

Chapter 6 Bending

6.0 PURE BENDING

Bending of beams is the next form of internal stress after uniform tensile/compressive stress and torsional shear stress. It is the most significant type of internal load transfer. What we want to derive in the present chapter is a relationship between the normal stresses in the beam cross-section and the applied bending moments. To achieve this we need to do three things:

- Determine some assumptions to simplify the analysis
- Determine the geometric relationships of the deformed beam
- Determine the load equilibrium conditions.

6.1 ASSUMPTIONS (SI&4th : 282-283; 5th : 282-283)

In order to be able to simplify the analysis it is best to define some assumptions about the beam, how it will deform.

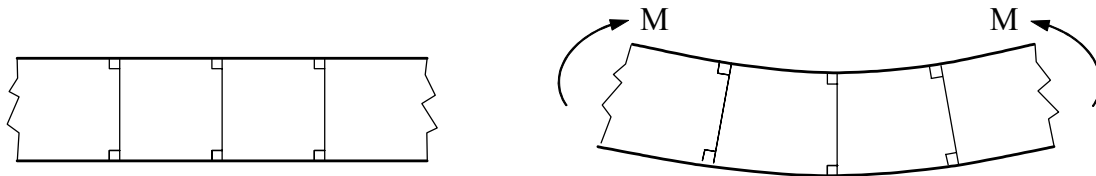


Fig. 6.1 Beam before and after a positive bending moment is applied

- Transverse planes before bending remain transverse after bending, Fig. 6.1, ie. no warping.
- Beam material is homogeneous and isotropic and obeys Hook's law with E the same in tension or compression.
- The beam is straight and has constant or slightly tapered cross section.
- Loads do not cause twisting or buckling. This is satisfied if the loading plane coincides with the section's symmetry axis.
- Applied load is pure bending moment (recall the B.M.D. in Assignment Q.4.3).

The definition for beams with applied positive and negative bending moments is as Fig. 6.2:

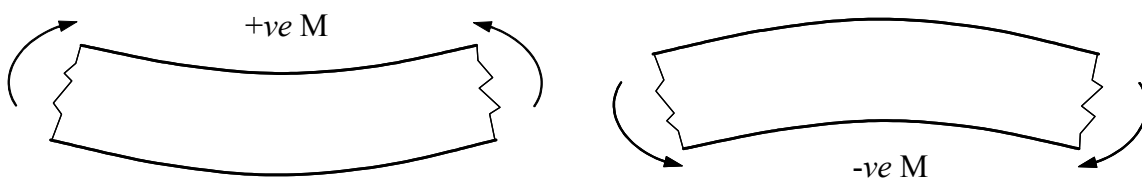


Fig. 6.2 Diagrams showing beam experiencing positive (left) and negative (right) bending moments

6.2 GEOMETRIC RELATIONSHIPS OF BEAM (SI&4th:283-285; 5th : 283-285)

To start the analysis we first must establish the nature of the deformation by observing how a beam deflects when a bending moment is applied to it.

When a beam is subjected to a pure bending moment, it will deform into a curved shape and this shape is the arc of a circle with a very large radius compared to the size of the beam. Let's now look at a segment of beam before and after the application of a positive bending moment:

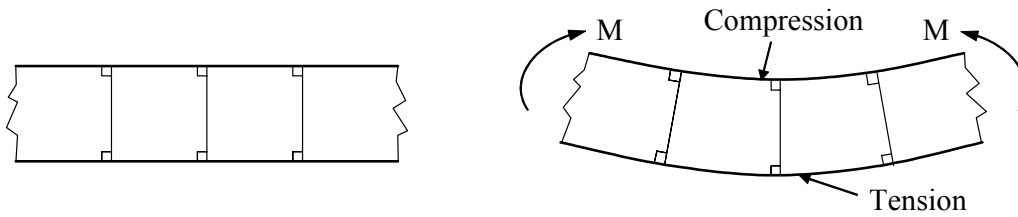


Fig. 6.3 Beam before and after a positive bending moment is applied indicating regions of +ve and -ve stresses

As you can see from Fig. 6.3, the fibres on the top surface are experiencing a compressive stresses, and those on the bottom a tensile stress. What this means is that at some point between these two surfaces, there must be a plane where the normal stresses and strains are **ZERO**. We call this plane the **Neutral Plane (N.P.)** or **Neutral Axis (N.A.)**.

Look now again at a small segment of beam before the application of a bending moment.

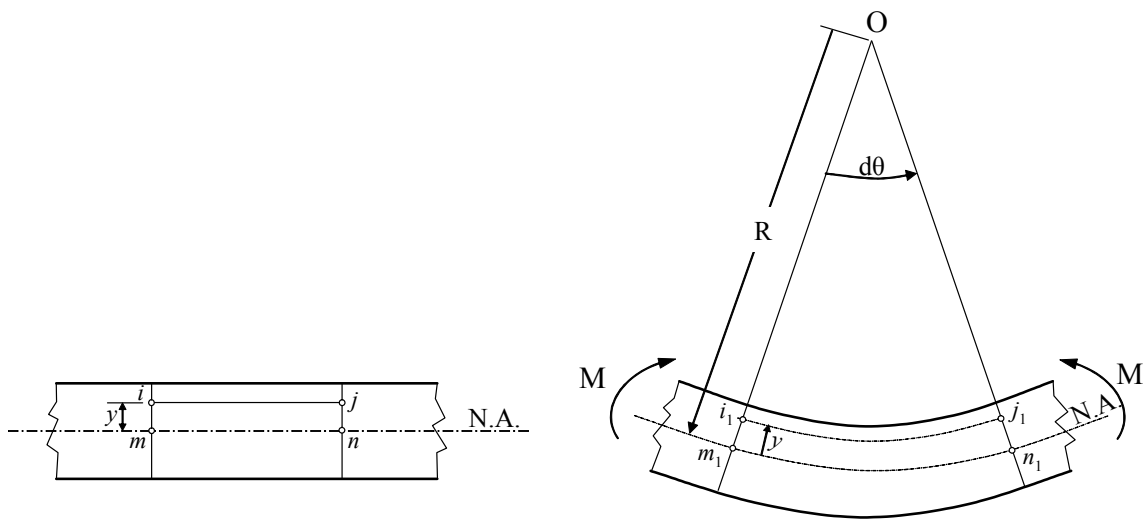


Fig. 6.4 Undeflected segment of beam **Fig. 6.5** Beam deformed by positive bending moment

