \times 0.018D} = \frac{M_b \times 12}{85{,}000 \times 0.018D}$$

$$N = \frac{M_b}{127.5\ D}$$

N, the number of strips of carbon fiber required, is calculated by substituting values for M_b and D at one-foot increments along the entire length of the mast. See figure A-6.

Obviously, the wall thickness cannot be zero at the ends of the mast. The ends must have sufficient thickness to withstand the shear loads inherent in bending. The shear loads have been calculated (figure A-5), but shear strength of carbon fiber perpendicular to the fiber axis is not a readily available figure. Little, if any, testing has been done to verify it; most research has been confined to the strength of laminates parallel to the direction of the fiber. To get around this problem without doing extensive testing, general practice is to make the wall thickness at the ends of the mast the same as at the point of highest fiber loading, in this case, at 18' above the base of the mast where N equals 44.8, say 45.

At that point

$$Nwt' = \pi Dt$$

$$t = \frac{Nwt'}{\pi D} = \frac{45 \times 12 \times 0.006}{\pi \times 9.625}$$

$$t = 0.107 \text{ inches thick}$$

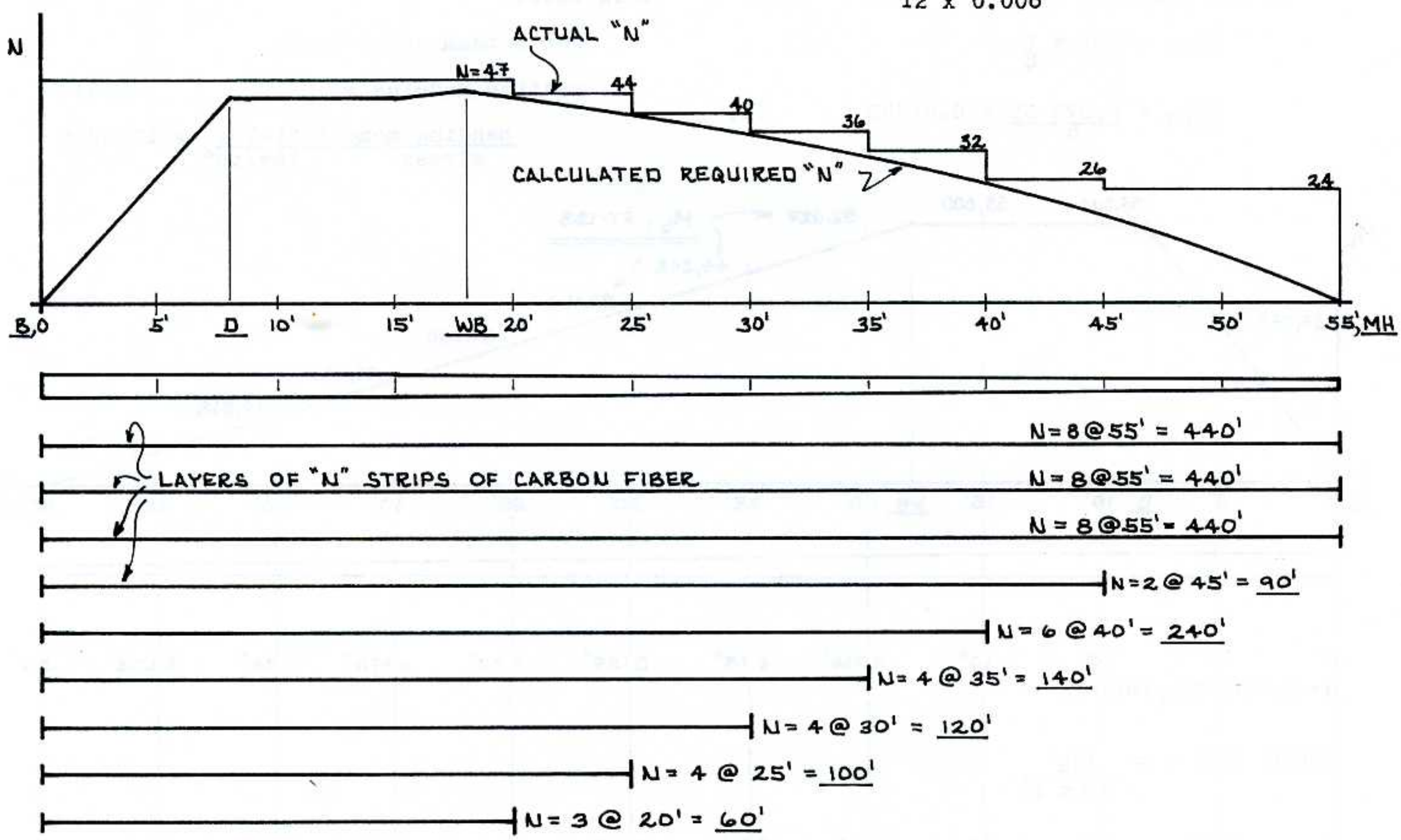
Substituting this value for t into the same equation for the ends of the mast, solve for N.

At the base

$$N = \frac{\pi \times 10 \times 0.107}{12 \times 0.006} = 46.7 \text{ strips}$$

At the masthead

$$N = \frac{\pi \times 5 \times 0.107}{12 \times 0.006} = 23.3 \text{ strips}$$



TOTAL AMOUNT OF CARBON FIBER REQUIRED = 2,070 FT.

