



A scaling law for form drag coefficients in incompressible turbulent flows

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ABSTRACT

We present a similarity law for form drag coefficients, which is obtained by a judicious utilization of an energy dissipation equation due to eddy viscosity for incompressible turbulent flow, the steady-state κ - ω turbulence model, and the Kolmogorov turbulence dissipation length scales. It is shown that the form drag coefficients of three geometrically similar vessels subjected to turbulent flows are scaled according to $\bar{C}_p = \{c_1 + c_2 \bar{Re}^{-1/3} + c_3 \bar{Re}^{-1}\}$ where $(\bar{C}_p, \bar{Re}, c_i (i = 1, 2, 3))$ are the form drag coefficient ratio, the mean flow Reynolds number ratio, and c_i are closure coefficients to be determined from existing geometrically similar vessels, respectively. The present theoretical form factor methodology is applied to predict the full scale form factor from the scaled towing tank experiment based on three simulation results, which show improved correlations compared to the empirical least-squares prediction methods.

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1. Introduction

A major goal in the design of aerospace and/or marine vehicles is to reduce drag emanating from skin friction as well as form drag (or pressure drag). Thanks to the century-old legacy of aerospace and marine vehicle design activities, there exist a host of base-line designs from which one can improve the efficiency of existing geometrically similar vehicle types, or design a new class of *scaled-up* vehicles. The concurrent increase in both the vehicle size and the operation speed inherently subject the modern air and marine transportation vehicles, as well as wind energy blades, to turbulent flow ranges. In scaling-up design iterations, an important design parameter that is needed is drag coefficients due to skin friction and form drag. Skin friction (more precisely skin friction coefficient) for a new design can be estimated based on the power and/or logarithmic laws (Hinze, 1975; ITTC, 1957; Prandtl, 1905; Schlichting, 1979; Prandtl, 1921; von Karman, 1934), among others. Hence, designers can estimate the drag due to skin friction in their assessment of overall aerodynamic or hydrodynamic performance.

When the aspect ratio (i.e., thickness-to-length ratio) is small such as thin plates, the form drag remains insignificant. However, when the vehicle cross section bulges out significantly as typically the case for surface ships, the form drag constitutes a significant

part of the total drag loss. It is generally accepted that the form drag in turbulent flows is caused by the viscous pressure applied on a vehicle surface. However, there exist scant proposed formulas or rational estimation procedures for the estimation of form drag coefficient. In ship hydrodynamics, for example, the prevailing ITTC practice determines the total viscous drag coefficient from a scaled-model test in which the wave-making resistance can be considered minimal or non-existent. The difference between the ITTC1957 skin friction line and the viscous drag coefficient is then deemed to be due to the form drag. This form drag coefficient which is expressed as a percentage of the skin friction coefficient, known as *form factor*, is then assumed to remain constant for the full model (ITTC, 1957). In an effort to improve the prediction of form factors, several investigators proposed various schemes ranging from a least-squares data fit (García-Gómez, 2000; Min and Kang, 2010) to CFD-based prediction (Kim and Menon, 1997; Kouh et al., 2009). To date, there exist no comparable theory or formula that one can rely to estimate form factors as is the case for skin friction.

The present study is an attempt to develop an engineering scaling law for estimating the power loss due to form drag in turbulent flows, which may be utilized in the design of geometrically similar full model vehicles, provided there exist form drag data for geometrically similar scale-model vehicles. In so doing, we observe that energy loss consists of two sources: the first due to skin friction near the wall/boundaries and the second due to the energy dissipation of locally isotropic Kolmogorov-scale (η) turbulence away from the boundaries or simply due to eddy viscosity.

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As for the skin friction loss, we accept that the near wall/boundary energy loss is accounted for by relying on the ITTC skin friction formula for surface vessel design. As for energy loss due to the viscous pressure – caused form drag in turbulent flows, we stipulate that the bulk of energy loss due to viscous pressure – caused form drag – is associated with eddy viscosity dissipation away from the submerged vessel surface. In other words, the dominant energy loss mechanisms away from the wetted surface are due to eddy viscosity dissipation, which in turn accounts for energy loss due to viscous pressure – caused form drag. This stipulation allows us to relate the form drag to the energy loss terms in turbulence in the energy equation (see, e.g., Stewart, 1942; Stewart and Townsend, 1951; Daly and Harlow, 1970; Wilcox, 2008).