Mark a longitudinal section with a distance y from the *Neutral Axis* as ij , and another section on the *Neutral Axis* as mn , as in Fig. 6.4. Initially, these sections are of equal length as they define the length between two transverse planes, i.e. $ij = mn$.

A pure positive bending moment M is then applied to the beam which makes the above section deform as Fig.6.5, where the applied bending moment causes the segment ij and mn to deform into concentric arcs i_1j_1 and m_1n_1 with an angle $d\theta$ between the segments i_1m_1 and j_1n_1 . The distance between these two arcs is still y .

Let R = Radius of curvature of the **Neutral Plane**, then the strain of segment i_1j_1 is defined as length i_1j_1 minus the original length ij over the original length ij , i.e.:

$$\epsilon_x = \frac{i_1j_1 - ij}{ij} \quad (6.1)$$

Now length mn and ij are defined as:

$$ij = mn = m_1n_1 = Rd\theta$$

and length i_1j_1 is defined as:

$$i_1j_1 = (R - y)d\theta$$

so the strain becomes:

$$\epsilon_x = \frac{(R - y)d\theta - Rd\theta}{Rd\theta} = -\frac{y}{R} \quad (6.2)$$

which indicates that the *strain is linearly varying with y*. And since stress is strain times Young's Modulus E then the stress can be defined by the following equation and is also linearly varying with y .

$$\sigma_x = E\varepsilon_x = -\frac{Ey}{R} \quad (6.3)$$

In effect, this equation has been derived using geometrical **compatibility** conditions; we need also to consider the equilibrium conditions of the beam cross-section.

6.3 APPLIED LOAD EQUILIBRIUM (SI&4th:286-294; 5th:286-294)

The stress equation (6.3) defined as above indicates that the normal stresses induced by the bending moment vary linearly through the depth of the beam. Look now at a drawing of the right hand end of the beam showing the normal stress distribution and applied bending moment as shown in Fig. 6.6.

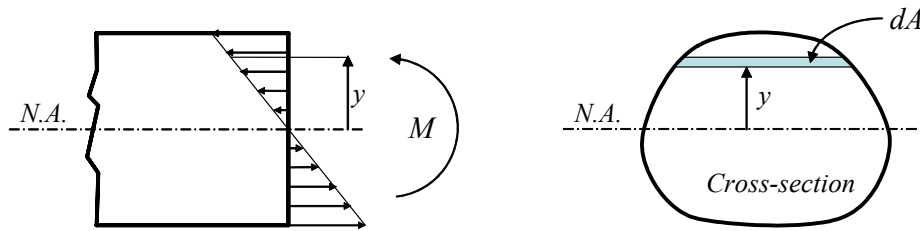


Fig. 6.6 Right hand end of beam showing applied bending moment M and normal stress distribution

Let $dF_x = \sigma_x dA$ be the component of force acting on the element of area dA . We now use equilibrium conditions on the stresses generated on the right hand side of the beam:

Force Equilibrium

$$\sum F_x = \int_A dF_x = \int_A \sigma_x dA = 0$$

Substituting the above equation (6.3) for stress gives:

$$\int_A \left(-\frac{Ey}{R} \right) dA = 0, \quad \text{or:} \quad \frac{E}{R} \int_A y dA = 0 \quad (6.4)$$

For materials with E constant the condition,

$$\int_A y dA = 0 \quad (6.5)$$

gives the origin for the “ y -axis” on the centroid of the section, i.e. the *location of the Neutral Plane*. We will look at what this means shortly

Moment Equilibrium

When equating moment equilibrium we have that the applied moment M must be equal to the moment generated internally by the normal stress caused by the external moment, such that:

$$\int_A dM = \int_A y \cdot (dF_x) = \int_A y \sigma_x dA = M \quad (6.6)$$

Substituting for the stress as in equation (6.3) gives :

$$M = \int_A y \sigma_x dA = \int_A -\frac{Ey^2}{R} dA = -\frac{E}{R} \int_A y^2 dA \quad (6.7)$$

We now define the term

$$I = \int_A y^2 dA \quad (6.8)$$

as the **Second Moment of Area** or **Moment of Inertia** of the beam about the *Neutral Axis* (which was actually defined by Eq. (6.5)). It is a measure of the **stiffness of the cross sectional shape** from a geometric point of view, without considering the material properties. Note that the unit of second moment of area is m^4 . Now substituting I into the equation for stress gives (from Eqs. (6.3) and (6.7))

$$\frac{\sigma_x}{y} = -\frac{M}{I} = -\frac{E}{R} \quad (6.9)$$

which is called **Engineer's Theory of Bending** (ETB). The standard form of writing and using this equation is:

$$\sigma_x = -\frac{My}{I} \quad (6.10)$$

So, if you know the applied bending moment, the location of the centroid and the section's second moment of area you can then find the stresses along the depth of the beam's section. Let's now define the location of the centroid and an easier equation to use than Eq. (6.5).

