

Owings: Aerodynamic Coefficients of Untwisted Circular Planforms

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SUMMARY

Aerodynamic coefficients calculated using Owings.

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1 Introduction

Calculations using Owings compared to historical values.

2 Flat Untwisted Symmetric (FUS)

Table 1: Flat Untwisted Symmetric.

Year	Author(s)	C_L/α	C_M/α	C_{Di}/α^2
1937	Kinner [17]	1.8174	-0.468	0.8244
1940	Schade [19]	1.7984	-0.4659	
1959	van Spiegel [20]	1.7902	-0.4663	0.8022
1959	Watkins et al [24]	1.7910	-0.4694	
1962	Levey & Wynter [18]	1.790	-0.4663	
1964	Boersma [1]	1.79002	-0.46617 3	0.80142
1967	Zandbergen et al [25]	1.7903		
1971	Jordan [9]	1.79002 30	-0.46617 35	
1974	Jordan [13]	1.79002 303	-0.46617 353	
1993	Tuck [22]	1.79035 0		
1996	Ichikawa [6]	1.790		0.801
1997	Standingford [21]	1.7900	-0.4661	
2020	Present	1.79002 30364 49760	-0.46617 35275 98550 0	0.80141 35373 70961
2020	Present: J4	1.79002 30364 50297 86	-0.46617 35275 98573 055	0.80141 35373 70840 657
2020	Present: VS11	1.79002 30364 50297 17314 55	-0.46617 35275 98573 02601 347	0.80141 35373 70840 81574 206

Table 2: Flat Untwisted Symmetric: Wing tip singularity strengths.

Year	Author(s)	$C_l(1)$
1971	Jordan [9]	1.5931
1974	Jordan [13]	1.59308 9
1997	Standingford [21]	
2020	Present: J4 ss	1.59306 579
2020	Present: VS11 ss	1.59306 579
2020	Present: J4 ssonb	1.59308 82174 3
2020	Present: VS11 ssonb	1.59308 82174 3
2020	Present: J4 ssonb1	1.59308 82181 6
2020	Present: VS11 ssonb1	1.59308 82181 2
2020	Present: J4 ssonb2	1.59308 82221 808
2020	Present: VS11 ssonb2	1.59308 82221 44
2020	Present: J4 ssonb3	1.59308 82221 25680
2020	Present: VS11 ssonb3	1.59308 82221 44

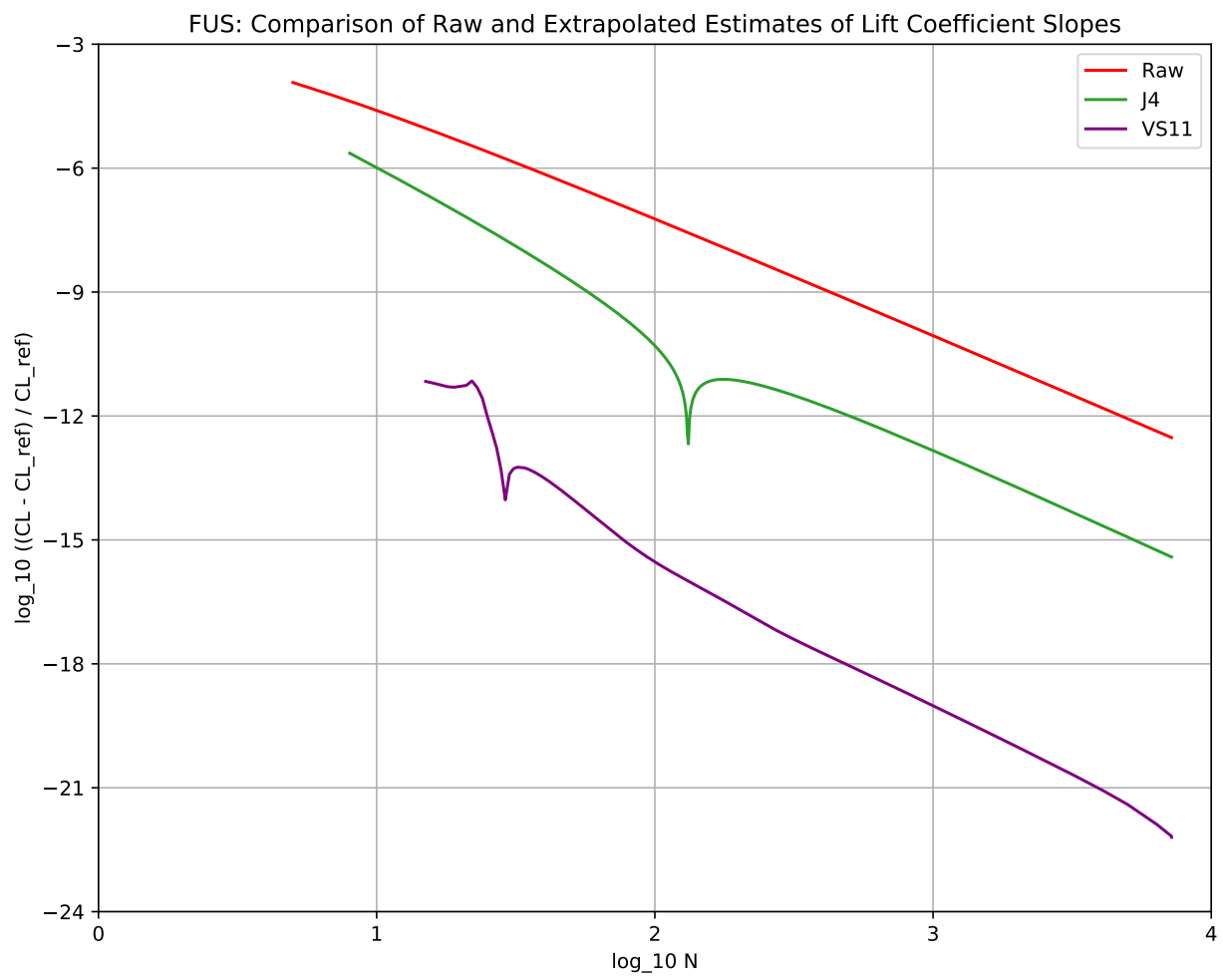


Figure 1: FUS: Lift coefficient slope.