It turns out that, in order to relate the energy loss terms of one scale to those of another scale, one needs the scaling relations of the kinematic eddy viscosity (ν_t), the turbulence kinetic energy (k), the Kolmogorov eddy scale (η), the mean flow length scale (ℓ_u), the mean flow velocity (U), and their interrelations. This puts us to employ a suitable turbulence model. We note that, although the turbulence kinetic energy (TKE) equation should, in principle, provide the needed interrelations among the preceding five variables, the TKE closure problem remains open problems; hence, we are unable to employ it in the present study. Among several proposed turbulence models (see, e.g., Landau and Lifshitz, 1959; Smagorinsky, 1963; Launder and Spalding, 1974; Hinze, 1975; Launder et al., 1975; Launder, 1989; Lilly, 1992; Spalart and Allmaras, 1992; Menter, 1992, 1994), we have chosen the k - ω turbulence model (Kolmogorov, 1942; Saffman, 1970; Wilcox, 1988, 2004), primarily because of our familiarity with it. We are aware of that what we are adopting is an isotropic turbulence model, not anisotropy turbulence theories (Biferale and Procaccia, 2005; Tong et al., 1990), by invoking the Kolmogorov hypothesis, viz., in turbulent flows, energy dissipation occurs in the Kolmogorov eddy scales for which isotropic turbulence assumption is assumed to be valid. Should anisotropic/intermittency theories prove to be applicable for form drag estimation, we believe the present methodology would apply. The rest of the paper is organized as follows.

Section 2 begins with the resistance expression that consists of the skin friction resistance and form drag (or viscous pressure) resistance. The ratio of the form drag for two geometrically similar vessels are then related to the ratio of the energy loss expressions. It is this ratio that the present paper seeks to determine.

Section 3 introduces the k - ω turbulence model together with the Kolmogorov scale to extract similarity-obeying conditions among the five variables ($\nu_t, \eta, k, \ell_u, U$). It is found that the k - ω turbulence model does not satisfy complete similarity requirements. To alleviate this difficulty, it is rearranged in powers of the small parameter $\epsilon_t = Re^{-1} \ll 1$ where Re is a characteristic Reynolds number. Similarity laws are then applied separately to each power of ϵ_t , which is analogous to a procedure adopted in classical asymptotic nonlinear analysis. A surprisingly simple relation is obtained for the ratio of eddy to mean-flow length scales given by $\bar{\ell}_t/\bar{\ell}_u = \bar{Re}^{-1/3}$ where the over bar denotes the ratios of appropriate variables for two geometrically similar vessels.

Section 4 derives a theoretical ratio of the form drag coefficients of two geometrically similar vessels solely in terms of their Reynolds number ratio. Section 5 summarizes the main results of the present study along with limitations of the present results and future work being carried out.

2. Problem statement

Our objective is to find a scaling law for the total resistance (R_{total}) consisting of the wave-making resistance (R_W), the

resistance due to skin friction (R_f) and due to form (or viscous pressure) drag emanating from the energy dissipation associated with the eddies (R_p) in incompressible turbulent flows. To this end, we express the total resistance (R_{total}) as

$$R_{total} = R_W + R_V$$

$$R_V = R_f + R_p = C_V \frac{1}{2} \rho U^2 A, \quad C_V = C_f + C_p \quad (2.1)$$

where ($C_W, C_V, C_f, C_p, \rho, U, A$) denote the wave-making resistance coefficient, the viscous resistance coefficient, the skin friction resistance coefficient, the form drag (or viscous pressure resistance) coefficient, the density, the mean-flow velocity, and the wetted surface area of the vessel of interest, respectively.

For subsequent analysis, we focus on the viscous drag, assuming that the viscous drag can be obtained either by assuming that the wave-making resistance is known or it can be effectively neglected by a careful low speed setup of the towing tank experiment. Hence, from now on we limit ourselves to the viscous drag only.

