SECTION 4 LAMINATES AND STIFFENERS

1 General

1.1 Definition

1.1.1 The purpose of this Chapter is to estimate the global laminate's elastic coefficients and the local distribution of stresses in the individual layers.

As a general rule, a laminate is made of stacked individual layers, which may differ in local orientation and in type.

1.2 General

1.2.1 The laminating process acts upon the mechanical characteristics of the final laminate. This influence is taken into account with the partial safety factor C_F as defined in Ch 4, Sec 3, [5.4.1].

1.2.2 All steps of laminating process are to be performed taking into account the recommendations of raw materials' manufacturers and of the requirements of Ch 12, Sec 5.

2 Determination of laminate's mechanical characteristics

2.1 General

2.1.1 The methodology to determine the mechanical characteristics of a laminate is described in Tab 1.

The aim of laminates' calculation approach defined in the present Rules is to determine, starting from external loads applied to a laminate, the local stresses in each individual layer in its own orthotropic axis system and to compare them with the local theoretical breaking stresses as defined in Ch 12, Sec 3, [5].

2.2 Software

2.2.1 All the calculations defined in the present chapter may be performed using the Society's program defined in Ch 1, Sec 4.

3 Laminate definition

3.1 Parameters

3.1.1 The laminate's main characteristics depend on:

- the type of individual layers
- the position of individual layers throught the laminate thickness
- the orientation of individual layers in relation to laminate's global axis.

3.2 Description

3.2.1 Type of individual layer

The type and properties of individual layers are defined in Ch 12, Sec 3.

Table 1 : Methodology

Step	Description	Rule's requirements
1	Estimation of elastic coefficients, mechanical and geometric characteristics of each individual layer in their local orthotropic axis	Ch 12, Sec 3, [2] and Ch 12, Sec 3, [3]
2	Description of the laminate to examine: • position of all individual layers • orientation of each layer in relation to the global laminate's in-plane axis	[3.2.2] and [3.2.3]
3	Calculation of rigidity and flexibility of individual layers in the laminate's global axis	[3.2.4]
4	For specific calculations (midship section modulus, global buckling or stiffeners), calculation of laminate's global elastic coefficients and mechanical characteristics in laminate's axis	[5]
5	Estimation of all external loads applied to the laminate	For panels: Ch 9, Sec 3 For stiffeners: Ch 9, Sec 4
6	Calculation of the laminate's global strain in its global axis, induced by loads estimated in previous step 5	[4.1]
7	Calculation of stresses and strains, for each individual layer, in the laminate's global axis	[4.2.1]
8	Calculation of stresses and strains, for each individual layer, in the individual layer's local axis	[4.2.2]
9	Estimation, for each individual layer, of the breaking criteria in their own local axis	Ch 12, Sec 3, [5]
10	Check of the safety coefficient (equal to the breaking stresses -step 9- divided by the local stresses in local axis -step 8-) in relation to Rule safety coefficients	Ch 4, Sec 3, [5.4]

3.2.2 Position of individual layer

Positions of all individual layers are described in Fig 1.

where:

AP : Median plane of the laminate, located at half the thickness of the laminate

e : Laminate's thickness, in mm

e_K : Thickness of individual layer K, in mm

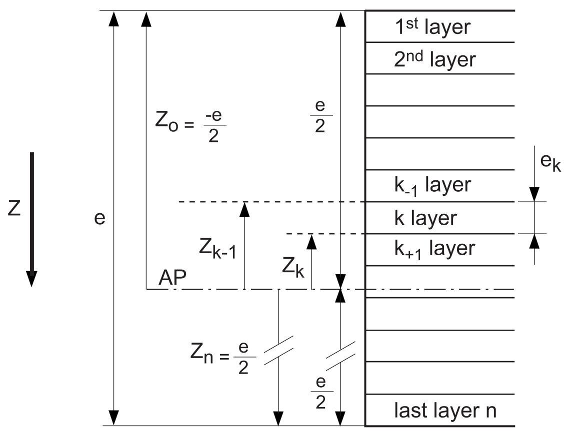
Z_K : Distance between AP and interface of layers K and K+1.

$$Z_K = \frac{-e}{2} + \sum_1^K e_i$$

Z_{K-1} : Distance between AP and interface of layers K and K-1.

$$Z_{K-1} = \frac{-e}{2} + \sum_1^{K-1} e_i$$

Figure 1 : Position of individual layer



3.2.3 Orientation of individual layer

The orientation between each individual layer's local axis and laminate's global axis is defined in Fig 2.