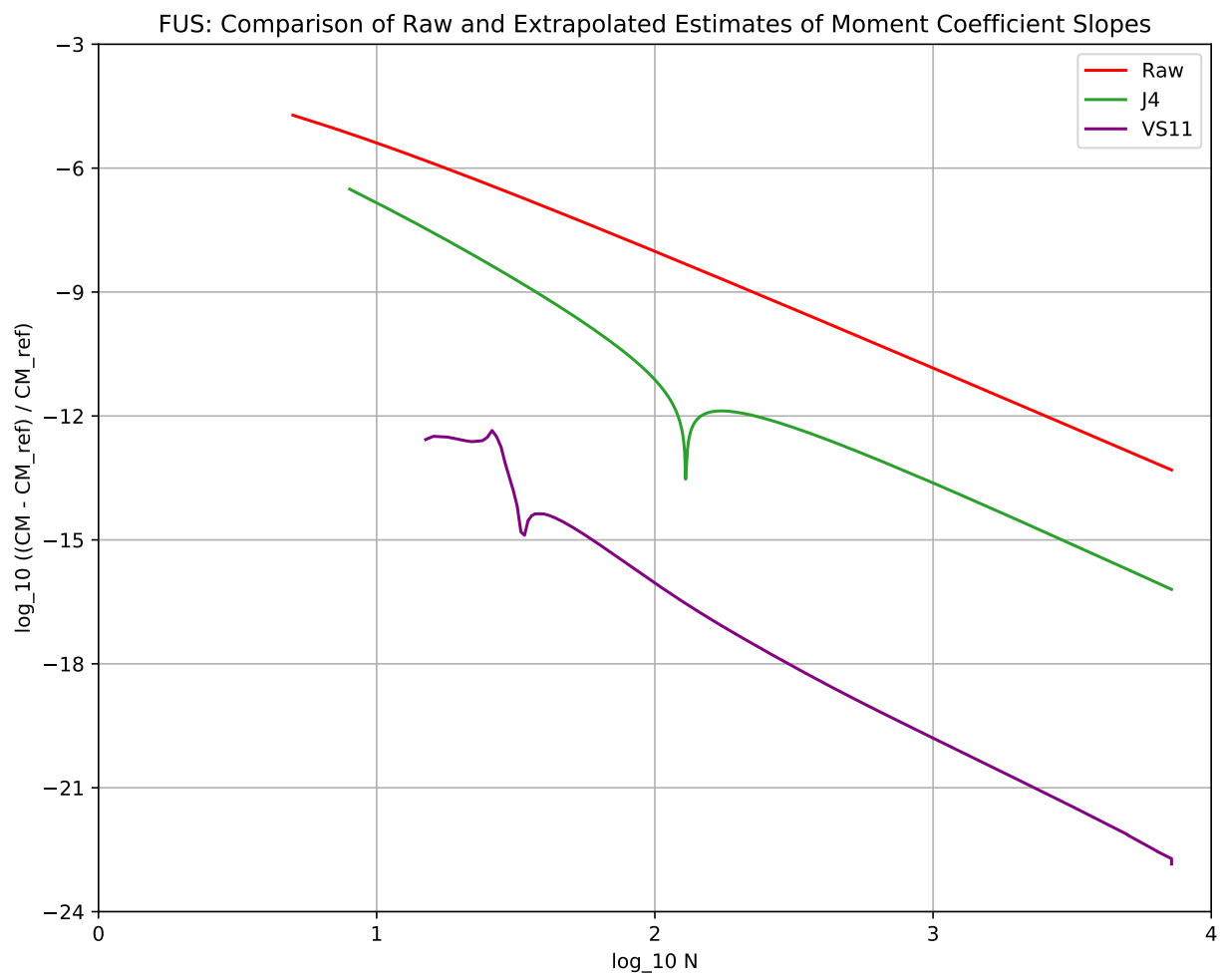


Figure 2: FUS: Moment coefficient slope.