ALSO, ADD INNER & OUTER LAYERS OF HOOP MATERIAL

FIG. A-7 LAMINATE SCHEDULE

With N calculated and known everywhere along the mast, a laminate schedule can be drawn in which the strips of carbon fiber are grouped into increments of five feet instead of every foot. Working in five-foot increments during actual building is much easier to control than it would be with one-foot increments. This is where production considerations begin to play a part in the design process. See figure A-7.

The above calculations refer only to the axial carbon fiber laminate. Material in the hoop direction is also required. The hoop material is usually a layer of E-glass cloth or unidirectional tape, one on the inside of the laminate and one on the outside. This hoop material sandwiches the carbon fiber layer, keeping all the carbon fiber aligned when under load as well as giving hoop strength to the mast. During hoop winding, the pressure of the hoop on the wet carbon fiber helps to squeeze the resin uniformly through the laminate. The outside layer also provides abrasion and impact resistance to the mast as well as a sound base on which to apply finish coats like pigmented resin or paint.

For hoop material, general practice is to use the equivalent of 24 oz/yd^2 of unidirectional E-glass in each layer. The actual number of plies per layer depends, of course, on the available material. For example, if 12 oz/yd^2 unidirectional tape is used, two plies (total 24 oz/yd^2) must be used in both the inner and outer layers of hoop. For areas where extra chafe is expected, for instance where the wishbone boom rides or at the maststep or partners, an extra 24 oz/yd^2 layer can be applied.

The laminate is complete and the corresponding weights of all the material can be calculated.

Carbon:

Total linear feet of carbon fiber tape = 2070 lin. ft

Weight at 0.05 lbs/ft = 103.5 lbs carbon fiber

Glass:

Inside area of mast

$$\pi \times \frac{10''}{12''/'} \times 15' = 39.3 \text{ ft}^2$$

$$\pi \times \frac{10''+5''}{2 \times 12''/'} \times 40' = \underline{78.5 \text{ ft}^2}$$

Total = 117.8 ft^2

Outside area of mast

O.D. @ base = 10" + 0.107" + 0.107" = 10.214"

O.D. @ masthead =

5" + 0.107" + 0.107" = 5.214"

$$\pi \times \frac{10.214''}{12''/'} \times 15' = 40.1 \text{ ft}^2$$

$$\pi \times \frac{10.214''+5.214''}{2 \times 12''/'} \times 40' = \underline{80.8 \text{ ft}^2}$$

Total 120.9 ft^2

Total area of glass (not including any chafe strips) = 238.7 ft^2

= 26.5 yds^2

Weight of glass

26.5 yds^2 x 24 oz/yd^2 = 636.0 oz

= 39.8 lbs glass

Total fiber weight:

103.5 lbs carbon fiber
39.8 lbs glass

143.3 lbs fiber

Total laminate weight at 60% fiber content by weight:

$$\frac{143.3 \text{ lbs fiber}}{0.6} = \underline{\underline{238.8 \text{ lbs}}}$$

Resin weight:

238.8 - 143.3 = 95.5 lbs resin

Therefore, the laminate for each mast will weigh just under 240 lbs. This figure increases after finishing coats of pigmented resin and paint are applied and the hardware is installed.

APPENDIX B

ENGINEERING AND DESIGN OF AN AIRFOIL-SECTION WINGMAST FOR THE SAME BOAT AS IN APPENDIX A

All design parameters are the same. The sail plan is the same except for the details shown in figure B-1. Note that the masts rotate and therefore are round at deck level and below and are of the same diameter as the masts in appendix A. Also note that the booms are of conventional design, not wishbone type.

The maximum bending moment for these masts will occur at the deck where the mast is round, as shown in figure B-2. The laminate here can be calculated exactly the same as shown in appendix A. The section chosen is shown in figure B-3 and is a 2:1 shape, meaning the section is twice as long as it is wide. However, the chord length C is as yet undetermined. Again, the laminate is very thin but made of high strength material. The section can again be considered a thin-walled tube.

Looking first at the elliptical leading edge in figure B-4.