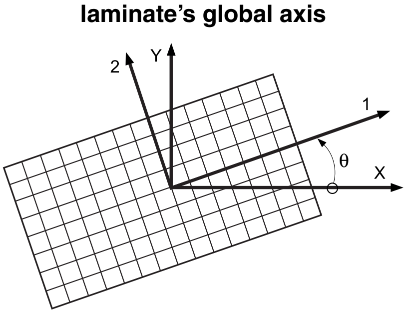
As a general rule, laminate's global reference axis are taken similar to the ship's reference axis:

X : Ship's longitudinal axis

Y : Axis perpendicular to the ship's longitudinal axis in laminate's plane

Note 1: The angle θ is considered positive from global axis to local axis (See Fig 2).

Figure 2 : Orientation of individual layer in relation to laminate's global axis



3.2.4 This orientation enables to calculate rigidity and flexibility of all individual layers in the laminate's global axis. The conversion's calculation is obtained by the following formulae:

$$[R]_K = \begin{bmatrix} R_{XX} & R_{XY} & R_{XZ} \\ R_{YX} & R_{YY} & R_{YZ} \\ R_{ZX} & R_{ZY} & R_{ZZ} \end{bmatrix}_K = T[\bar{R}]_K T'^{-1}$$

$$[S]_K = \begin{bmatrix} S_{XX} & S_{XY} & S_{XZ} \\ S_{YX} & S_{YY} & S_{YZ} \\ S_{ZX} & S_{ZY} & S_{ZZ} \end{bmatrix}_K = T'[\bar{S}]_K T^{-1}$$

where:

$[\bar{R}]_k$ and $[\bar{S}]_k$: Rigidity and flexibility matrix of an individual layer K in local axis as defined in Ch 12, Sec 3, [4.1.2] and Ch 12, Sec 3, [4.1.3]

$[R]_k$ and $[S]_k$: Rigidity and flexibility matrix of an individual layer K in global axis

T and T′ : Transfer matrixes

$$T = \begin{bmatrix} (\cos\theta)^2 & (\sin\theta)^2 & -2\cos\theta\sin\theta \\ (\sin\theta)^2 & (\cos\theta)^2 & 2\cos\theta\sin\theta \\ (\cos\theta\sin\theta) & (-\cos\theta\sin\theta) & ((\cos\theta)^2 - (\sin\theta)^2) \end{bmatrix}$$

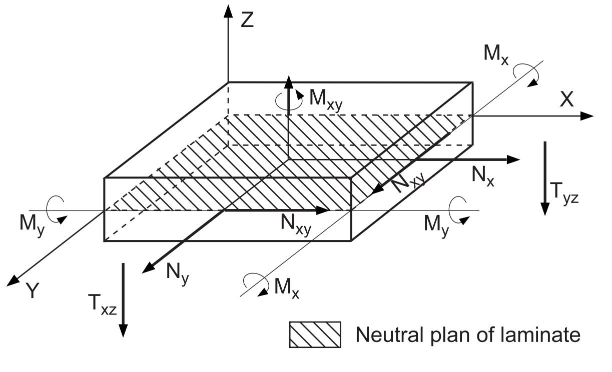
$$T' = \begin{bmatrix} (\cos\theta)^2 & (\sin\theta)^2 & -\cos\theta\sin\theta \\ (\sin\theta)^2 & (\cos\theta)^2 & \cos\theta\sin\theta \\ (2\cos\theta\sin\theta) & (-2\cos\theta\sin\theta) & ((\cos\theta)^2 - (\sin\theta)^2) \end{bmatrix}$$

4 Laminate analysis under external loading

4.1 Laminate in-plane strain under external loading

4.1.1 As a general rule, a laminate can be loaded by forces and moments as described in Fig 3.

Figure 3 : Forces and moments loading



These external loads are expressed as a function of the median plane of the laminate, per meter of width:

$$\begin{bmatrix} N_X \\ N_Y \\ N_{XY} \\ M_X \times 10^3 \\ M_Y \times 10^3 \\ M_{XY} \times 10^3 \end{bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \cdot \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \\ K_x \\ K_y \\ K_{xy} \end{bmatrix}$$

where matrix [ABD] represents the global rigidity of the laminate:

A_{ij} : Tensile rigidity (matrix 3x3)

$$A_{ij} = \sum_1^n (R_{ij})_K \cdot e_K$$

B_{ij} : Coupling effect tensile and bending (matrix 3x3)

$$B_{ij} = \frac{1}{2} \cdot \sum_1^n (R_{ij})_K \cdot (Z_K^2 - Z_{K-1}^2)$$

D_{ij} : Bending rigidity (matrix 3x3)

$$D_{ij} = \frac{1}{3} \cdot \sum_1^n (R_{ij})_K \cdot (Z_K^3 - Z_{K-1}^3)$$

$\varepsilon°_x$: Tensile or compressive strain of the median plane of the laminate in X direction

$\varepsilon°_y$: Tensile or compressive strain of the median plane of the laminate in Y direction

$\gamma°_{xy}$: Shear strain of the median plane of the laminate in XY plan

K_x : Curvate deformation of the median plane of the laminate around Y axis

K_y : Curvate deformation of the median plane of the laminate around X axis

K_{xy} : Curvate deformation of the median plane of the laminate around Z axis

N_X, N_Y, N_{XY}: Used for specific calculation as defined in step 4 of Tab 1. Global loads applied at the median plane of the laminate (see Fig 3) induced by the global loadings applied to the hull girder structure as defined in Ch 9, Sec 2, [1].

For laminate loaded by local external pressure, N_X, N_Y and N_{XY} are to be taken equal to 0.

M_X, M_Y, M_{XY}:Local flexural moments applied to the laminate (see Fig 3) as defined in Ch 9, Sec 3, [7].

$M_{XY} = 0$ in general case.

4.1.2 Strains and curvate deformations can be expressed by reversing the matrix [ABD] by the following formula:

$$\begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \\ K_x \\ K_y \\ K_{xy} \end{bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix}^{-1} \cdot \begin{bmatrix} N_X \\ N_Y \\ N_{XY} \\ M_X \times 10^{-3} \\ M_Y \times 10^{-3} \\ M_{XY} \times 10^{-3} \end{bmatrix}$$

4.2 Analysis of an individual layer in a laminate

4.2.1 Individual layer's strains in laminate global axis

From laminate's membrane and bending strains, calculated in [4.1], the membrane and bending strains for each individual layer are given by the following formula:

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix}_K = \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix} + \begin{bmatrix} K_x \\ K_y \\ K_{xy} \end{bmatrix} \cdot \frac{Z_K + Z_{K-1}}{2}$$

Note 1: ε_x, ε_y, and γ_{xy} are calculated at mid-thickness of each individual layer, except for core materials where ε_x, and ε_y are to be calculated at each interface of the core.

4.2.2 Individual layer's strains and stresses in local axis

Strains of an individual layer can be defined in its local axis by the following formula:

$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix}_K = T'^{-1} \cdot \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix}_K$$

with T' defined in [3.2.4] for every individual layer.

The local stresses in an individual layer expressed in local axis, at mid-thickness, are defined by the following formula (except for core material, see [4.2.1], Note 1):

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix}_K = [\overline{R}] \cdot \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix}_K$$

where $[\overline{R}]$ is defined in [3.2.4] for every individual layer.

4.3 Laminate's interlaminar shear analysis

4.3.1 General

The interlaminar shear stresses τ_{yz} and τ_{xz}, located between each individual layers are induced by shear loads T_{YZ} and T_{XZ} normal to the median plane of the laminate, as defined in Fig 3.

4.3.2 The interlaminar shear stresses, in the global X and Y directions of the laminate, between two layers K and K-1, are determined by the following formulae:

$$\begin{bmatrix} \tau_{yz} \\ \tau_{xz} \end{bmatrix}_K = \begin{bmatrix} H_{44} & H_{45} \\ H_{54} & H_{55} \end{bmatrix}_K \cdot \begin{bmatrix} T_{yz} \\ T_{xz} \end{bmatrix}$$

where:

T_{YZ} and T_{XZ} : Applied shear loads, normal to the median plane of laminate (see Fig 3) and defined in Ch 9, Sec 3, [7].