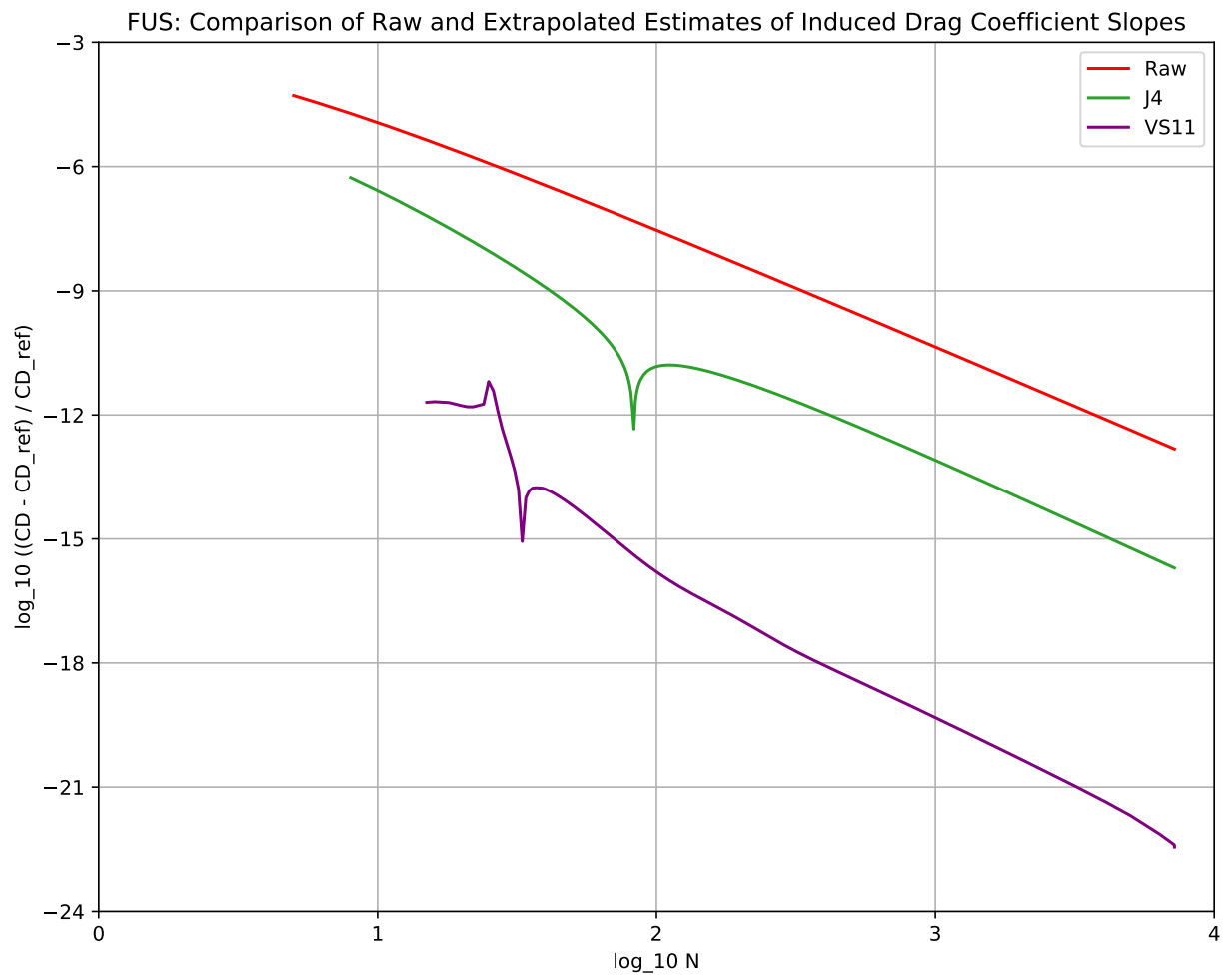


Figure 3: FUS: Induced drag coefficient slope.