6.4 DETERMINATION OF NEUTRAL AXIS (SI&4th: 775-777; 5th: 775-777)

Look at an arbitrary symmetrical beam cross section, Fig. 6.7:

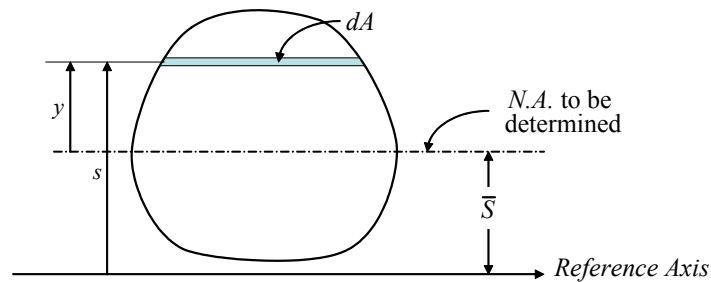


Fig. 6.7 Determination of Neutral Axis of an arbitrary cross section

Centroid of an Arbitrary Area In order to find the centroid it is often best to find it in reference to the bottom of the beam cross section. If we do this and because the centroid equation (6.5) is integrated about the neutral plane we firstly need to change the axis from y to s .

Changing beam axis from y (distance away from *Neutral Plane*) to s (distance away from bottom of beam) has that:

$$y = s - \bar{S}$$

Substituting into Eq. (6.5) gives:

$$\int_A (s - \bar{S}) dA = \int_A s dA - \int_A \bar{S} dA = 0$$

but since \bar{S} is the distance to the centroid, it is a constant and can be taken out of the integral equation, giving :

$$\bar{S} \int_A dA = \int_A s dA$$

and dividing by total area gives:

$$\bar{S} = \frac{\int_A s dA}{\int_A dA} \quad (6.11)$$

Composite Areas However as most engineering beams are made of several simpler regular shapes, for which you know the areas and the centroids of these areas, then this equation can be used in finite summation form instead of integral form as in (6.11).

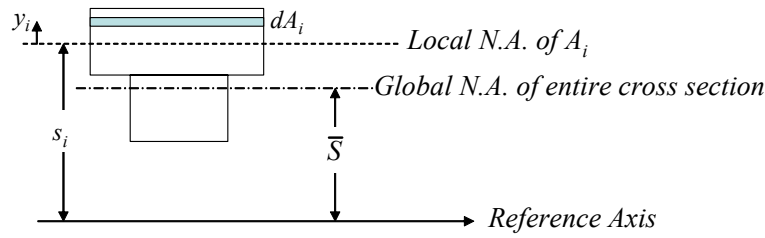


Fig. 6.8 Parallel axis method for composite area

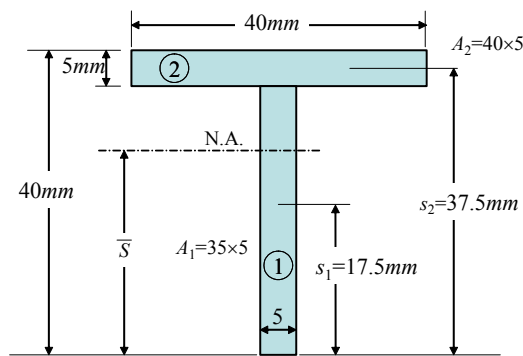
From Fig.6.8:

$$\int_A s dA = \sum \int_{A_i} (s_i + y_i) dA_i = \sum \left(\int_{A_i} s_i dA_i + \int_{A_i} y_i dA_i \right) = \sum \left(s_i \int_{A_i} dA_i + 0 \right) = \sum s_i A_i$$

$$\therefore \bar{S} = \frac{\sum s_i A_i}{\sum A_i} \quad (6.12)$$

where s_i represents the reference coordinate for the centroid of each part and A_i is its area. We can now use Eqs. (6.8), (6.10) and (6.12) to determine the stresses in a beam under an applied bending moment.

Example 6.1: Determine the second moment of area I for the following T-shaped cross section.



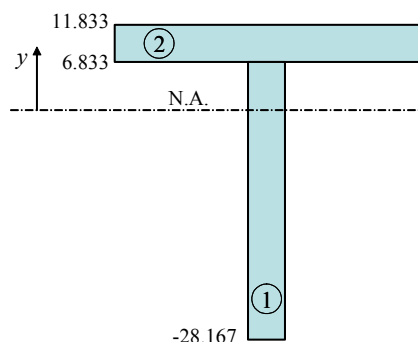
For all these type of problems it is best to follow the following methodology when solving them:

Step 1: Determine the location of the Neutral Plane

Using Eq. (6.12), do this by firstly subdividing the beam's cross section into regular geometric shapes. In this example the beam can be divided into two rectangular sections 1 and 2 as shown. Substituting the areas of each of the rectangles that make this shape as well as the distances to their respective centroids gives:

$$\bar{S} = \frac{s_1 A_1 + s_2 A_2}{A_1 + A_2} = \frac{17.5 \times (35 \times 5) + 37.5 \times (40 \times 5)}{(35 \times 5) + (40 \times 5)} = 28.167 \text{ mm}$$

Step 2: Coordinate transformation Once the position of the centroid (neutral plane) for the section has been found, re-draw the section with all *new coordinates* about the neutral axis:



Step 3: Determine the section's Second Moment of Area

$$I = \int_A y^2 dA = \int_{A_1} y^2 dA + \int_{A_2} y^2 dA$$

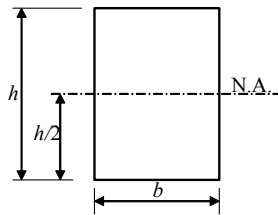
which for the section becomes
$$I = \int_{6.883}^{11.833} y^2 40 dy + \int_{-28.167}^{6.833} y^2 5 dy$$

$$I = 17837 + 37777 = 55613 \text{ mm}^4 \text{ or } I = 55614 \times 10^{-12} \text{ m}^4$$

