SECTION 3 INDIVIDUAL LAYER

1 General

1.1 Application

1.1.1 General

The present Section deals with the methodology to estimate the five elastic coefficients and the six breaking stresses requested to define the breaking strength of an individual layer.

The in-plane elastic coefficients to take into account are:

- a longitudinal Young's modulus
- a transverse Young's modulus
- two Poisson's coefficients
- a shear modulus.

The theoretical breaking stresses to estimate are:

- in-plane longitudinal tensile and compressive breaking stresses
- in-plane transverse tensile and compressive breaking stresses
- in-plane shear breaking stress
- interlaminar shear breaking stress.

Two geometric parameters are also to be defined:

- the individual layer's thickness
- the individual layer's density (or weight per surface unit).

1.1.2 Methodology

Coefficients, breaking stresses and geometric parameters defined in [1.1.1] are based on the Society experience and take into account:

- the type of raw material as defined in Ch 12, Sec 2
- the fibre/resin mix ratio
- the laminating and curing processes used for the composite work
- the type of stress in relation to the reinforcement's orientation.

Whatever the type of reinforcement making up the individual layer, the first step of the methodology consists in estimating the elastic coefficients of a unidirectional (UD) fabric having same raw materials and content of fibre than the considered individual layer to calculate.

Where unusual individual layers are used (due to specific raw materials or laminating process), the Society may request mechanical tests to be performed in order to evaluate elastic coefficients and/or breaking stresses and compare them to the present Rule theoretical approach.

1.1.3 Symbols

Symbols used in the formulae of the present Section are:

C_{eq} : Woven balance coefficient for woven rovings. See [3.2.2].

e : Individual layer thickness, in mm

$E_{f0°}$: Longitudinal Young's modulus of fibre, in MPa (see Note 1)

$E_{f90°}$: Transversal Young's modulus of fibre, in MPa (see Note 1)

E_m : Young's modulus of resin, in MPa (see Note 1)

G_f : Shear modulus of fibre, in MPa (see Note 1)

G_m : Shear modulus of resin, in MPa (see Note 1)

m : Total mass per square meter of individual layer, in gr/m^2

M_f : Content in mass of fibre in an individual layer, in %

M_m : Content in mass of resin in an individual layer, in %

P_f : Total mass per square meter of dry reinforcement fabric, in g/m^2

V_f : Content in volume of fibre in an individual layer, in %

V_m : Content in volume of resin in an individual layer, in %

ν_f : Poisson's coefficient of fibre

ν_m : Poisson's coefficient of resin

ρ : Density of an individual layer

ρ_f : Density of fibre

ρ_m : Density of resin.

Note 1: Minimum mechanical characteristics are given, for information only, in Ch 12, Sec 2, Tab 1and Ch 12, Sec 2, Tab 2.

2 Geometrical and physical properties of an individual layer

2.1 fibre/resin mix ratio

2.1.1 The fibre/resin mix ratio of an individual layer can be expressed in:

- mass or volume, and
- resin or reinforcement.

The contents in mass are obtained from the following formulae:

- M_f = fibres' mass (gr/m²)/individual layer's mass (gr/m²)
- M_m = resin's mass (gr/m²)/individual layer's mass (gr/m²)
- V_f and V_m are defined in [1.1.3].

$$V_f = \frac{(M_f/\rho_f)}{(M_f/\rho_f) + ((1 - M_f)/\rho_m)}$$

$$V_m = 1 - V_f$$

$$M_f = \frac{(V_f \times \rho_f)}{(V_f \times \rho_f) + ((1 - V_f) \times \rho_m)}$$

$$M_m = 1 - M_f$$

with all parameters defined in [1.1.3].

2.1.2 The resin/fibre mix ratio is to be specified by the shipyard and depends on the laminating process.

For information only, the common ratio values are given in Tab 1.

2.2 Individual layer's thickness

2.2.1 The individual layer's thickness, in mm, can be expressed from the fibre's content, in mass or in volume, by the following formulae:

$$e = \frac{\left(P_f \cdot \left(\frac{1}{\rho_f} + \frac{1 - M_f}{M_f \cdot \rho_m}\right)\right)}{1000}$$

$$e = \frac{P_f/(V_f \cdot \rho_f)}{1000}$$

with all parameters defined in [1.1.3].

2.3 Mass, voluminal mass and density of an individual layer

2.3.1 The density of an individual layer is obtained by the following formula:

$$\rho = \rho_f \times V_f + \rho_m \times (1 - V_f)$$

with all parameters defined in [1.1.3].