$[H_{44}]_k$, $[H_{45}]_k$, $[H_{54}]_k$, $[H_{55}]_k$: Shear constants of layer k

$$H_{44} = \left[C_{yz}\right]_{k,5} - \left[R_{21}\ R_{22}\ R_{23}\right]_k \cdot \begin{bmatrix} Z_{k-1}\cdot B'_{12} + \dfrac{Z_{k-1}^2}{2}\cdot D'_{12} \\ Z_{k-1}\cdot B'_{22} + \dfrac{Z_{k-1}^2}{2}\cdot D'_{22} \\ Z_{k-1}\cdot B'_{32} + \dfrac{Z_{k-1}^2}{2}\cdot D'_{32} \end{bmatrix}$$

$$H_{45} = \left[C_{yz}\right]_{k,6} - \left[R_{21}\ R_{22}\ R_{23}\right]_k \cdot \begin{bmatrix} Z_{k-1}\cdot B'_{13} + \dfrac{Z_{k-1}^2}{2}\cdot D'_{13} \\ Z_{k-1}\cdot B'_{23} + \dfrac{Z_{k-1}^2}{2}\cdot D'_{23} \\ Z_{k-1}\cdot B'_{33} + \dfrac{Z_{k-1}^2}{2}\cdot D'_{33} \end{bmatrix}$$

$$H_{54} = \left[C_{xz}\right]_{k,6} - \left[R_{11}\ R_{12}\ R_{13}\right]_k \cdot \begin{bmatrix} Z_{k-1}\cdot B'_{13} + \dfrac{Z_{k-1}^2}{2}\cdot D'_{13} \\ Z_{k-1}\cdot B'_{23} + \dfrac{Z_{k-1}^2}{2}\cdot D'_{23} \\ Z_{k-1}\cdot B'_{33} + \dfrac{Z_{k-1}^2}{2}\cdot D'_{33} \end{bmatrix}$$

$$H_{55} = \left[C_{xz}\right]_{k,4} - \left[R_{11}\ R_{12}\ R_{13}\right]_k \cdot \begin{bmatrix} Z_{k-1}\cdot B'_{11} + \dfrac{Z_{k-1}^2}{2}\cdot D'_{11} \\ Z_{k-1}\cdot B'_{21} + \dfrac{Z_{k-1}^2}{2}\cdot D'_{21} \\ Z_{k-1}\cdot B'_{31} + \dfrac{Z_{k-1}^2}{2}\cdot D'_{31} \end{bmatrix}$$

$[C_{YZ}]_k$, $[C_{XZ}]_k$: Local coefficients of each individual layer k

$$\left[C_{yz}\right]_k = \left[C_{yz}\right]_{k-1} + \left(\left[R_2\right]_k - \left[R_2\right]_{k-1}\right)\cdot\left[M\right]_k$$

$$\left[C_{xz}\right]_k = \left[C_{xz}\right]_{k-1} + \left(\left[R_1\right]_k - \left[R_1\right]_{k-1}\right)\cdot\left[M\right]_k$$

where:

$$\left[M\right]_k = \left[\left(Z_{k-1}\cdot A' + \frac{Z_{k-1}^2}{2}\cdot C'\right)\left(Z_{k-1}\cdot B' + \frac{Z_{k-1}^2}{2}\cdot D'\right)\right]$$

$[C_{xz}]_k$ and $[C_{yz}]_k$: Shear distribution coefficient (matrix [1x6]) for the layer k

$[C_{xz}]_{k-1}$ and $[C_{yz}]_{k-1}$: Shear distribution coefficient (matrix [1x6]) for the layer k-1

$[R_1]_k$ and $[R_1]_{k-1}$: First line of matrix of rigidity [R] for layers k and k-1 as defined in [3.2.4]

$[R_2]_k$ and $[R_2]_{k-1}$: Second line of matrix of rigidity [R] for layers k and k-1 as defined in [3.2.4]

A′, B′, C′ and D′ : Obtained by reversing matrix [ABD] defined in [4.1.1] according the following formula:

$$\begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} = \begin{bmatrix} A_{ij} & B_{ij} \\ B_{ij} & D_{ij} \end{bmatrix}^{-1}$$

Note 1: For the first layer (k=1), coefficients are as follows:

$$\left[C_{yz}\right]_{k-1} = \left[C_{xz}\right]_{k-1} = 0$$

$$\left[R_2\right]_{k-1} = \left[R_1\right]_{k-1} = 0$$

4.3.3 The interlaminar stresses (expressed in the local orthotropic axis of the individual layer) between two layers k and k-1 are calculated by the following formula:

$$\begin{bmatrix} \tau_{23} \\ \tau_{13} \end{bmatrix}_k = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \cdot \begin{bmatrix} \tau_{yz} \\ \tau_{xz} \end{bmatrix}_k$$

Transfer matrix T is defined in [3.2.4].

Note 1: τ_{23} and τ_{13} are identical to the interlaminar shear stresses τ_{IL1} and τ_{IL2} located between layers k and k_{-1}.