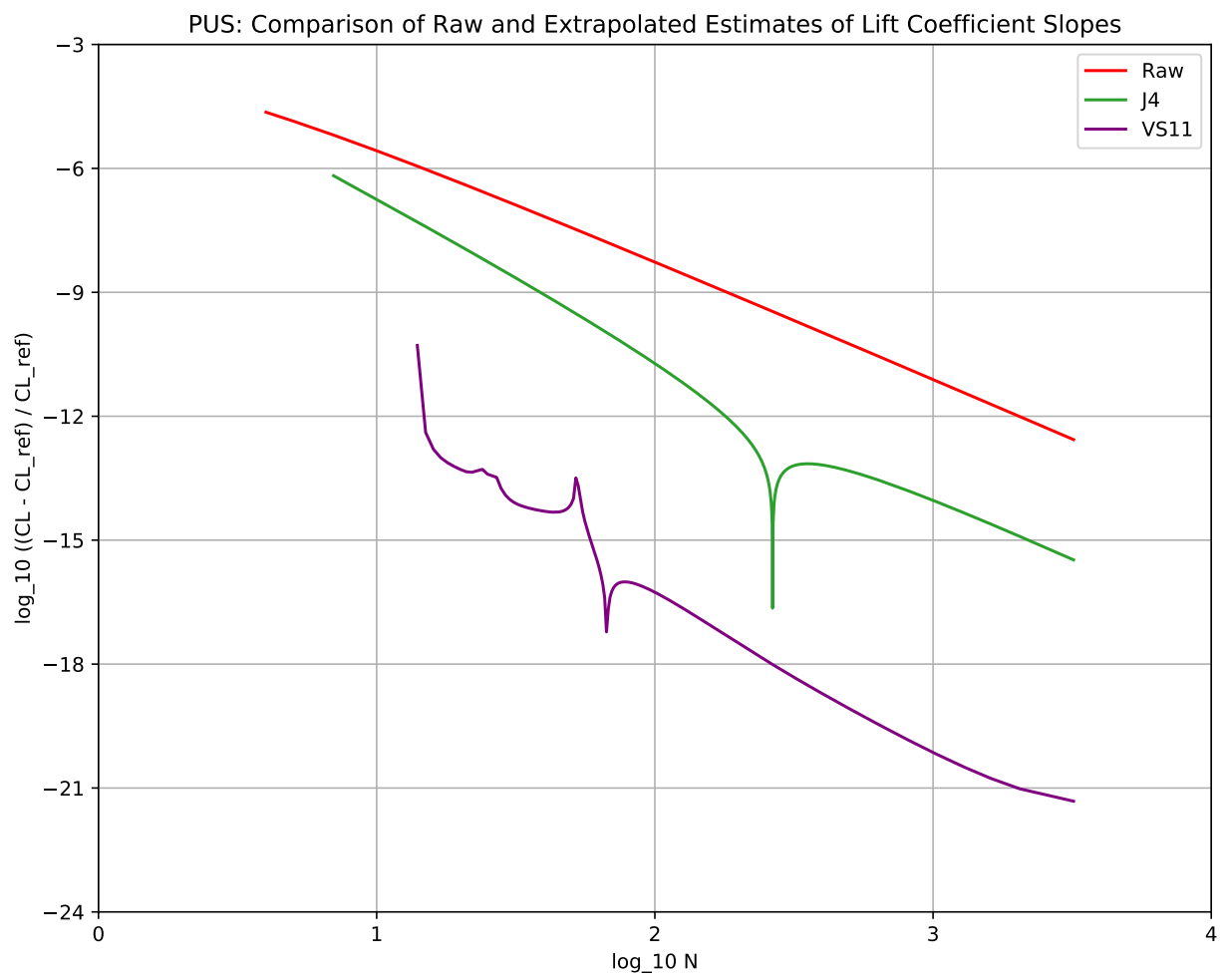


Figure 4: PUS: Lift coefficient slope.

3 Parabolic Untwisted Symmetric (PUS)

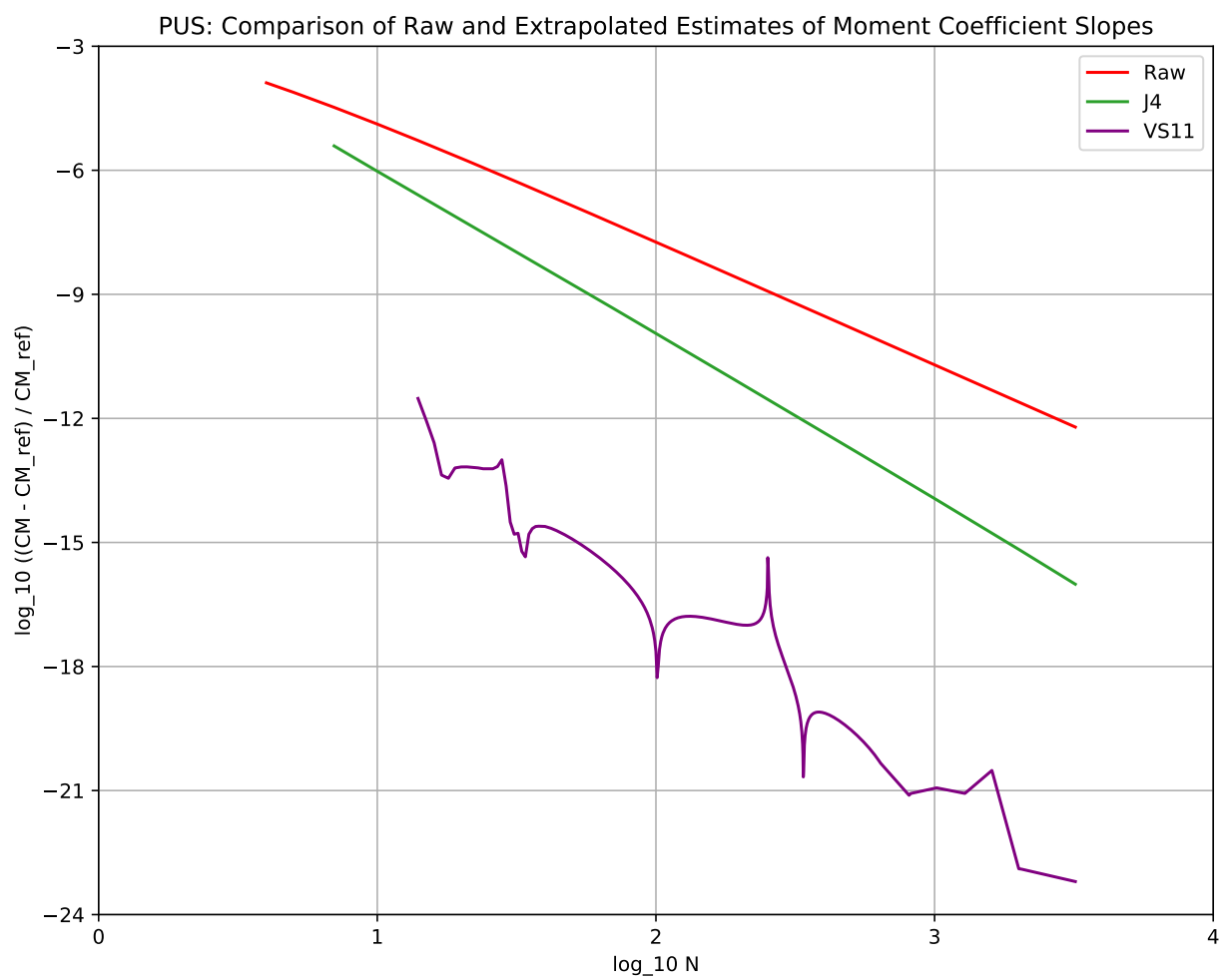


Figure 5: PUS: Moment coefficient slope.