3 Elastic coefficient of an individual layer

3.1 Unidirectionals

3.1.1 Reference axis

The reference axis system for a unidirectional is as follows (see Fig 1):

- 1 : axis parallel to the fibre's direction
- 2 : axis perpendicular to the fibre's direction
- 3 : axis normal to plane containing axis 1 and 2, leading to direct reference axis system.

The reference axis for an elementary fibre is defined as follows (see Fig 2):

- 0° : Longitudinal axis of the fibre
- 90° : Transverse axis of the fibre.

Figure 1 : Reference axis for unidirectionals

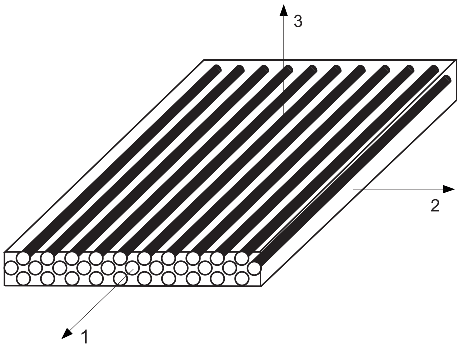
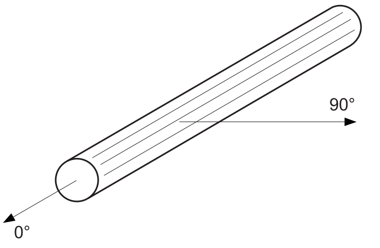


Figure 2 : Reference axis of an elementary fibre



3.1.2 Elastic coefficients

The elastic coefficients of an unidirectional are estimated by the following formulae, with all parameters defined in [1.1.3]:

- Longitudinal Young's modulus E_{UD1}, in MPa:

$$E_{UD1} = C_{UD1} \times (E_{f0°} \times V_f + E_m \times (1 - V_f))$$

- Transverse Young's moduli E_{UD2} and E_{UD3}, in MPa:

$$E_{UD2} = E_{UD3} = C_{UD2} \times \left(\left(\frac{E_m}{1 - \nu_m^2}\right) \times \frac{1 + 0,85 \cdot V_f^2}{(1 - V_f)^{1,25} + \frac{E_m}{E_{f90°}} \times \frac{V_f}{1 - \nu_m^2}}\right)$$

- Shear moduli, in MPa:

$$G_{UD12} = G_{UD13} = C_{UD12} \cdot G_m \times \frac{1 + \eta \cdot V_f}{1 - \eta \cdot V_f}$$

$$\text{with} \quad \eta = \frac{\left(\frac{G_f}{G_m}\right) - 1}{\left(\frac{G_f}{G_m}\right) + 1}$$

$$G_{UD23} = 0,7 \cdot G_{UD12}$$

- Poisson's coefficients:

$$\nu_{UD13} = \nu_{UD12} = C_{UD\nu} \times (\nu_f \times V_f + \nu_m \times (1 - V_f))$$

$$\nu_{UD21} = \nu_{UD31} = \nu_{UD12} \times \frac{E_{UD2}}{E_{UD1}}$$

$$\nu_{UD23} = \nu_{UD32} = C_{UD\nu} \times (\nu_f' \times V_f + \nu_m \times (1 - V_f))$$

$$\text{with} \quad \nu_f' = \nu_f \cdot \frac{E_{f90°}}{E_{f0°}}$$

The coefficients C_{UD1}, C_{UD2}, C_{UD12} and $C_{UD\nu}$ are experimental coefficients taking into account the specific characteristics of fibre's type. They are given in Tab 2.

Table 1 : Resin / fibre mix ratios (in %)

Laminating Process		V_f	M_f		
			Glass	Carbon	Para-aramid
Hand Lay-up	Mat	from 15 to 20	from 25 to 35	-	-
	Roving	from 25 to 40	from 40 to 60	from 35 to 50	from 30 to 45
	Unidirectional	from 40 to 50	from 60 to 70	from 50 to 60	from 45 to 55
Infusion		45	60	55	50
Pre-pregs		from 55 to 60	from 60 to 70	from 65 to 70	from 60 to 65

Table 2 : Coefficients C_{UD1} , C_{UD2}, C_{UD12} and C_{UDv}

	E-glass	R-Glass	Carbon HS	Carbon IM	Carbon HM	Para-aramid
C_{UD1}	1	0,9	1	0,85	0,9	0,95
C_{UD2}	0,8	1,2	0,7	0,8	0,85	0,9
C_{UD12}	0,9	1,2	0,9	0,9	1	0,55
C_{UDv}	0,9	0,9	0,8	0,75	0,7	0,9

3.2 Woven Rovings

3.2.1 Reference axis

The reference axis defined for woven rovings are the same than for unidirectionals with the following denomination:

- 1 : axis parallel to warp direction
- 2 : axis parallel to weft direction
- 3 : axis normal to plane containing axis 1 and 2, leading to direct reference axis system.