A_{le} = cross-sectional area for a tubular half ellipse

$$A_{le} = \frac{\pi}{2} t(a+b) \qquad (12)$$

I_{yle} = moment of inertia for a tubular half ellipse about its base

$$I_{yle} = \frac{\pi}{8} t(3a^2b+a^3) \qquad (13)$$

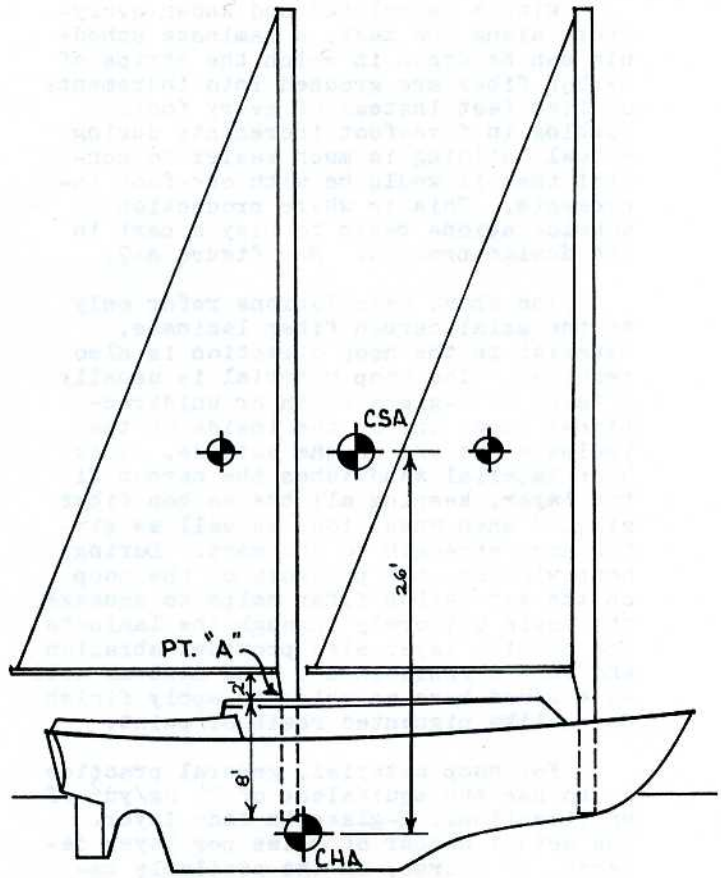


FIG. B-1

For the trailing edge, figure B-5.

$$A_{te} = 2at \qquad (14)$$

$$I_{yte} = \left[\frac{1}{12}ta^3+at\left(\frac{a}{2}\right)^2\right] \times 2$$