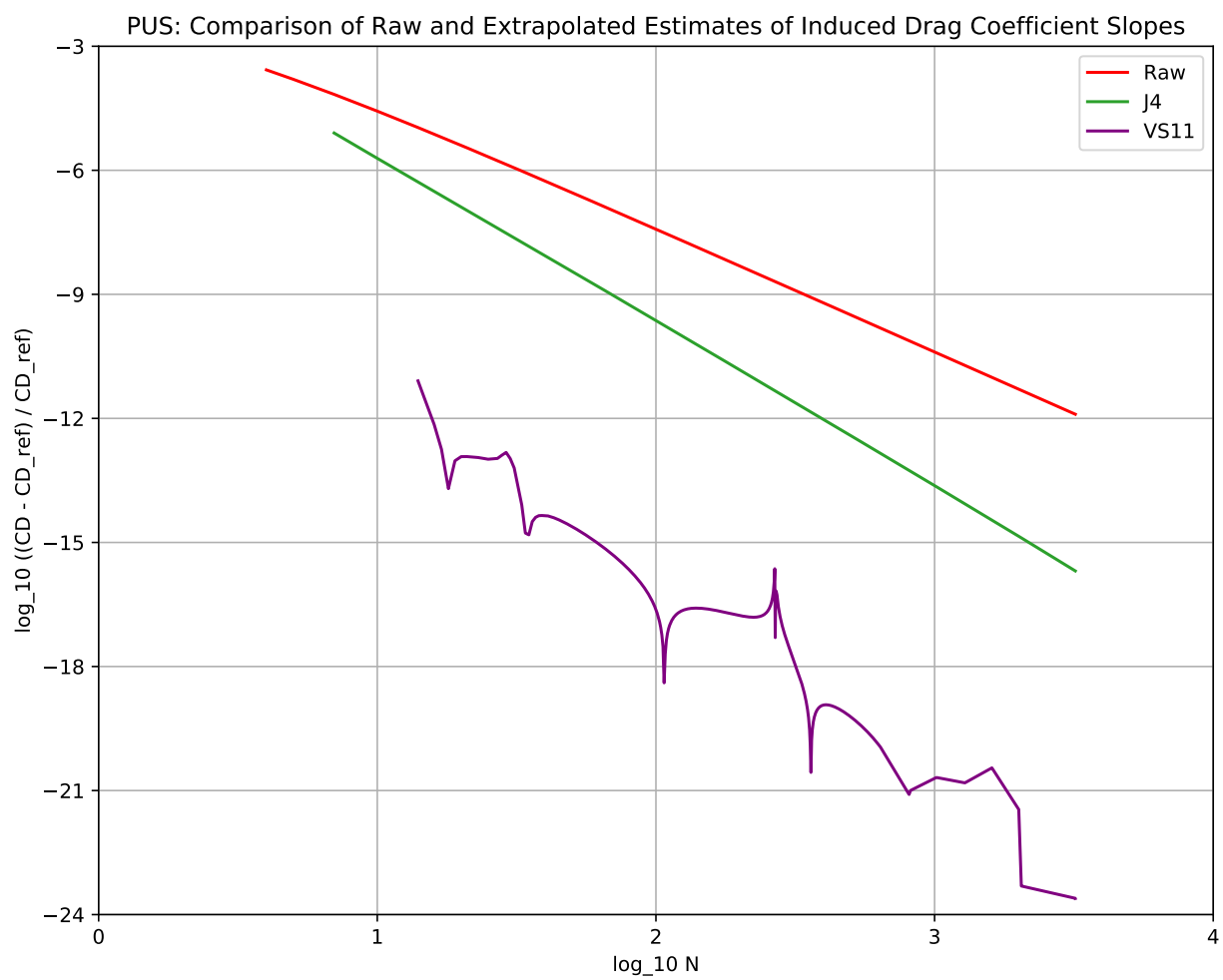


Figure 6: PUS: Induced drag coefficient slope.

Table 3: Parabolic Untwisted Symmetric.

Year	Author(s)	C_L/α	C_M/α	C_{Di}/α^2
1937	Kinner [17]	-1.872	-0.438	0.492
1940	Schade [19]	-1.8872	-0.4382	
1959	van Spiegel [20]	-1.8652	-0.4388	0.4744
1964	Boersma [1]	-1.86469 4	-0.43380 2	0.47589
2005	Tuck & Lazauskas [23]	-1.86842	-0.44003	
2020	Present	-1.86469 41103 9479	-0.43880 22815 60196	0.47589 32093 8835
2020	Present: J4	-1.86469 41103 94291 482	-0.43880 22815 59926 002	0.47589 32093 87756 335
2020	Present: VS11	-1.86469 41103 94292 10405 4206	-0.43880 22815 59925 95917 89410	0.47589 32093 87756 23839 44372