3.2.2 Woven balance coefficient C_{eq}

The woven balance coefficient is equal to the mass ratio of dry reinforcement in warp direction to the total dry reinforcement of woven fabric.

3.2.3 Elastic coefficients

The elastic coefficients of woven rovings as individual layers are estimated by the following formulae:

- Young's modulus in warp direction E_{T1}, in MPa:

$$E_{T1} = \frac{1}{e} \cdot \left(A_{11} - \frac{A_{12}^2}{A_{22}}\right)$$

- Young's modulus in weft direction E_{T2}, in MPa:

$$E_{T2} = \frac{1}{e} \cdot \left(A_{22} - \frac{A_{12}^2}{A_{11}}\right)$$

- Out-of-plane Young's modulus E_{T3}, in MPa:

$$E_{T3} = E_{UD3}$$

- Shear moduli G_{12}, G_{23} and G_{13}, in MPa:

$$G_{T12} = \frac{1}{e} \cdot A_{33} \qquad \text{and} \qquad G_{T23} = G_{T13} = 0,9 \cdot G_{T12}$$

- Poisson's coefficients:

$$\nu_{T12} = \frac{A_{12}}{A_{22}}$$

$$\nu_{T21} = \nu_{T12} \cdot \frac{E_{T2}}{E_{T1}}$$

$$\nu_{T32} = \nu_{T31} = (\nu_{UD32} + \nu_{UD31})/2$$

$$\nu_{T13} = (\nu_{UD23} + \nu_{UD13})/2$$

where:

$$A_{11} = e \cdot (C_{eq} \cdot Q_{11} + (1 - C_{eq}) \cdot Q_{22})$$

$$A_{22} = e \cdot (C_{eq} \cdot Q_{22} + (1 - C_{eq}) \cdot Q_{11})$$

$$A_{12} = e \cdot Q_{12}$$

$$A_{33} = e \cdot Q_{33}$$

with:

$$Q_{11} = E_{UD1}/(1 - (\nu_{UD12} \cdot \nu_{UD21}))$$

$$Q_{22} = E_{UD2}/(1 - (\nu_{UD12} \cdot \nu_{UD21}))$$

$$Q_{12} = (\nu_{UD21} \cdot E_{UD1})/(1 - (\nu_{UD12} \cdot \nu_{UD21}))$$

$$Q_{33} = G_{UD12}$$

Note 1: Parameters with suffix UD are defined in [3.1].

3.3 Chopped Strand Mats

3.3.1 General

A chopped strand mat is made of cut fibres, random arranged and supposed uniformly distributed in space. It is assumed as isotropic material.

3.3.2 Elastic coefficients

Isotropic assumption makes possible to define only three elastic coefficients obtained by the following formulae:

- Young's moduli, in MPa:

$$E_{mat1} = E_{mat2} = \frac{3}{8} \cdot E_{UD1} + \frac{5}{8} \cdot E_{UD2}$$

$$E_{mat3} = E_{UD3}$$

- Poisson's coefficient is as all isotropic materials:

$$\nu_{mat12} = \nu_{mat21} = \nu_{mat32} = \nu_{mat13} = 0,3$$

- Shear moduli, in MPa:

$$G_{mat12} = E_{mat1}/(2 \cdot (1 + \nu_{mat21}))$$

$$G_{mat23} = G_{mat31} = 0,7 \cdot G_{UD12}$$

Where parameter with suffix UD are defined in [3.1].

Note 1: Parameters with suffix UD are defined in [3.1].

3.4 Combined fabrics

3.4.1 Combined fabrics, as defined in Ch 12, Sec 2, [3.3.6], are to be considered as a series of individual layers such as unidirectionals, woven rovings or chopped strand mats. Each component is analysed as defined in [3.1], [3.2] or [3.3] accordingly to type of reinforcement fabric.

4 Rigidity and flexibility of an individual layer

4.1 In-plane characteristics

4.1.1 General

Rigidity and flexibility of an individual layer need to be determined to perform the mechanical calculations of a laminate, made of several individual layers, as defined in Ch 12, Sec 4.

4.1.2 Rigidity

The rigidity $\overline{R}$, defined in the individual layer coordinate system, is as follows:

$$[\sigma]_{1,2} = [\overline{R}] \cdot [\varepsilon]_{1,2}$$

where [σ] is the matrix of in-plane stresses, [ε] is the matrix of in-plane strains and $[\overline{R}]$ local matrix of rigidity.