$$I_{yte} = \frac{2}{3}a^3t \qquad (15)$$

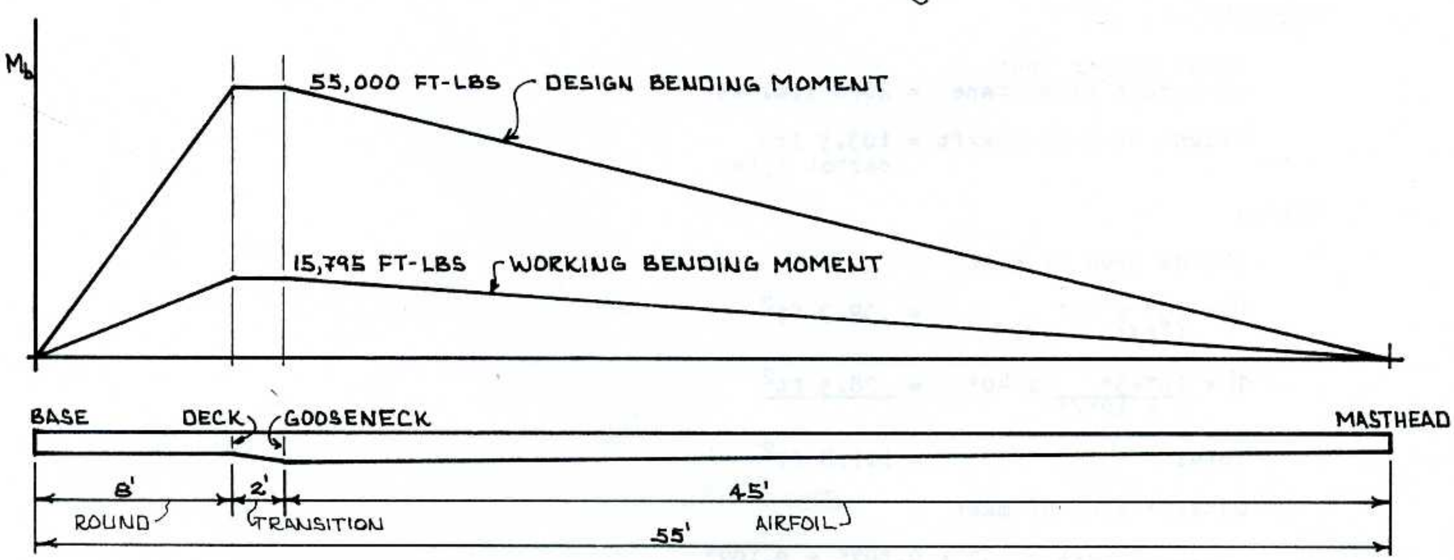


FIG. B-2 BENDING MOMENT DIAGRAM

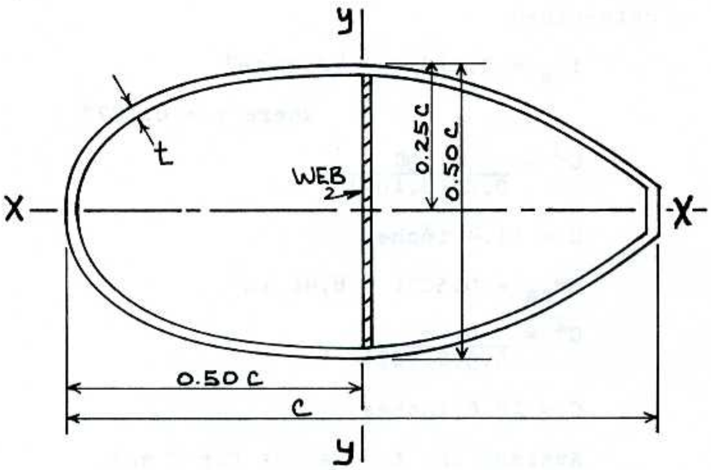


FIG. B-3 PROPOSED SECTION SHAPE

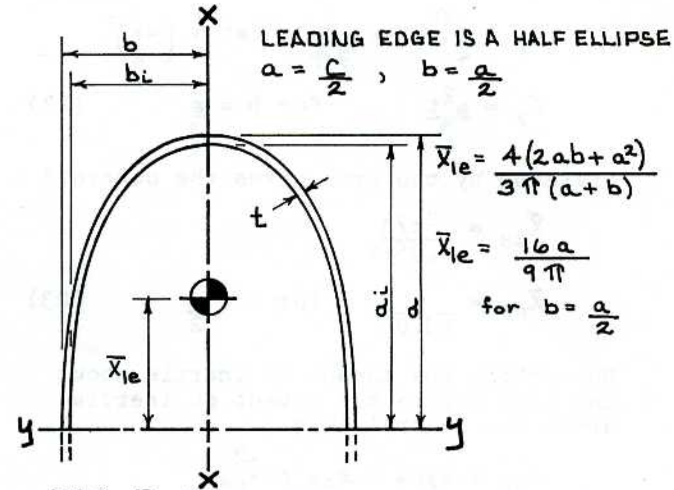


FIG. B-4 LEADING EDGE SHAPE

Sum equations (12) with (14) and (13) with (15) to get the total area and moment of inertia about the line y-y.

Total area = $A_{cs} = \frac{\pi}{2}t(a+b) + 2at$ (16)

Total moment of inertia =

$$I_{ycs} = \frac{\pi}{8}t(3a^2b+a^3) + \frac{2}{3}a^3t \qquad (17)$$

We have specified that this is a 2:1 shape, so $b = \frac{a}{2}$. Substituting this into the equations (16) and (17),

$$A_{cs} = \frac{\pi}{2}t(a+\frac{a}{2}) + 2at$$

$$A_{cs} = at(\frac{3\pi}{4}+2) = 4.35at \qquad (18)$$

and

$$I_{ycs} = \frac{\pi}{8}t(3a^2\frac{a}{2}+a^3) + \frac{2}{3}a^3t$$

$$I_{ycs} = a^3t(\frac{5\pi}{16}+\frac{2}{6}) = 1.65a^3t \qquad (19)$$

The moment of inertia must be related to the neutral axis. First find the centroid of the section along its major axis. From figure B-4, the centroid of the leading edge is

$$\bar{X}_{le} = \frac{4(2ab+a^2)}{3\pi(a+b)} \qquad (20)$$

$$\bar{X}_{le} = \frac{16a}{9\pi} \qquad \text{for } b = \frac{a}{2}$$

and the centroid for the trailing edge is

$$\bar{X}_{te} = -\frac{a}{2} \qquad (21)$$

Sum moments about the line y-y

$$M_y = A_{le} \times \bar{X}_{le} + A_{te} \times \bar{X}_{te}$$

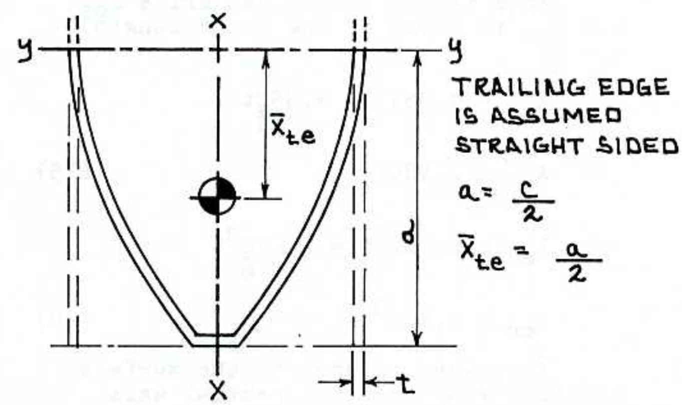


FIG. B-5 TRAILING EDGE SHAPE

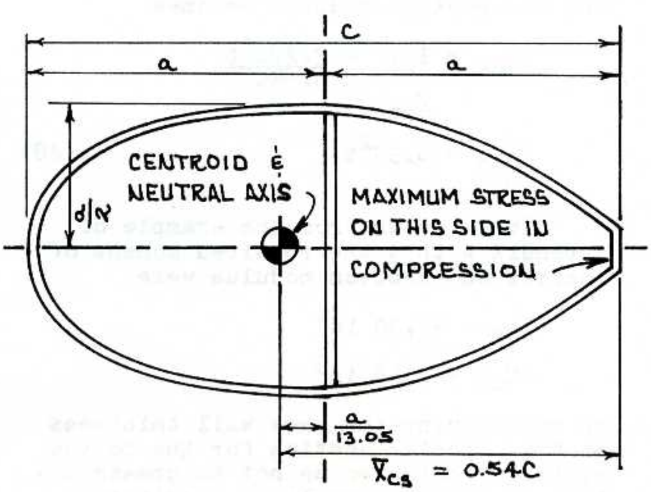


FIG. B-6 CENTROID OF SECTION

$$M_y = \frac{3}{4}\pi at \times \frac{16a}{9\pi} + 2at \times \left(-\frac{a}{2}\right)$$

$$M_y = \frac{a^2t}{3} \qquad \text{for } b = \frac{a}{2} \qquad (22)$$

Dividing by the area gives the centroid

$$\bar{X}_{cs} = \frac{a^2t/3}{4.35at}$$

$$\bar{X}_{cs} = \frac{a}{13.05} \qquad \text{for } b = \frac{a}{2} \qquad (23)$$