(Note that, $1 \text{ mm} = 1 \times 10^{-3} \text{ m}$, $\therefore (1 \text{ mm})^4 = 1 \text{ mm}^4 = (1 \times 10^{-3} \text{ m})^4 = 1 \times 10^{-12} \text{ m}^4$)

Regular Shapes:

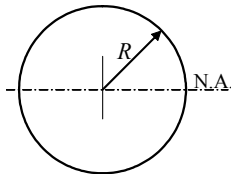
You can use the following standard solutions to the second moments of areas of regular shapes. These results can also be found inside the Front Cover of your textbook.



a) Rectangle Sections

$$I = \int_{-\frac{h}{2}}^{\frac{h}{2}} y^2 b dy = \frac{bh^3}{12}$$

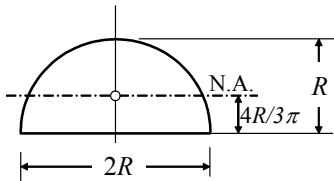
$$A = bh$$



b) Circular Sections

$$I = \frac{\pi R^4}{4} = \frac{\pi D^4}{64}$$

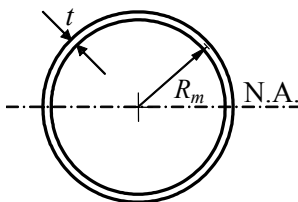
$$A = \pi R^2$$



c) Semi-Circular Sections

$$I = 0.110 R^4$$

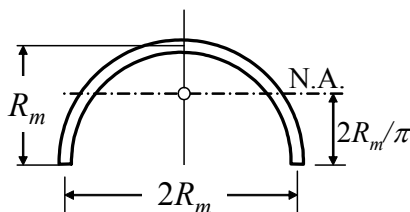
$$A = \pi R^2 / 2$$



d) Thin Tubular Sections for $t < R/10$

$$I = \pi R_m^3 t$$

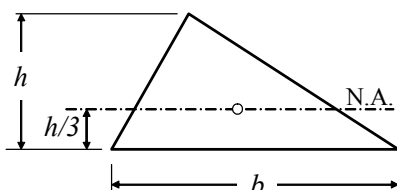
$$A = 2\pi R_m t$$



e) Half-Thin Tubular Sections for $t < R/10$

$$I = 0.095 \pi R_m^3 t$$

$$A = \pi R_m t$$



f) Triangular Sections

$$I = \frac{bh^3}{36}$$

$$A = \frac{1}{2} bh$$

Having said that you can use the above standard solutions to determine the second moment of area, it is fine if the centroid of the section you are analyzing lies on the neutral plane of the whole cross section. As you have seen from the examples just done (Example 6.1), the individual centroids of each of the shapes that the cross section was divided in, did not lie at the centroid of the cross section. For this reason, and to save the trouble of always having to determine an integral to find the second moment of area, we need to look at the derivation of the *Parallel Axis theorem* for evaluating the second moment of area.

6.5 PARALLEL AXIS THEOREM (SI&4th: 778-781; 5th: 778-781)

If the second moment of an area is known about its centroid, we can use the parallel axis theorem to find I about a corresponding parallel axis, Fig. 6.8.

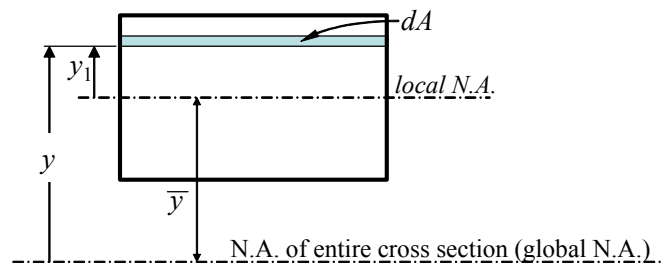


Fig. 6.8 Known section a distance \bar{y} above neutral axis of section

The equation for the second moment of area can be computed by the integral as:

$$I = \int_A y^2 dA \quad (6.13)$$

But the distance y about the *global Neutral Axis* of the entire cross section is given by:

$$y = y_1 + \bar{y}$$

Substituting this gives:

$$I = \int_A (y_1 + \bar{y})^2 dA = \int_A y_1^2 dA + 2\bar{y} \int_A y_1 dA + \bar{y}^2 \int_A dA$$

The first term is the local second moment of area, the second is zero because y_1 passes through the local centroid (Eq. (6.5)) and the third is the distance between the local and global centroids squared times the area of this shape, such that:

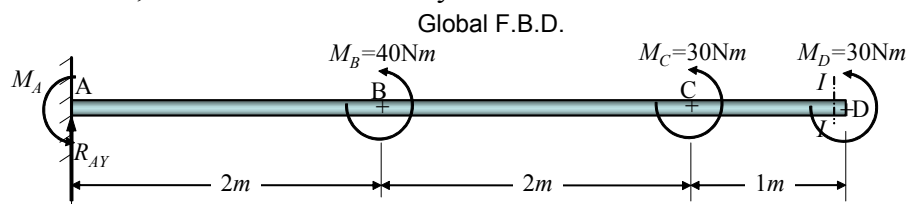
$$\underline{I_{N.P.} = I_{local} + \bar{y}^2 A} \quad (6.14)$$

For a cross-section consisted of a number of regular shapes, the total second moment of area can be computed as