Or under matrix notation:

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} = \begin{bmatrix} \overline{R}_{11} & \overline{R}_{12} & 0 \\ \overline{R}_{21} & \overline{R}_{22} & 0 \\ 0 & 0 & \overline{R}_{33} \end{bmatrix} \cdot \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix} \quad \text{with} \quad \overline{R} = \begin{vmatrix} \overline{R}_{11} & \overline{R}_{12} & 0 \\ \overline{R}_{21} & \overline{R}_{22} & 0 \\ 0 & 0 & \overline{R}_{33} \end{vmatrix}$$

Elements of matrix of rigidity are specific to types of reinforcement and define in Tab 3.

4.1.3 Flexibility

The flexibility $\overline{S}$, defined in the individual layer coordinate system, is as follow:

$$[\varepsilon]_{1,2} = [\overline{S}] \cdot [\sigma]_{1,2}$$

where σ and ε are defined in [4.1.2] and $[\overline{S}]$ local individual layer flexibility matrix.

Or under matrix notation:

$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix} = \begin{bmatrix} \overline{S}_{11} & \overline{S}_{12} & 0 \\ \overline{S}_{21} & \overline{S}_{22} & 0 \\ 0 & 0 & \overline{S}_{33} \end{bmatrix} \cdot \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} \qquad \overline{S} = \begin{vmatrix} \overline{S}_{11} & \overline{S}_{12} & 0 \\ \overline{S}_{21} & \overline{S}_{22} & 0 \\ 0 & 0 & \overline{S}_{33} \end{vmatrix}$$

Elements of matrix of flexibility are specific to types of reinforcement and are defined in Tab 4.

5 Individual layer breaking stress criteria

5.1 General

5.1.1 The individual layer breaking criteria are in relation with the elastic coefficients defined in [3], taking into account the type and the direction of the stresses.

5.2 Definitions

5.2.1 Breaking stresses

The individual layer breaking criteria are defined, in MPa, as the maximum breaking stresses of the individual layer in its local coordinate system, and are obtained by the following formulae:

$\sigma_{br1} = \varepsilon_{br1} \cdot E_1 \cdot Coef_{res}$

$\sigma_{br2} = \varepsilon_{br2} \cdot E_2 \cdot Coef_{res}$

$\tau_{br12} = \gamma_{br12} \cdot G_{12} \cdot Coef_{res}$

$\tau_{brIL1} = \gamma_{brIL23} \cdot G_{23} \cdot Coef_{res}$

$\tau_{brIL2} = \gamma_{brIL13} \cdot G_{13} \cdot Coef_{res}$

where :

E_1, E_2, G_{12}, G_{13}, G_{23} : Elastic coefficients defined in [3], in N/mm², for the individual layer considered according to type of reinforcement (unidirectionals, woven rovings, mats)

ε_{br1} : Theoretical breaking strain, in %, in traction or compressive, of an individual layer in the direction 1 of its local coordinate system

ε_{br2} : Theoretical breaking strain, in %, in traction or compressive, of an individual layer in the direction 2 of its local coordinate system

γ_{br12} : Theoretical in-plane breaking shear strain, in %, of an individual layer

γ_{brIL} : Theoretical interlaminar breaking shear strain, in %, of an individual layer

$Coef_{res}$: Coefficient taking into account the adhesive quality of the resin system.

All breaking strains and coefficients are given in Tab 5 or Tab 6, as applicable.

Table 3 : Elements of matrix of rigidity

	For Unidirectionals	For Woven Rovings	For Mats	Core material
$\overline{R}_{11}$	$E_{UD1}/(1 - \nu_{UD12} \cdot \nu_{UD21})$	$E_{T1}/(1 - \nu_{T12} \cdot \nu_{T21})$	$E_{mat}/(1 - \nu_{mat}^2)$	$E_1/(1 - \nu_{12} \cdot \nu_{21})$
$\overline{R}_{22}$	$E_{UD2}/(1 - \nu_{UD12} \cdot \nu_{UD21})$	$E_{T2}/(1 - \nu_{T12} \cdot \nu_{T21})$	$E_{mat}/(1 - \nu_{mat}^2)$	$E_2/(1 - \nu_{12} \cdot \nu_{21})$
$\overline{R}_{12}$	$\nu_{UD21} \cdot E_{UD1}/(1 - \nu_{UD12} \cdot \nu_{UD21})$	$\nu_{T21} \cdot E_{T1}/(1 - \nu_{T12} \cdot \nu_{T21})$	$\nu_{mat} \cdot E_{mat}/(1 - \nu_{mat}^2)$	$\nu_{21} \cdot E_1/(1 - \nu_{12} \cdot \nu_{21})$
$\overline{R}_{21}$	$\nu_{UD12} \cdot E_{UD2}/(1 - \nu_{UD12} \cdot \nu_{UD21})$	$\nu_{T12} \cdot E_{T2}/(1 - \nu_{T12} \cdot \nu_{T21})$	$\nu_{mat} \cdot E_{mat}/(1 - \nu_{mat}^2)$	$\nu_{12} \cdot E_2/(1 - \nu_{12} \cdot \nu_{21})$
$\overline{R}_{33}$	G_{UD12}	G_{T12}	G_{mat12}	G_{12}