For a class of geometrically similar vessels, we wish to find a scaling functional ($\bar{C}(\bar{Re}, \bar{\ell})$) that relates the viscous resistance coefficient of one scale to that of another scale in the form of

$$C_V^{(a)} = \bar{C}(\bar{Re}, \bar{\ell}) C_V^{(b)}, \quad \bar{Re} = \frac{Re^a}{Re^b}, \quad \bar{\ell} = \frac{\ell^a}{\ell^b} \quad (2.2)$$

and similarly for the skin friction and form drag coefficient, where superscripts (a, b) refer to two different geometrically similar vessels, and (ℓ^a, ℓ^b) are characteristic lengths of the two vessels.

In this paper we adopt the log-law skin friction coefficient (C_f) that is expressed as (cf., Schlichting, 1979)

$$C_f = \alpha (\log_{10} Re - \beta)^\gamma \quad (R_f = C_f \frac{1}{2} \rho U^2 A) \quad (2.3)$$

where (α, β, γ) are constants proposed by various investigators.

What remains to be done is the determination of the form drag coefficient (C_p) that is needed for the computation of the form drag (or eddy-making resistance) expressed as

$$R_p = C_p \frac{1}{2} \rho U^2 A \quad (2.4)$$

The difficulty in expressing the eddy-making resistance in terms of mean-flow speed (U) is that the turbulence kinetic energy (TKE) associated with the eddies (k) is given by

$$k = \frac{1}{2} \sum_{i=1}^3 \langle u_i' u_i' \rangle \quad (2.5)$$

where u_i' is the fluctuating turbulence velocity, and $\langle \cdot \rangle$ denotes averaging operator.

Since our objective is to find a scaling law governing the eddy-making resistance coefficients, we express the ratio of form drag coefficients from (2.4) as

$$\bar{C}_p = \frac{C_p^s}{C_p^f} = \left[\frac{R_p^s}{R_p^f} \right] \left[\frac{1}{\bar{\rho} \bar{U}^2 \bar{A}} \right], \quad \bar{\rho} = \frac{\rho^s}{\rho^f}, \quad \bar{U} = \frac{U^s}{U^f}, \quad \bar{A} = \frac{A^s}{A^f} \quad (2.6)$$

where superscripts (s, f) refer to two geometrically similar scales.

The above equation, circuitous it may seem, leads us to find \bar{C}_p if we know the ratio of the eddy-making resistance. This is derived as follows.

The energy conservation for incompressible flows can be expressed as (Stewart, 1942; Wilcox, 2008)

$$\frac{\partial}{\partial t} \left[\rho \left(\frac{1}{2} u_i u_i + k \right) \right] + \frac{\partial}{\partial x_j} \left[\rho u_j \left(\frac{1}{2} u_i u_i + k \right) \right]$$

$$= \frac{\partial}{\partial x_j} \left[u_i (\tau_{ij}^u + \rho \tau_{ij}^t) + \rho (\nu + \sigma^* \nu_t) \frac{\partial k}{\partial x_j} \right] \quad (2.7a)$$

$$\tau_{ij}^u = 2\mu S_{ij}, \quad \tau_{ij}^t = 2\nu_t S_{ij} - \frac{2}{3} k \delta_{ij}, \quad S_{ij} = \frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \quad (2.7b)$$

In the above energy equation, internal energy and enthalpy are omitted as we address an isothermal case; (μ, ν_t) are the fluid viscosity and the kinematic eddy viscosity, respectively; and, σ^* is a closure coefficient.

Of the above energy equation (2.7a), we recognize that energy dissipation terms due solely to turbulence (E_t) is given by

$$E_t = \rho \frac{\partial}{\partial x_j} \left[2\nu_t u_i S_{ij} + (\nu + \sigma^* \nu_t) \frac{\partial k}{\partial x_j} \right] \quad (2.8)$$