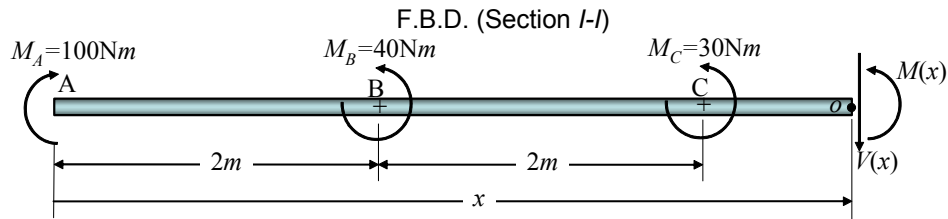
$$\underline{I_{N.P.} = \sum (I_{i\,local} + \bar{y}_i^2 A_i)} \quad (6.15)$$

here \bar{y}_i is the distance between the global and local centroid (N.A.) of area A_i . For the previous example if Eq. (6.15) was used instead of Eq. (6.8), you would have the following:

Example 6.2: Several concentrated external bending moments are applied over a cantilever beam as shown. The cross-sectional area is the same as in Example 6.1. Please use parallel axis theorem to determine I , and then further to analyze the maximum normal stresses in the beam.



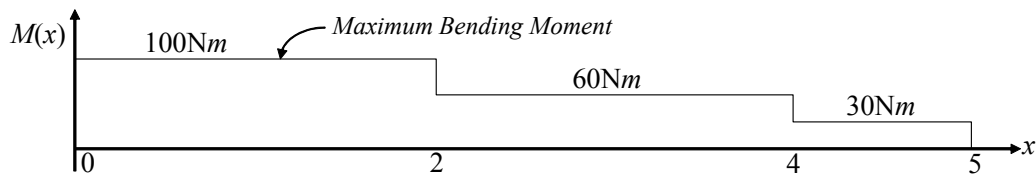
Step 1: Plot the bending moment diagram to determine the bending moment peaks



Equilibrium for FBD of beam cut just before RHS (*Section I-I*) and take moments about RHS:

$$\sum M_O = 0 = -100\langle x \rangle^0 + 40\langle x - 2 \rangle^0 + 30\langle x - 4 \rangle^0 + M(x) = 0$$

$$M(x) = 100\langle x \rangle^0 - 40\langle x - 2 \rangle^0 - 30\langle x - 4 \rangle^0$$

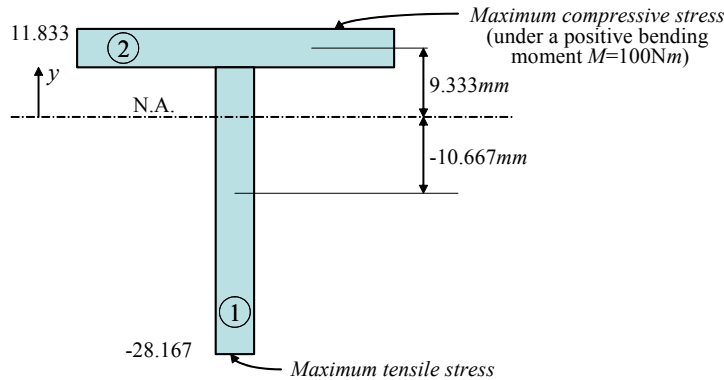


From the bending moment diagram, the bending moments at all cross sections are positive and the peak occurs between sections *A* and *B* with $M_{max} = 100\text{Nm}$.

Step 2: Determine the location of the Neutral Plane As in Example 6.1:

$$\bar{S} = \frac{s_1 A_1 + s_2 A_2}{A_1 + A_2} = \frac{17.5 \times (35 \times 5) + 37.5 \times (40 \times 5)}{(35 \times 5) + (40 \times 5)} = 28.167\text{mm}$$

Step 3: Determine the distances between the global and local N.A.



Step 4: Determine I using Parallel Axis Theorem, Eq. (6.15)

$$I = (I_1 + \bar{y}_1^2 A_1) + (I_2 + \bar{y}_2^2 A_2)$$

$$I = \left(\frac{5 \times 35^3}{12} + (-10.667)^2 \times (35 \times 5) \right) + \left(\frac{40 \times 5^3}{12} + (9.33)^2 \times (40 \times 5) \right) = 55614.6\text{mm}^4$$

or $I = 55614 \times 10^{-12} \text{m}^4$

Step 5 Compute the normal peak stresses

Substituting this value for I and the maximum bending moment into Eq. (6.10) gives:

$$\sigma_{max} = -\frac{M_{max} y}{I} = -\frac{100y}{55614 \times 10^{-12}}$$

and the maximum stresses occur at the furthest points (fibre) away from the neutral plane:

at $y = 11.833 \text{ mm}$, $\sigma_{y=0.011833} = -21.280 \text{ MPa}$, the compressive stress peak and at $y = -28.167 \text{ mm}$, $\sigma_{y=-0.028167} = 50.648 \text{ MPa}$, the tensile stress peak.

Remarks: For a non-symmetrical cross section (w.r.t. N.A.), it is worth noting that the maximum compressive or tensile stresses may correspond to other bending moment peaks than the maximum positive one. That is to say that it may be very necessary for an analyst to carefully examine several sections with both positive and negative moment peaks.

6.6 PRINCIPLE OF SUPERPOSITION (SI&4th: 133; 5th: 133)

Used to determine the stress or displacement at a point in a structural component which is subjected to several types of loads.

The *Principle of Superposition* states that the resultant stress or displacement at a point can be determined by finding the stress or displacement caused by each load *separately* on the structure, and then add their contributions.

6.7 COMBINED LOADINGS (Bending and Tension) (SI&4th: 409; 5th: 409)

We now need to look at how to analyze a beam with both a compressive or tensile load and a bending moment acting simultaneously. We call such analysis Combined Loadings. Before we do this though we need to define the Principle of Superposition

If a structural element is applied a tensile or compressive axial force and a bending moment simultaneously, as Fig. 6.10, we can determine the resultant stresses from the loading condition by using the principle of superposition as in the Table 6.1.