Converting the moment of inertia about the line y-y to the moment of inertia about the neutral axis

$$I_{cs} = I_{ycs} - A_{cs} \times \bar{X}_{cs}^2$$

$$I_{cs} = 2.15a^3t - 4.35at \times \left(\frac{a}{13.05}\right)^2$$

$$I_{cs} = 2.12a^3t \qquad (24)$$

From figure B-6 we can write A_{cs} and I_{cs} in terms of the chord length C; $a = \frac{C}{2}$.

$$A_{cs} = 4.35at = 4.35\frac{C}{2}t$$

$$A_{cs} = 2.18Ct \qquad (25)$$

$$I_{cs} = 2.12a^3t = 2.12\frac{C^3}{8}t$$

$$I_{cs} = 0.27C^3t \qquad (26)$$

$\bar{X}_{cs}$, when refered to the surface farthest away from the neutral axis (in this case the trailing edge undergoing compression), becomes

$$\bar{X}_{cs} = 1.077a = 1.077\frac{C}{2}$$

$$\bar{X}_{cs} = 0.54C \qquad (27)$$

and the section modulus becomes

$$SM_{cs} = \frac{I_{cs}}{\bar{X}_{cs}} = \frac{0.27C^3t}{0.54C}$$

$$SM_{cs} = 0.37C^2t \qquad (28)$$

It is known from the example of appendix A that the required moment of inertia and section modulus were

$$I_{cs} = 42.30 \text{ in}^4$$

$$SM_{cs} = 8.46 \text{ in}^3$$

By maintaining the same wall thickness as the circular section for the bottom airfoil section so as not to create any thinner areas where local buckling could occur, the chord length can be determined

$$I_{cs} = 0.27C^3t = 42.30 \text{ in}^4$$

where t = 0.107"

$$C^3 = \frac{42.30}{0.27(0.107)}$$

C = 11.4 inches

$$SM_{cs} = 0.5C^2t = 8.46 \text{ in}^3$$

$$C^2 = \frac{8.46}{0.5(0.107)}$$

C = 12.6 inches

Average the two values for C and round it off to the nearest whole inch, say 12 inches for the bottom chord. The top chord should be 75% of the bottom chord, therefore 9 inches. The taper from bottom to top is uniformly linear. Knowing the chord length for every section along the mast, N, the number of strips of carbon fiber, can then be calculated.

Equating the areas as was done in appendix A,

$$A_{cs} = 2.18Ct = Nwt' \qquad (29)$$

$$t = \frac{Nwt'}{2.18C}$$

Substituting for t in equation (28)

$$SM_{cs} = 0.5C^2\frac{Nwt'}{2.18C} \qquad (30)$$

Recalling that w = 12" and t' = 0.006" for the carbon fiber tape

$$SM_{cs} = \frac{0.5C^2N(12)(0.006)}{2.18C}$$

$$SM_{cs} = 0.0165CN$$

Also recall that

$$SM_{cs} = \frac{M_b12}{\sigma}$$

Therefore

$$\frac{M_b12}{\sigma} = 0.0165CN \qquad (31)$$

Solving for N

$$N = \frac{Mb12}{\sigma\, 0.0165C} = \frac{Mb12}{85{,}000(0.0165)C}$$

$$N = \frac{M_b}{86.4C} \qquad (32)$$

Figure B-7 shows the calculation for N at five-foot increments all along the mast by substituting values for M_b and C into equation (32). Figure B-8 shows the laminate schedule. On the