Table 4 : Elements of matrix of flexibility

	For Unidirectionals	For Woven Rovings	For Mats	Core material
$\bar{S}_{11}$	$1/E_{UD1}$	$1/E_{T1}$	$1/E_{mat}$	$1/E_1$
$\bar{S}_{22}$	$1/E_{UD2}$	$1/E_{T2}$	$1/E_{mat}$	$1/E_2$
$\bar{S}_{12}$	$-\nu_{UD21}/E_{UD2}$	$-\nu_{T21}/E_{T2}$	$-\nu_{mat}/E_{mat}$	$-\nu_{21}/E_2$
$\bar{S}_{21}$	$-\nu_{UD12}/E_{UD1}$	$-\nu_{T12}/E_{T1}$	$-\nu_{mat}/E_{mat}$	$-\nu_{12}/E_1$
$\bar{S}_{33}$	$1/G_{UD12}$	$1/G_{T12}$	$1/G_{mat12}$	$1/G_{12}$

Table 5 : Theoretical breaking strains, in %

		Strains		Reinforcement fibres type					
				E Glass	R Glass	HS Carbon	IM Carbon	HM Carbon	Para-aramid
Reinforcement fabrics' type	Unidirectionals	Tensile	ε_{br1}	2.7	3.1	1.2	1.15	0.7	1.7
			ε_{br2}	0.42	0.35	0.85	0.65	0.4	0.65
		Compressive	ε_{br1}	1.8	1.8	0.85	0.65	0.45	0.35
			ε_{br2}	1.55	1.1	2.3	2.3	2.1	2
		Shear	γ_{br12}	1.8	1.5	1.6	1.7	1.8	2
			γ_{br13}, γ_{brIL2}	1.8	1.5	1.6	1.7	1.8	2
			γ_{br23}, γ_{brIL1}	2.5	1.8	1.9	1.85	1.8	2.9
	Woven	Tensile	ε_{br1}	1.8	2.3	1	0.8	0.45	1.4
			ε_{br2}	1.8	2.3	1	0.8	0.45	1.4
		Compressive	ε_{br1}	1.8	2.5	0.85	0.8	0.5	0.42
			ε_{br2}	1.8	2.5	0.85	0.8	0.5	0.42
		Shear	γ_{br12}	1.5	1.5	1.55	1.6	1.85	2.3
			γ_{br13}, γ_{brIL2}	1.8	1.8	1.55	1.6	1.85	2.9
			γ_{br23}, γ_{brIL1}	1.8	1.8	1.55	1.6	1.85	2.9
	Mats	Tensile	ε_{br1}	1.55	NA	NA	NA	NA	NA
			ε_{br2}	1.55	NA	NA	NA	NA	NA
		Compressive	ε_{br1}	1.55	NA	NA	NA	NA	NA
			ε_{br2}	1.55	NA	NA	NA	NA	NA
		Shear	γ_{br12}	2	NA	NA	NA	NA	NA
			γ_{br13}, γ_{brIL2}	2.15	NA	NA	NA	NA	NA
			γ_{br23}, γ_{brIL1}	2.15	NA	NA	NA	NA	NA

Table 6 : Coefficient $Coef_{res}$

Resin systems		
Polyester	Vinylester	Epoxy
0.8	0.9	1

5.2.2 As a general Rule, the mechanical characteristics of the individual layer are also depending on the laminating process. To simplify the breaking criteria, the influence of the process is taken into account by means of a dedicated safety coefficient defined in Ch 4, Sec 3, [5.4.1].

5.2.3 Other maximum breaking stresses of an individual layer may be taken into account, provided that representative mechanical tests are submitted to the Society.

The elastic coefficients and theoretical individual layer breaking criteria may be computed by the Bureau Veritas dedicated software, as defined in Ch 1, Sec 4.