4 Flat Untwisted Antisymmetric (FUA)

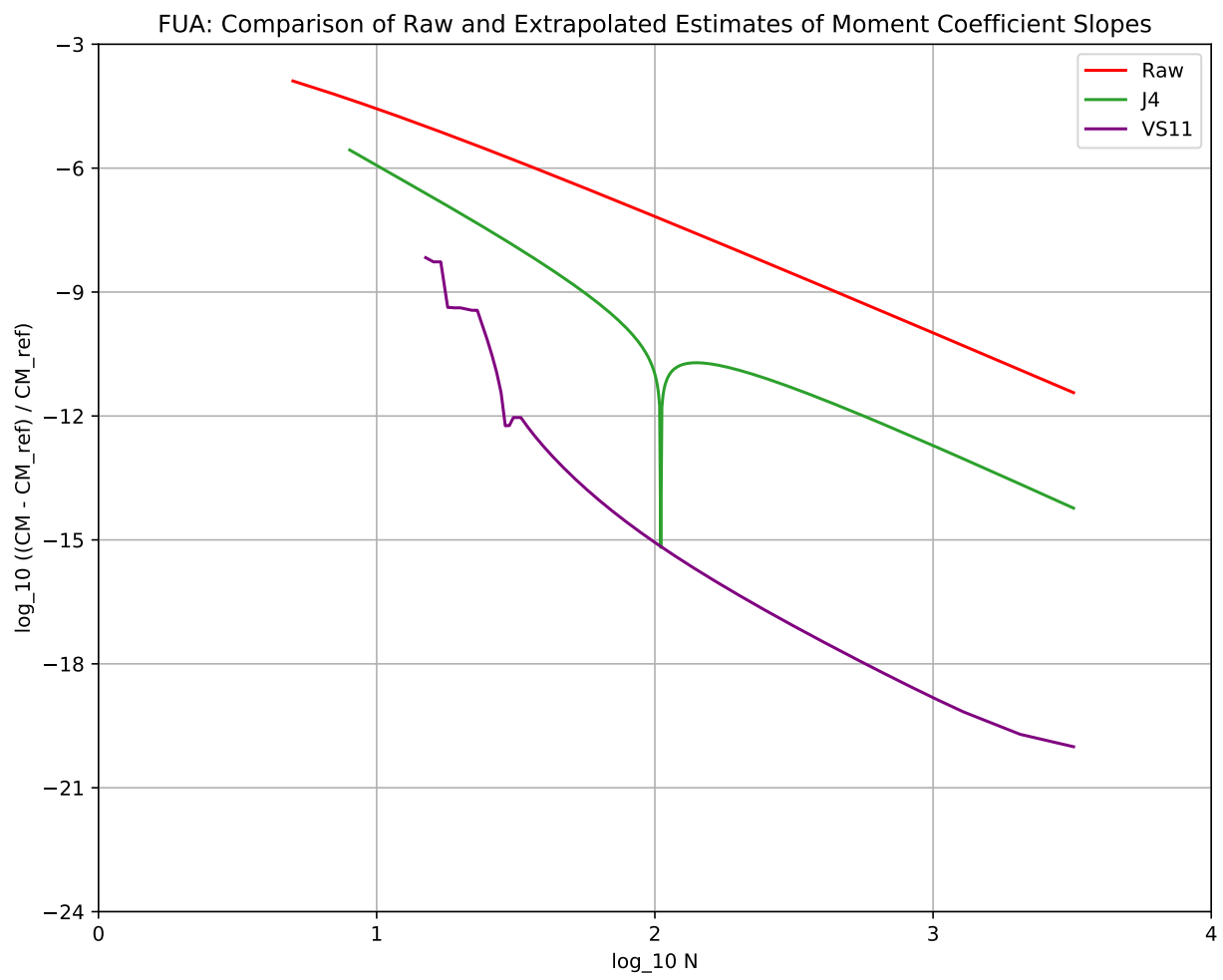


Figure 7: FUA: Moment coefficient slope.