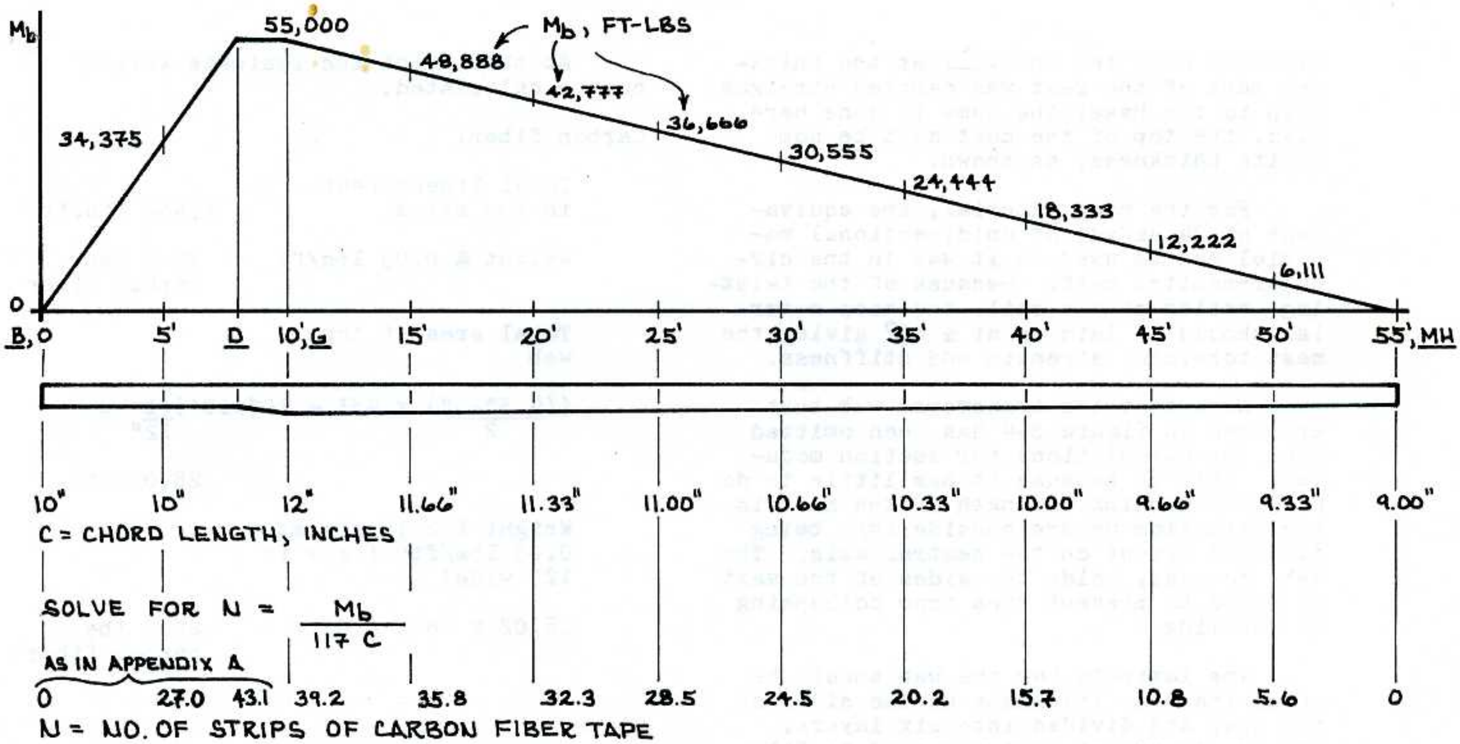
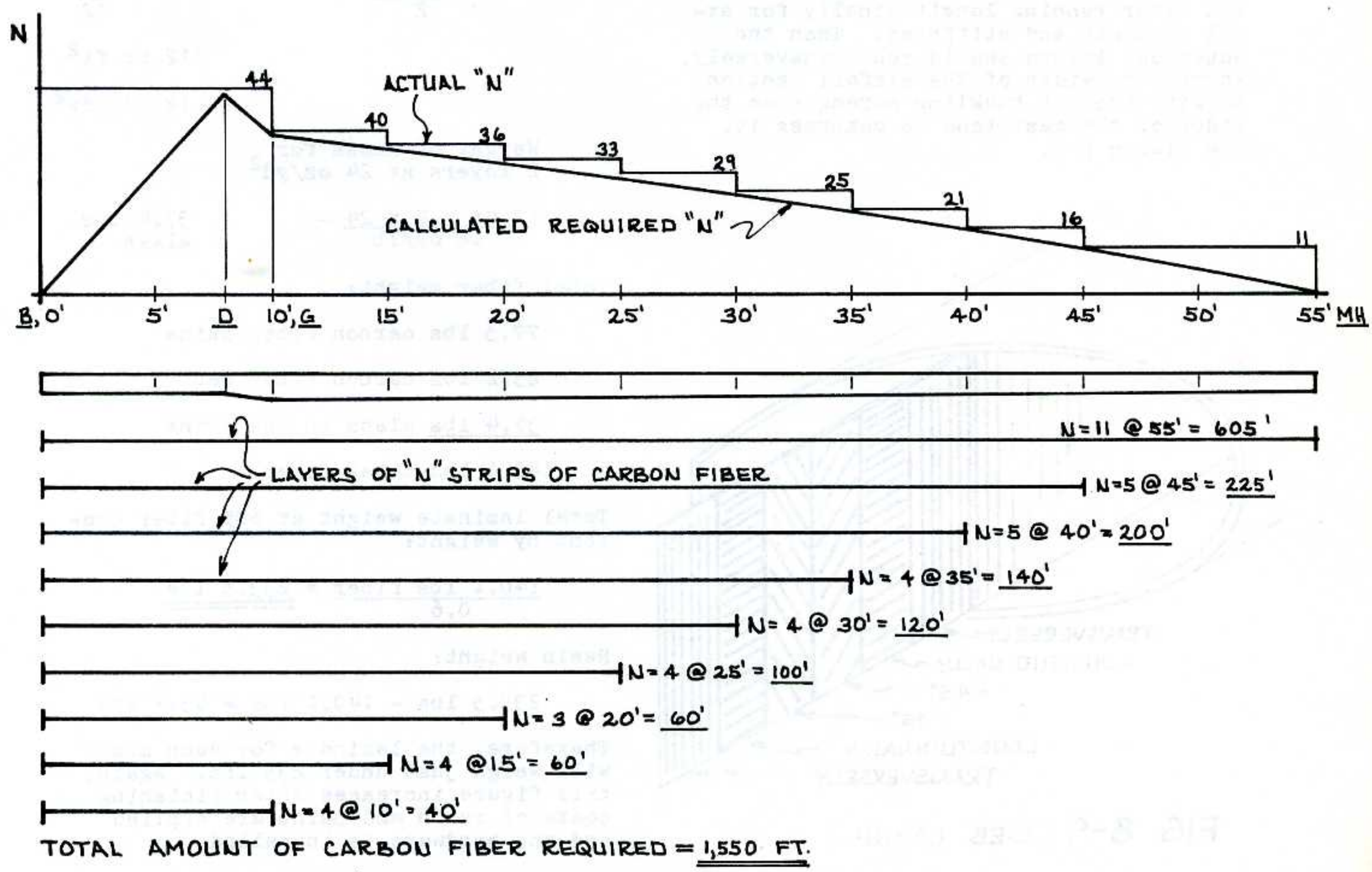