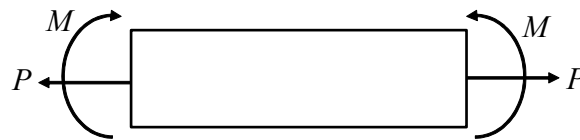


Fig. 6.10 Beam with applied bending moment M and tensile load P

Table 6.1 Superposition method to analyze the combined loads:

	Stresses Produced by Each Load Individually	Stress Distributions	Stresses
Tensile Load			$\sigma_P = P/A$
Bending Load			$\sigma_M = -My/I$
Both Tensile and Bending Loads			$\sigma_M = P/A - My/I$

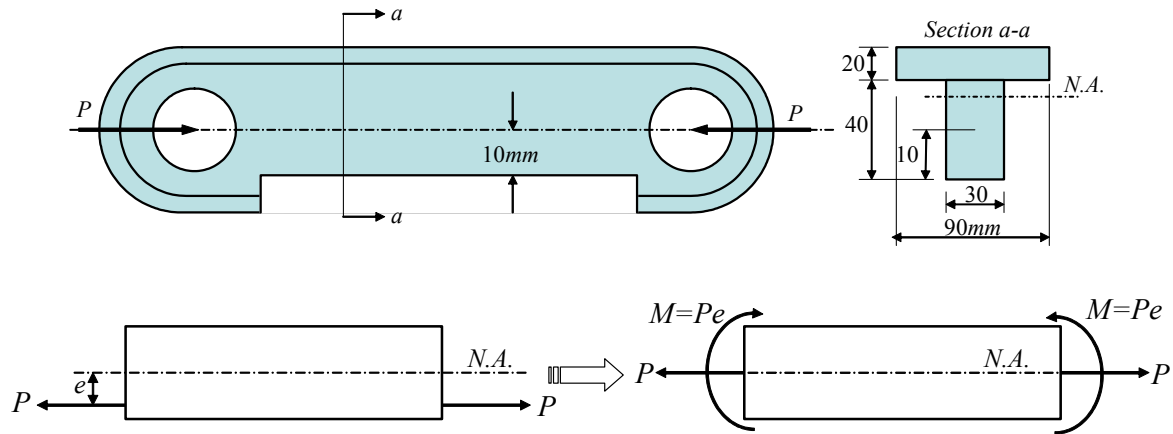


Fig. 6.11 Beam with an eccentric tensile load P

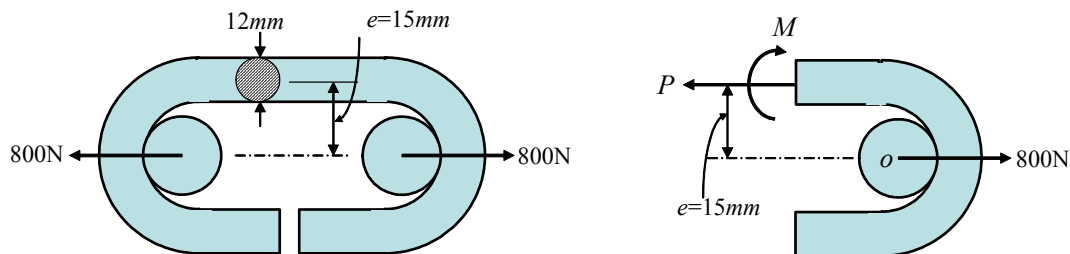
The combined loading cases can be created by applying an axial load on a beam a distance away from the beam's the Neutral Plane/Axis as illustrated in Fig. 6.11. This loading can be considered as a bending moment of magnitude equal to the applied force multiplied by its distance from the *Neutral Plane* (i.e. Pe) and the tensile or compressive load P .

Example 6.3 An open-link chain is obtained by bending low-carbon steel rods of 12-mm diameter into the shape shown in the figure of this example. Knowing that the chain carries a load of 800N, determine the maximum tensile and compressive stresses in the straight portion of a link.

Step 1: Determine the internal force and bending moment from section equilibrium of F.B.D.

$$+\rightarrow \sum F_x = 0 = -P + 800 = 0 \quad \therefore P = 800\text{N}$$

$$\curvearrowright \sum M = 0 = P \times e - M = 0 \quad \therefore M = Pe = 800 \times 0.015 = 12\text{Nm}$$



Step 2: Second moment of area

$$\text{Area of cross section: } A = \pi R^2 = 3.14159 \times 0.006^2 = 113.1 \times 10^{-6} \text{m}^2$$

$$\text{Second moment of area: } I = \frac{\pi R^4}{4} = \frac{\pi \times 0.006^4}{4} = 1.018 \times 10^{-9} \text{m}^4$$

(Note that if the cross section is not a regular shape, you may need to use the steps in Examples 6.1 and 6.2 to determine I)

Step 3: Superposition of P and M

The stress distribution due to the centric tensile force P is uniform as

$$\sigma_a = \frac{P}{A} = \frac{800}{113.1 \times 10^{-6}} = 7.07\text{MPa}$$

The stress distribution due to the bending moment M is linear with a maximum stress as

$$\sigma_m = -\frac{My}{I} = \pm \frac{12 \times 0.006}{1.018 \times 10^{-9}} = \pm 70.7\text{MPa}$$

It is worth noting that the maximum bending normal stress σ_m is about 10 times of the average tensile normal stress σ_a . This means that the eccentric load can cause a substantial change in stress of the structural element.