5 Global laminate parameters for specific calculation

5.1 Global rigidity

5.1.1 General

Global tensile and shear rigidity expressed in the present Article are used for specific calculations such as stiffeners or midship section modulus.

Main directions X and Y of the global laminate are defined in [3.2.3].

5.1.2 Tensile and shear rigidity

The global tensile rigidity of a laminate can be expressed in its two main directions X and Y by the following formulae:

- In X direction:

$$E_x = \frac{1}{(A'_{11} \times th)}$$

- In Y direction:

$$E_y = \frac{1}{(A'_{22} \times th)}$$

The global shear rigidity of a laminate can be expressed in its plane by the following formula:

$$G_{xy} = \frac{1}{(A'_{33} \times th)}$$

The distances V_x and V_y between the global neutral axis of a laminate and the edge of the first individual layer, in mm, are defined in its two main directions X and Y by the following formulae:

$$V_x = \left(\sum E_{xi}\cdot th_i \cdot Z_i\right) / \left(\sum E_{xi}\cdot th_i\right)$$

$$V_y = \left(\sum E_{yi}\cdot th_i \cdot Z_i\right) / \left(\sum E_{yi}\cdot th_i\right)$$

with:

A'_{11}, A'_{22}, A'_{33} : defined in [4.3.1]

th : Laminate's total thickness, in mm

E_{xi} : Tensile rigidity of each individual layer in X direction in global reference axis system, in N/mm², equal to $1/S_{XX}$ as defined in [3.2.4]

E_{yi} : Tensile rigidity of each individual layer in Y direction in global reference axis system, in N/mm², equal to $1/S_{YY}$ as defined in [3.2.4]

th_i : Thickness of each individual layer, in mm, as defined in Ch 12, Sec 3, [2.2.1]

Zi : Distance between edge of the laminate and mid-thickness of layer i, in mm.

5.1.3 Bending rigidity

The global bending rigidity of a laminate can be expressed, in N.mm²/mm, in its two main directions X and Y by the following formulae.

- In X direction:

$$[EI]_x = \frac{1}{D'_{11}}$$

- In Y direction:

$$[EI]_y = \frac{1}{D'_{22}}$$

with:

D'_{11}, D'_{22} : defined in [4.3.1].

5.2 Laminate's weight

5.2.1 The total laminate weight per square meter of a laminate, in kg/m², is equal to:

$$W = \sum_{1}^{n} \frac{P_{fi}}{M_{fi}}$$

with:

P_{fi} : Mass per square meter of dry reinforcement fabric of each individual layer, defined in Ch 12, Sec 3, [1.1.3] of the present chapter.

M_{fi} : Content in mass of fibre for each individual layer, defined in Ch 12, Sec 3, [2.1.1] of the present chapter.

6 Stiffener's analysis

6.1 General

6.1.1 As a general rule, a composite stiffener is made of an attached plating, a web and a flange. All these elements are to be considered as independent laminate characterized by their own:

- global tensile rigidity E_{xi} and shear rigidity G_{XYi}, defined in [5.1], in the longitudinal direction of the stiffener
- thickness th_i,
- global neutral axis position V_{xi}, as defined in [5.1],
- width for associated platings and flanges, height for webs.

Note 1: Where associated plating is a sandwich structure, its rigidity is to be calculated without taking into account the sandwich core.

6.2 Calculations' parameters

6.2.1 Neutral axis

The neutral axis V, in mm, of a stiffener is to be calculated by the following formula:

$$V = \sum E_{xi} \cdot S_i \cdot Z_{xi} / \sum E_{xi} \cdot S_i$$

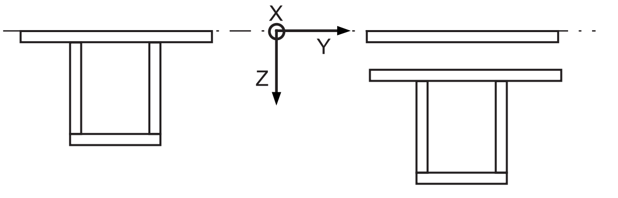
where:

E_{xi} : Global tensile rigidity of each element (flange, web and associated plating) of a stiffener, in N/mm², in the longitudinal direction of the stiffener

S_i : Section of each element of the stiffener, in mm²

Z_{xi} : Distance, in mm, between outer-face of associated plating and neutral axis of each element of the stiffener (see Fig 4).