FIG. B-7 DETERMING N. VALUES SHOWN ARE AT 5' INCREMENTS FOR CLARITY; ACTUAL CALCULATION TO BE DONE AT 1' INCREMENTS.



TOTAL AMOUNT OF CARBON FIBER REQUIRED = 1,550 FT.

ALSO, INCLUDE INNER & OUTER LAYERS OF HOOP MATERIAL AND THE WEB.

FIG. B-8 LAMINATE SCEDULE

circular mast the material at the thickest part of the mast was carried straight down to the base; the same is done here. Also, the top of the mast must be some finite thickness, as shown.

For the hoop material, the equivalent of 24 oz/yd^2 of unidirectional material can be used as it was in the circular-section mast. Because of the twisting action of the sail, the hoop material should be laid in at $\pm 45^o$ giving the mast torsional strength and stiffness.

Note that the transverse web that appeared in figure B-4 has been omitted from the calculations for section modulus. This is because it has little to do with the bending strength of the mast in the direction we are considering, being situated almost on the neutral axis. The web, however, holds the sides of the mast in place to prevent them from collapsing or buckling.

The laminate for the web should be about the same thickness as the sides of the mast and divided into six layers. The two innermost layers should be fiber running at $\pm 45^o$ for shear strength between the tension and compression loads in the sides of the mast. Either side of this core should be two layers of carbon fiber running longitudinally for axial strength and stiffness. Then the outermost layers should run transversely, across the width of the airfoil section, to give the web buckling strength as the sides of the mast tend to compress it. See figure B-9.

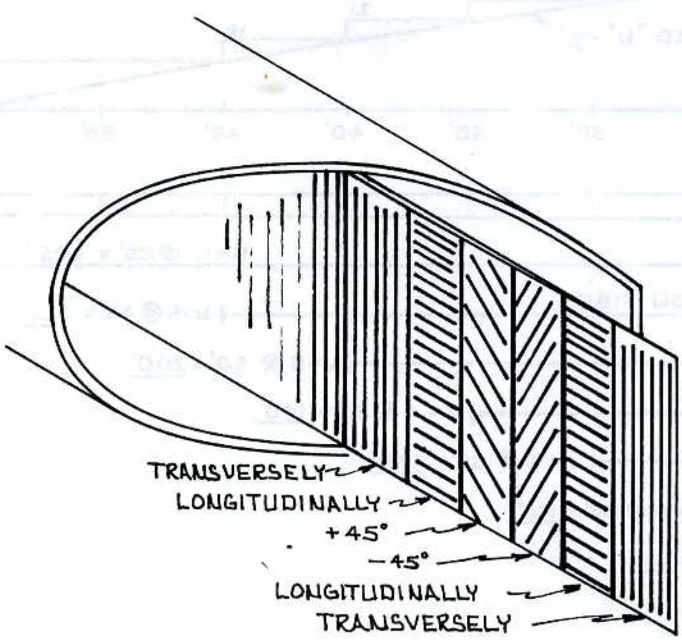


FIG. B-9 WEB LAYUP

At this point the laminate weight can be calculated.

Carbon fiber:

Total linear feet in the skins = 1,550 lin.ft

Weight @ 0.05 lbs/ft = 77.5 lbs carbon fiber

Total area of the web

$$\left(\frac{4.5"+6"}{2} \times 45' + 10" \times 10'\right)\frac{ft}{12"} =$$

28.02 ft^2

Weight for 18 ply at 0.05 lbs/ft^2 (tape is 12" wide)

28.02 x 18 x 0.05 = 25.2 lbs carbon fiber

Glass:

The surface area of the mast

$$\left(2.18\left(\frac{12"+9"}{2}\right) \times 45' + \pi \times 10" \times 10'\right)\frac{ft}{12} =$$

112.02 ft^2

= 12.45 yds^2

Weight of glass for 2 layers at 24 oz/yd^2

$$\frac{12.45 \times 2 \times 24}{16 \text{ oz/lb}} =$$ 37.4 lbs glass

Total fiber weight:

77.5 lbs carbon fiber skins

25.2 lbs carbon fiber web

37.4 lbs glass in the skins

140.1 lbs total fiber

Total laminate weight at 60% fiber content by weight:

$$\frac{140.1 \text{ lbs fiber}}{0.6} = 233.5 \text{ lbs}$$

Resin weight:

233.5 lbs - 140.1 lbs = 93.4 lbs

Therefore, the laminate for each mast will weigh just under 235 lbs. Again, this figure increases after finishing coats of resin and paint are applied and the hardware is installed.