Superposing these two distributions, the largest tensile and compressive stresses in the section are found to be respectively

$$\sigma = \frac{P}{A} \pm \frac{My}{I} = 7.07 \pm 70.70$$

$$\underline{\sigma_{tensile} = 77.8 \text{ MPa}} \quad (\text{occurs in the inner fibre})$$

$$\underline{\sigma_{compressive} = -63.6 \text{ MPa}} \quad (\text{occurs in the outer fibre})$$

So far we have been looking at beams which are made up of a homogeneous material, this is not always the case. You can have beams where the material properties vary through the depth of the material. Such beams are called composite beams, and we are now going to look at how we can modify them so that they can be analysed by the equations and procedures that have already been derived.

6.8 COMPOSITE BEAMS (SI&4th: 315-321; 5th: 315-321)

Beams made of two or more different materials are referred to as *composite beams*. Such beams can be made of wood with straps of steel at the top and bottom surfaces, or concrete beams reinforced with steel. The reason for manufacturing such beams is to develop structures that can support loads more efficiently. Look at a beam made from 3 materials as shown in Fig. 6.12:

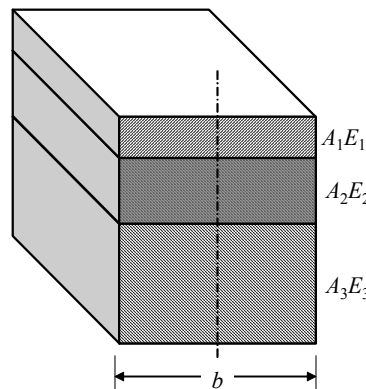


Fig. 6.12 Beam made from three materials of different Young's Modulus of Elasticity

Equivalent Second Moment of Area

Going back to the definition of the Neutral Axis equation, (6.4), if E is not a constant, Eq. (6.4) becomes:

$$\frac{1}{R} \int_A E y dA = 0 \quad (6.16)$$

but because the beam is made from 3 materials with different Young's Modulus, we can divide this into 3 integrals:

$$\int_{A_1} E_1 y dA_1 + \int_{A_2} E_2 y dA_2 + \int_{A_3} E_3 y dA_3 = 0 \quad (6.17)$$

For this beam, the term dA is given by:

$$dA_1 = dA_2 = dA_3 = b dy$$

If the cross section was made from the same material we would not have this problem. So let's pretend that the beam is made from material 1.

We can achieve this by dividing through by E_1 .

$$\text{Then} \quad E_1 \left(\int_{A_1} \left(\frac{E_1}{E_1} b \right) y dy + \int_{A_2} \left(\frac{E_2}{E_1} b \right) y dy + \int_{A_3} \left(\frac{E_3}{E_1} b \right) y dy \right) = 0 \quad (6.18)$$

We define the terms:

$$\underline{\underline{b_{3_{e1}} = \frac{E_1}{E_3} b; \quad b_{2_{e1}} = \frac{E_1}{E_2} b; \quad b_{1_{e1}} = \frac{E_1}{E_1} b}} \quad (6.19)$$

as the Equivalent Sections of Material 1, where E_1/E_1 , E_2/E_1 and E_3/E_1 are termed as material **transformation factors**. What has been done is to change each section to property number 1 by altering their widths. This beam made from an equivalent material 1 would look as Fig. 6.13:

By using the dimensions of the equivalent beam cross section, the position of the Neutral Axis can be calculated in the way as if the beam was made from one material property, using Eq. (6.12).

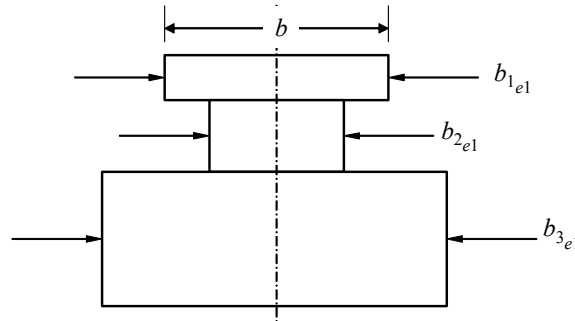


Fig. 6.13 Equivalent beam cross section

The second moment of area can now be found for this equivalent beam cross section. Using Eq. (6.13) you have that:

$$\underline{\underline{I_{eq1} = \int_{A_1} y^2 b_{1_{e1}} dy + \int_{A_2} y^2 b_{2_{e1}} dy + \int_{A_3} y^2 b_{3_{e1}} dy}} \quad (6.20)$$

Also you can compute the second moment of area by the parallel axis method as Eq. (6.15).

Actual Stresses

What we need to determine, however is the stresses through the composite beam, to do this look at the strains through the depth.

Although the cross section has been converted to an equivalent section of a homogeneous material, the strains in this section must be the same to those of the real material for this assumption to work. This means that, say looking at the bottom fibre of the beam cross section, the strains there must be the same in the real beam with material property 3 as in the equivalent beam of material property 1. Such that:

$$\varepsilon_{3_{bottomFibre}} = \varepsilon_{1_{bottomFibre}} = \frac{\sigma_{1_{eq}}}{E_1} = \frac{My_3}{I_{eq1} E_1} \quad (6.21)$$

But as we are interested in determining the stress in material 3 then:

$$\sigma_3 = E_3 \varepsilon_3 = E_3 \frac{My}{I_{eq1} E_1} = \left(\frac{E_3}{E_1} \right) \frac{My}{I_{eq1}} = \frac{My}{I_{eq3}} \quad (6.22)$$

which gives that:

$$\underline{\underline{I_{eq3} = I_{eq1} \left(\frac{E_1}{E_3} \right)}} \quad (6.23)$$

Similarly $\underline{\underline{I_{eq2} = I_{eq1} \left(\frac{E_1}{E_2} \right)}} \quad (6.24)$