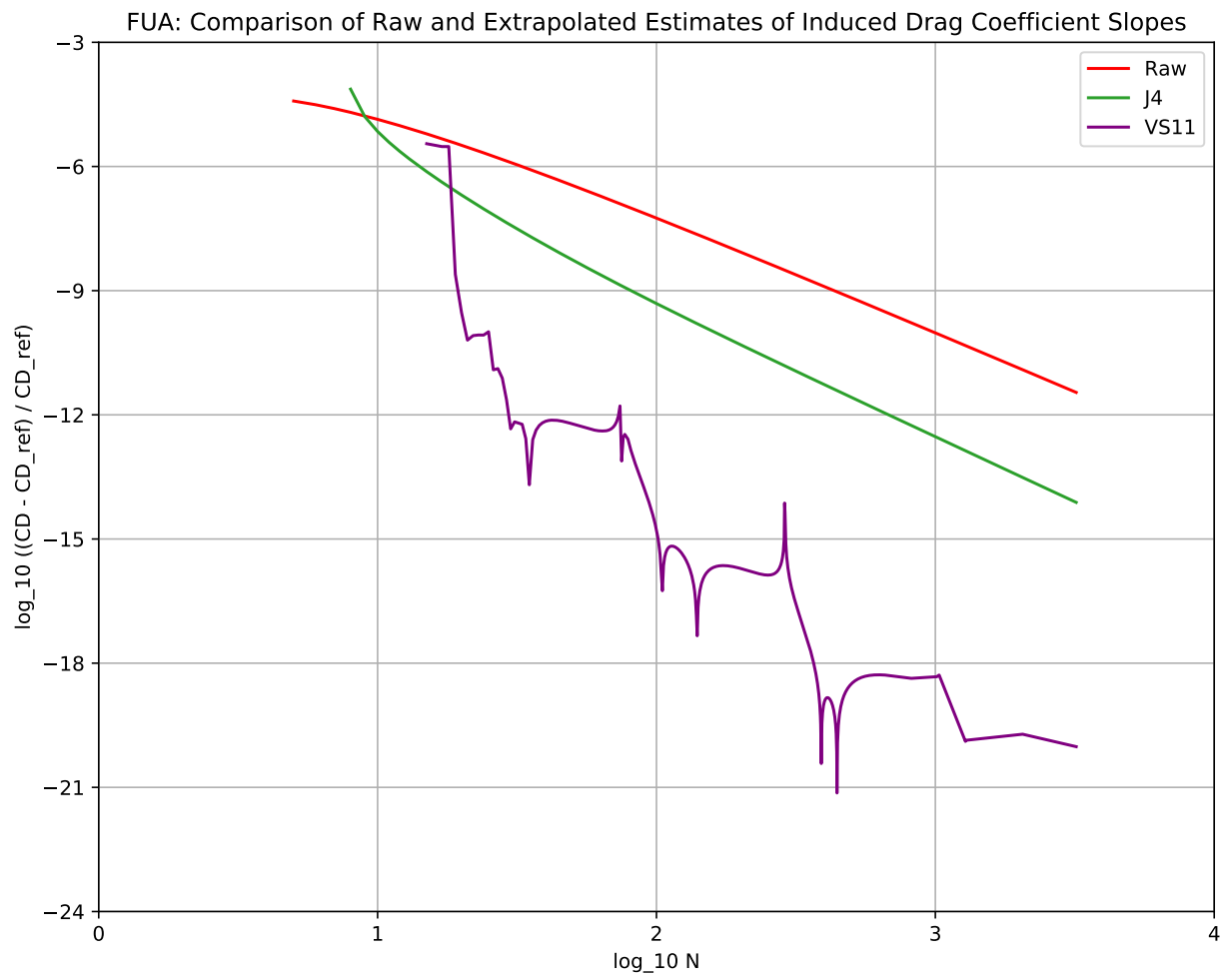


Figure 8: FUA: Induced drag coefficient slope.

Table 4: Flat Untwisted Antisymmetric.

Year	Author(s)	C_M/α	C_{Di}/α^2
1937	Kinner [17]	-0.254	
1940	Schade [19]	-0.2552	
1959	van Spiegel [20]	-0.245	0.2408
1964	Boersma [1]	-0.24496 2	0.24005 0
2020	Present	-0.24496 24555 43590	0.24004 86339 9187
2020	Present: J4	-0.24496 24555 42689 27	0.24004 86339 92715 14
2020	Present: VS11	-0.24496 24555 42690 71352 458	0.24004 86339 92713 30408 824