Figure 4 : Stiffener model



6.2.2 Bending rigidity and global inertia of stiffener

The rigidity, in N.mm², of the stiffener is given by the following formula:

$$[EI] = \sum E_{xi} \cdot (I_i + S_i \cdot d_i^2)$$

where:

d_i : Distance, in mm, between neutral axis of each element of a stiffener and the stiffener's one.

$$d_i = Z_{xi} - V$$

S_i : Section, in mm², as defined in [6.2.1]

I_i : Proper inertia of each element of the stiffener, in mm⁴, in relation to the longitudinal axis of the stiffener X.

6.2.3 Strains

The strain of each element of a stiffener submitted to a bending moment M is estimated in the longitudinal direction of the stiffener, in percent by:

$$\varepsilon_{xi} = \frac{M \cdot d_i}{[EI]} \cdot 10^8$$

where:

E_{xi} : As defined in [6.2.1]

d_i : As defined in [6.2.2]

[EI] : As defined in [6.2.2]

M : Bending moment, in kN.m, applied to the stiffener as defined in Ch 9, Sec 4, [2.2].

Note 1: Attention is to be paid to the sign of the bending moment which condition the type of stresses (tensile or compressive) in the attached the plating and the flange of the stiffener.

6.2.4 Strains and stresses in attached plating and flange induced by moments

As a general rule, the stains and tensile or compressive stresses induced by flexural moment are only determined in the attached plating and the flange of the stiffener. These local strains, in %, and stresses, in MPa, in each layer of each element of a stiffener are given by the following formulae:

$\varepsilon_1 = (\cos\theta)^2\,\varepsilon_{xi}$

$\varepsilon_2 = (\sin\theta)^2\,\varepsilon_{xi}$

$\gamma_{12} = -2\sin\theta\cos\theta\,\varepsilon_{xi}$

and

$$\sigma_1 = (\overline{R_{11}}\cdot\varepsilon_1 + \overline{R_{12}}\cdot\varepsilon_2)/100$$

$$\sigma_2 = (\overline{R_{21}}\cdot\varepsilon_1 + \overline{R_{22}}\cdot\varepsilon_2)/100$$

$$\tau_{12} = (\overline{R_{33}}\cdot\gamma_{12})/100$$

$\overline{R}_{11}$, $\overline{R}_{12}$, $\overline{R}_{21}$ and $\overline{R}_{22}$ are defined in Ch 12, Sec 3, [4.1.2].

θ is the orientation of an individual layer in relation to the logitudinal axis X of the stiffener as shown in Fig 4.

6.2.5 Strains and stresses in web induced by shear force

As a general rule, shear strain, in percent, is only calculated in the web of the stiffener and is given by the following formula:

$$\gamma_{xy} = \frac{T}{\left(\sum S_i\cdot G_{XYi}\right)10^5}$$

where:

T : Shear force applied to the stiffener as defined in Ch 9, Sec 4, [2.2], in kN

S_i : Shear area of each part of the webs contributing to the stiffener shear strength, in mm²

G_{XYi} : Shear rigidity of each part of the webs contributing to shear strength in global axis, in N/mm².

Strains and stresses of the individual layers of the web contributing to shear strength of the stiffener are given by the following formulae:

$$\varepsilon_1 = \sin\theta\cdot\cos\theta\cdot\gamma_{xy}$$

$$\varepsilon_2 = -\sin\theta\cdot\cos\theta\cdot\gamma_{xy}$$

$$\gamma_{12} = ((\cos\theta)^2-(\sin\theta)^2)\cdot\gamma_{xy}$$

and

$$\sigma_1 = (\overline{R_{11}}\cdot\varepsilon_1 + \overline{R_{12}}\cdot\varepsilon_2)/100$$

$$\sigma_2 = (\overline{R_{21}}\cdot\varepsilon_1 + \overline{R_{22}}\cdot\varepsilon_2)/100$$

$$\tau_{12} = (\overline{R_{33}}\cdot\gamma_{12})/100$$

Note 1: ε_1, ε_2 and σ_1, σ_2 are only calculated under the shear strain of web.

7 Transverse ship section analysis

7.1 General

7.1.1 The main characteristics of a transverse ship section can be estimated with the same approach than defined in [6] for a stiffener.

In this case, all typical laminates constituing the transverse section is to be considered as independent laminate characterized as defined in [6.1.1].

For shear strength calculation, vertical section or vertical projection of all parts of transverse ship section have to be taken into account.