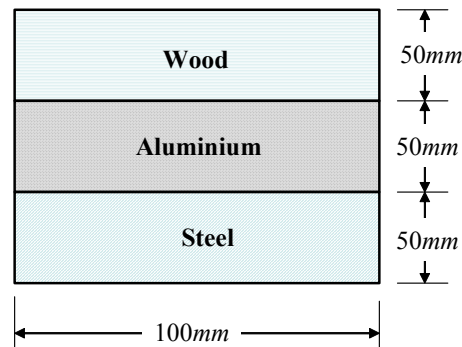
So the general term becomes

$$\underline{\underline{I_{eqj} = I_{eqi} \left(\frac{E_i}{E_j} \right)}} \quad (6.25)$$

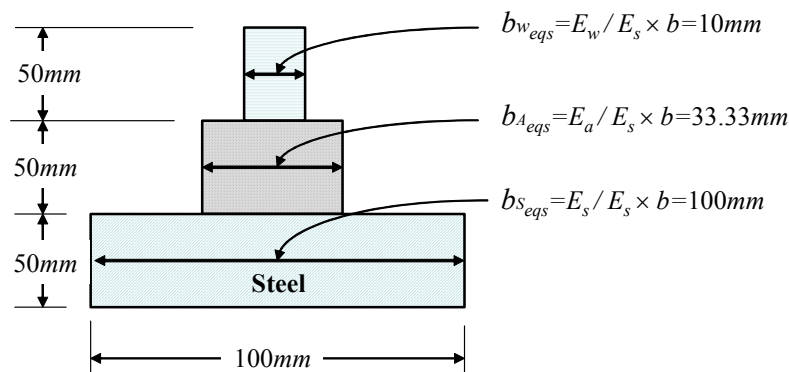
This gives the stresses in the *actual* materials as

$$\underline{\underline{\sigma_3 = \left(\frac{E_3}{E_1} \right) \sigma_{3eq} = \left(\frac{E_3}{E_1} \right) \left(-\frac{My}{I_{eq1}} \right)}} \text{ and } \underline{\underline{\sigma_2 = \left(\frac{E_2}{E_1} \right) \sigma_{2eq} = \left(\frac{E_2}{E_1} \right) \left(-\frac{My}{I_{eq1}} \right)}} \quad (6.26)$$

Example 6.4: Determine the maximum stresses in each material if a moment of 20 Nm is applied to the composite beam shown, where $E_{al} = E_S / 3$ and $E_W = E_S / 10$:



Step 1: Transform this to an equivalent section for one of the three materials.
In this case we select **steel**.

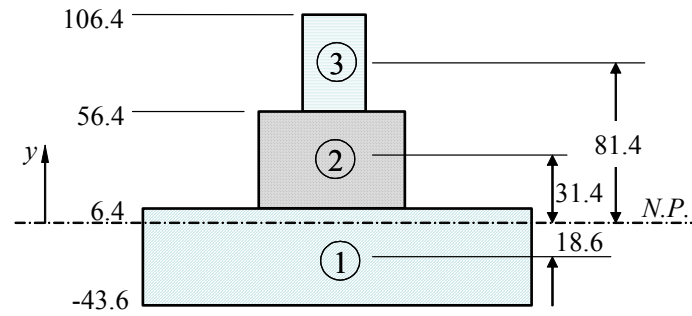


Step 2: Determine the position of Neutral Plane

$$\bar{S} = \frac{\sum s_i A_i}{\sum A_i} = \frac{25 \times (100 \times 50) + 75 \times (33.33 \times 50) + 125 \times (10 \times 50)}{(100 \times 50) + (33.33 \times 50) + (10 \times 50)} = 43.6mm$$

Which gives that: $\underline{\underline{\bar{S} = 43.6mm}}$

Step 3: Coordinate transformation and re-draw the section with vertical distances about N.P.



Step 4: Determine I_{eqS}

$$I_{eqS} = I_1 + I_2 + I_3$$

From the *Parallel Axis Theorem*:

$$I_i = I_{i_{Local}} + \bar{y}_i^2 A_i$$

whereas
$$I_{i_{Local}} = \frac{bh^3}{12}$$

So:
$$I_1 = \frac{100 \times 50^3}{12} + (18.6)^2 \times (100 \times 50) = 2771467 \text{ mm}^4$$

And similarly: $I_2 = 1990325 \text{ mm}^4$, and $I_3 = 3417147 \text{ mm}^4$

So adding these three values together and converting the results to **metres** gives

Therefore:
$$I_{eqS} = I_1 + I_2 + I_3 = 8.1789 \times 10^{-6} \text{ m}^4$$

Step 5: Determine Stresses in each material

In Steel :

$$\sigma_S = -\frac{My_S}{I_{eqS}} = -\frac{20 \times (-0.0436)}{8.1789 \times 10^{-6}} = 106.6 \text{ kPa}$$

In Aluminium;

$$I_{eqAl} = I_{eqS} \left(\frac{E_S}{E_{Al}} \right) = (8.1789 \times 10^{-6}) \times (3) = 24.537 \times 10^{-6} \text{ m}^4$$

and:

$$\sigma_{Al} = -\frac{My_{Al}}{I_{eqAl}} = -\frac{20 \times 0.0564}{24.537 \times 10^{-6}} = -45.97 \text{ kPa}$$

and in Wood:

$$I_{eqW} = I_{eqS} \left(\frac{E_S}{E_W} \right) = (8.1789 \times 10^{-6}) \times (10) = 24.537 \times 10^{-5} \text{ m}^4$$

$$\sigma_W = -\frac{My_W}{I_{eqW}} = -\frac{20 \times 0.1064}{8.1789 \times 10^{-5}} = -26.018 \text{ kPa}$$