5 Parabolic Untwisted Antisymmetric (PUA)

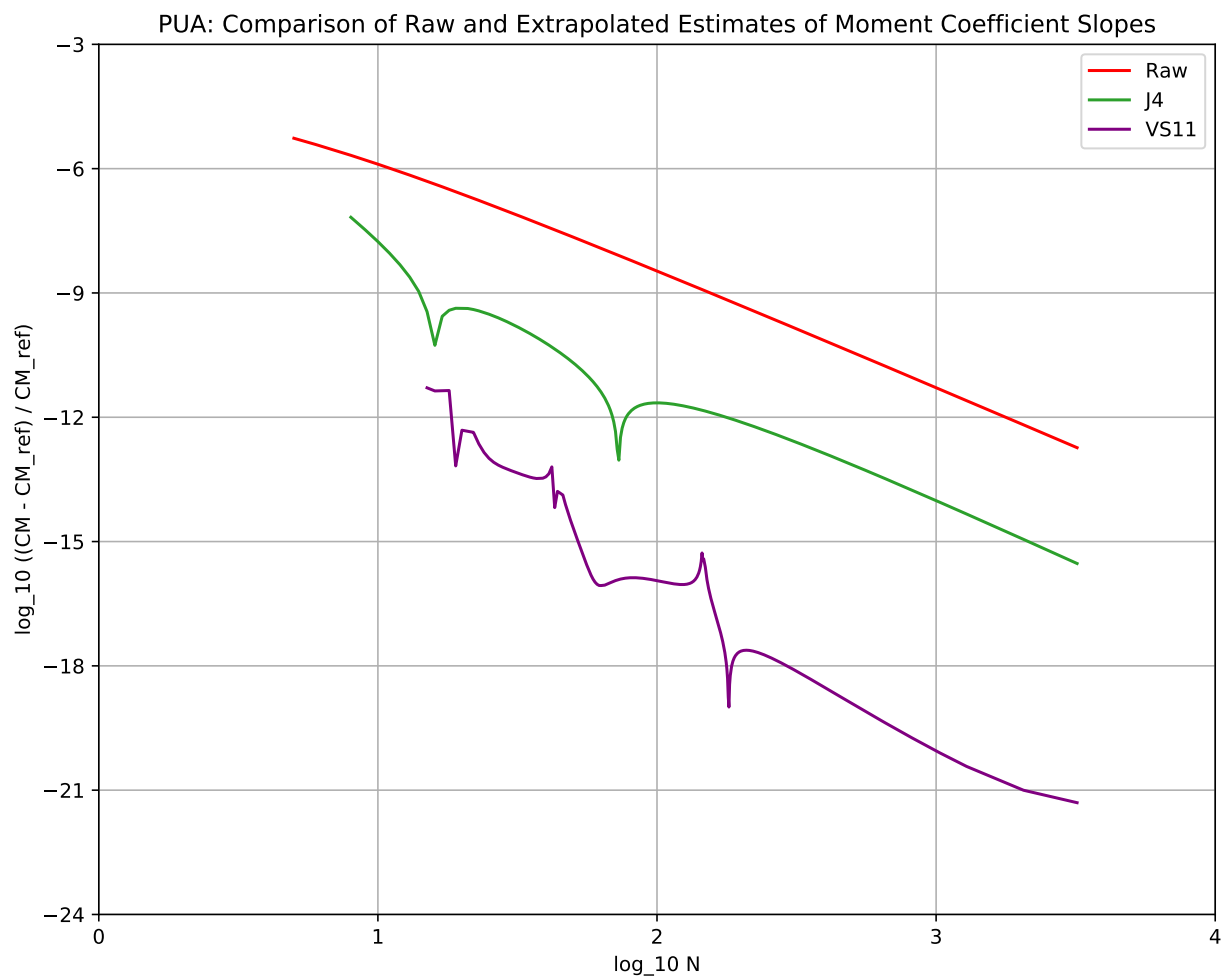


Figure 9: PUA: Moment coefficient slope.

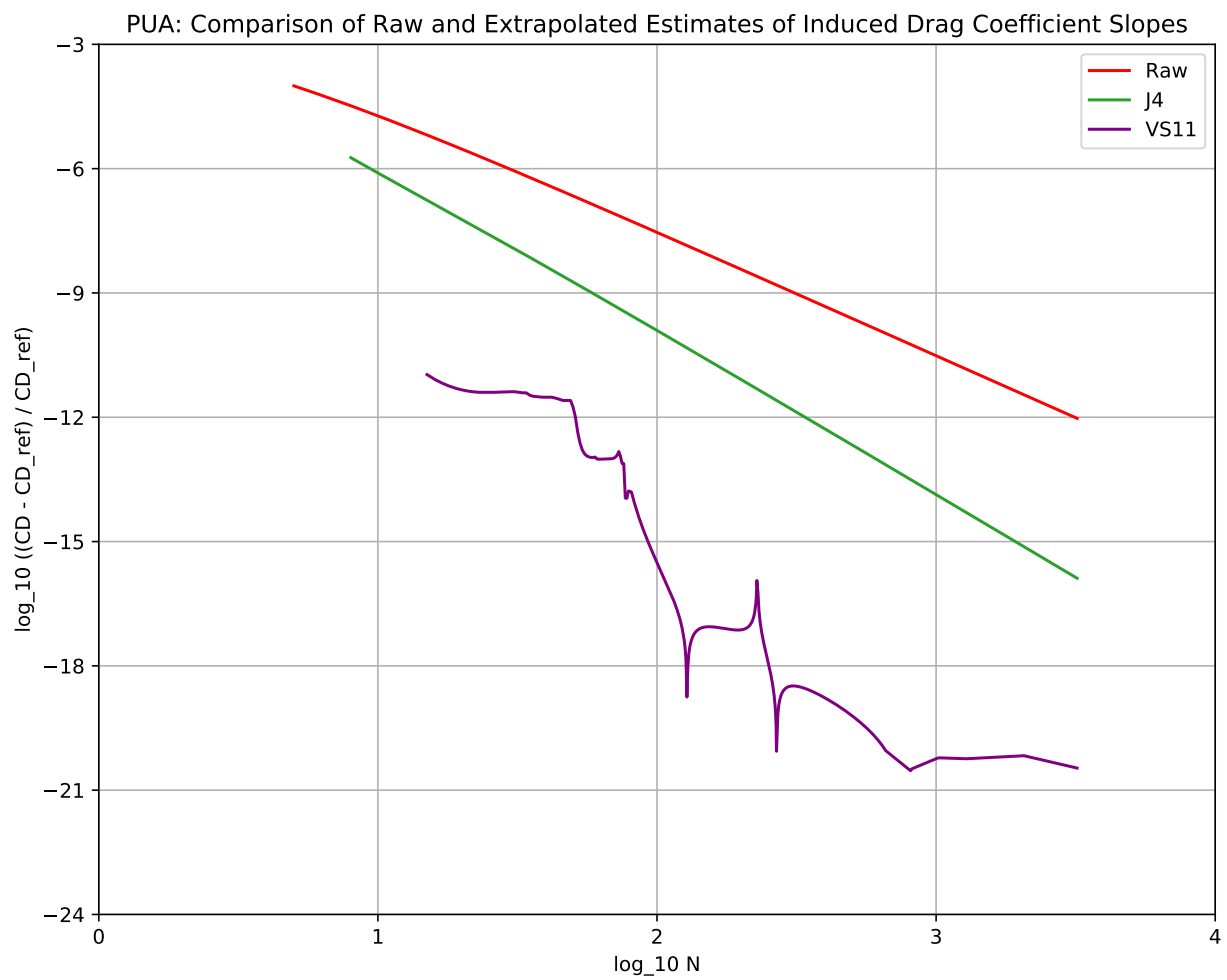


Figure 10: PUA: Induced drag coefficient slope.

Table 5: Parabolic Untwisted Antisymmetric.

Year	Author(s)	C_M/α	C_{Di}/α^2
1959	van Spiegel [20]	-0.1152	0.062
1964	Boersma [1]	-0.11503 67	0.06146 61
2020	Present	-0.11503 67447 05578 8	0.06146 60997 758
2020	Present: J4	-0.11503 67447 05557 5838	0.06146 60997 75832 1829
2020	Present: VS11	-0.11503 67447 05557 61782 75540	0.06146 60997 75832 17496 9510

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