ADDENDUM TO
"DESIGN AND ENGINEERING ASPECTS OF FREE-STANDING MASTS AND WINGMASTS"
6th CHESAPEAKE SAILING YACHT SYMPOSIUM, MARCH 1983

AUTHOR'S NOTE:
The following inquiry came to me from a student of naval architecture in England requesting design and engineering information regarding free-standing masts. As far as I know, this paper is the only one that describes the really practical design and engineering aspects of such masts. The paper was not without errors, and some time ago, I prepared a clean and corrected copy that could be photocopied and mailed to any interested parties. Now in this day of computer files and the Internet, I have scanned the paper into this .pdf file and am attaching this note. In my response to the student in England, I prepared the following remarks describing my thoughts on this science. I have deleted any personal references to the student, and offer these remarks to all others that may be interested.

Eric W. Sponberg
Naval Architect,
St. Augustine, FL
2 February 2011

From: Eric W. Sponberg [mailto:ewsponberg@comcast.net]
Sent: Tuesday, February 01, 2011 10:10 AM
Subject: RE: Incoming message from website--D&E of Free-standing Masts & Wingmasts

Dear ---------,

Thank you for your interest in free-standing masts. I wrote probably the only definitive paper ever on free-standing masts, for the 6th Chesapeake Sailing Yacht Symposium in 1983. There were a few errors in it as it was printed, so I created a corrected copy and have scanned it into a PDF file. This is attached. There are some other things that you should note about this paper for consideration when designing free-standing masts:

1. The mechanical properties of composite laminates shown in Table II on page 74 are very "ideal". You don't get properties like that in real life boatbuilding. I'd cut those numbers at least in half.

2. On page 80, on section shape buckling, I discuss the need to find a way to determine the necessary thickness required to prevent section shape buckling. In my subsequent research, I have determined that for unidirectional laminates such as cantilevered masts, whose laminate is not more than 80% unidirectional fiber, nor less than 50% unidirectional, then the minimum wall thickness of a solid (non-cored) laminate required to prevent buckling is 3% of inside diameter. That is, the minimum t/ID ratio = 0.03. This applies to the mast wall thickness at any height, any diameter. I use the inside diameter because practically all my designs start with the controlled surface as the inside surface of the mast (that is, the masts are laid up over male mandrels).

3. Related to #1, on page 85, Table A-3, I show carbon fiber laminate properties which were typical of my engineering at the time. However, I have found these to be overly optimistic, and I typically use properties that are about half these amounts.

4. On pages 86 and 87 in Appendix A, I show how the boat's maximum righting moment is used as the definitive load for the mast. Although I go through a calculation on page 85 to determine wind loads in the sails, it really is the righting moment load of the hull that is the definitive load for the mast. All you need to do is determine this maximum righting moment of the boat, spread that as a constant between the partners and the gooseneck, and then taper that moment load in a straight line down to each end. You will still need a factor of safety; I typically use a factor of safety of 3.0, either by multiplying the max. righting moment load by the FoS and using the full strength capabilities of the laminate, or use the max. righting moment load and divide the full strength properties by 3.0 reducing them to allowable stresses. That is how I still do it.

5. In Appendix B, I show similar calculations for a wingmast. In determining the section shape properties (I and SM) of a wing shape, I approximate the trailing edge as straight sided (Fig. B-5, page 91). Nowadays, it is relatively easy to calculate the actual section shape properties (by spreadsheet, or by AutoCad "mass properties" command), so there is no need for shape approximations anymore.

6. I do not show in this paper how to do deflection calculations. That is a process that I developed later, after this paper was written. Typically, a free-standing mast is tapered, both in section shape and wall thickness. Therefore, section area, I and SM vary all along the mast. If you follow regular cantilever beam engineering and calculus principles, you can do a double integration of M/(ExI) all along the mast to determine the tip deflection. I will leave it up to you and your professors to figure out how that is done. Be sure that you "normalize" the calculation to take into account that the mast heel and the deck partners do not move—they are stationary. I do this on a spreadsheet. By the way, the amount of deflection that you are looking for live-load sailing conditions, not max. righting moment conditions. So you have to change the moment load down from the maximum righting moment load to a regular sailing load, say and Force 4 or Force 5 wind speed.

7. I offer this paper to you AND your school. Please make a copy of this paper, and this email which contains these guidelines, and give it to your engineering library for all future student use.

Good luck on your design.

Best regards,

Eric

Eric W. Sponberg
Naval Architect
President
Sponberg Yacht Design Inc.

Sent: Sunday, January 30, 2011 3:00 PM
To: ewsponberg@sponbergyachtdesign.com
Subject: Incoming message from website

Name: ----------------------
Company: Southampton Solent University
Telephone:
Email:
Address:
Address2:
City:
State:
Zip:
Country:

Comments:
Dear Mr. Sponberg, I am a second year Yacht Design student at Southampton Solent University. I am currently designing a 25ft sloop for amateur construction using strip planking techniques. During some lectures about mast analysis, as part of a structural analysis unit at university, I was surprised by the amount of compression produced by conventional rigs in the mast. This is why I started researching about free standing masts, and apparently you are one of the main supporter and experts of this type of rigging. I am starting to consider this as a possible solution for my project, so I would like to estimate loads and required sections for a free standing wing mast. Where can I find some good engineering information about free standing rig design? Most of the books I have skimmed trough do not consider this option, can you suggest me something? Best Regards -----------------