Remark 1. The above energy dissipation is valid for isotropic turbulence. We adopt this expression by invoking the Kolmogorov hypothesis that states the energy dissipation in turbulence which occurs due to viscosity when the eddy size reaches the Kolmogorov scale for which turbulence becomes largely isotropic. In other words, we assume that the two turbulence variables, (ν_t, k) , are associated with the Kolmogorov eddy size when (2.8) is applicable. What remains to be done is to relate the eddy resistance (R_p) to the above energy dissipation (E_t). This is accomplished by invoking a hypothesis stated in Introduction: *In turbulent flow ranges, far-field (that is, away from the vehicle surface) eddy viscosity-driven energy loss indirectly accounts for the bulk of energy loss due to viscous pressure-caused form drag.* This hypothesis allows us to relate the eddy-making resistance (R_p) to the energy dissipation due to turbulence (E_t) as

$$dR_p u_t \propto E_t dV, \quad dR_p = C_p \frac{1}{2} \rho U^2 dA, \quad u_t = \sqrt{k} \quad (2.9)$$

where u_t is an average fluctuating velocity for isotropic turbulence flow.

Substituting (2.9) into (2.6), we obtain

$$\bar{C}_p = \left[\frac{E_t^s}{E_t^f} \right] \left[\frac{\bar{\rho}}{\rho U^2 u_t} \right], \quad \bar{u}_t = \frac{u_t^s}{u_t^f} = \sqrt{\frac{k^s}{k^f}}, \quad \frac{dV^s}{dV^f} = \bar{\rho}_u^3, \quad \frac{dA^s}{dA^f} = \bar{\rho}_u^2 \quad (2.10)$$

The above equation shows that the scaling of the eddy-making resistance coefficients depends on the ratio of the energy dissipation due to turbulence scales. On examining Eq. (2.8), this ratio depends on scaling and spatial variations of three flow variables (ν_t, \bar{U}, k) . This is addressed in the next section.

3. Determination of eddy viscosity-related similarity parameters

The difficulty in scaling of turbulence-caused parameters is manifested in the fact that the eddy viscosity ν_t depends in a complex manner on the turbulent flow characteristics (Landau and Lifshitz, 1959; Hinze, 1975). In the present study of a plethora of turbulence models proposed by various investigators (Hinze, 1975; Landau and Lifshitz, 1959; Launder and Spalding, 1974; Launder et al., 1975; Launder, 1989; Lilly, 1992; Menter, 1992, 1994) we employ the steady-state κ - ω turbulence model (Kolmogorov, 1942; Saffman, 1970; Wilcox, 1988, 2004) to identify a functional relation of the eddy viscosity and flow characteristics for incompressible turbulent flow:

$$(\mathbf{u} - (\nu + \sigma_k \nu_t) \nabla) \cdot \nabla k - \sigma_k \nabla k \cdot \nabla \nu_t + \beta \omega k - \nu_t g = 0 \quad (3.1a)$$

$$(\mathbf{u} - (\nu + \sigma_\omega \nu_t) \nabla) \cdot \nabla \omega - \sigma_\omega \nabla \omega \cdot \nabla \nu_t + \beta^* \omega^2 - \alpha g = 0 \quad (3.1b)$$

$$\nu_t = \mu_t / \rho = k / \omega \quad g = \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \frac{\partial u_i}{\partial x_j}, \quad i, j = 1, 2, 3. \quad (3.1c)$$

where k is the turbulence kinetic energy, ω is the specific dissipation rate, ν_t is the kinematic eddy viscosity, and $(\sigma_k, \sigma_\omega, \beta, \beta^*, \alpha)$ are empirically determined closure coefficients

amassed from various turbulence kinetic theories, and computational and experimental correlations (Wilcox, 2004).

It should be noted that the use of the steady-state κ - ω turbulence model is in keeping with part of Kolmogorov's original hypothesis: "... the fine pulsations of the higher orders are subjected to approximately space-isotropic statistical régime. Within small time-intervals it is natural to consider this régime approximately steady even in the case, when the flow in the whole is not steady (Kolmogorov, 1991, p. 10)." Note also that the above κ - ω turbulence model (3.1a)–(3.1c) remains valid away from the wall. At and near the wetted wall, special constitutive laws are invoked, which is configured to capture the log-law of skin friction.

It turns out that the κ - ω turbulence model (3.1a)–(3.1c) that we are about to utilize to obtain similarity relations does not satisfy complete similarity laws. To overcome this, first we express the κ - ω model as

$$f_0(k, \omega, \nu_t) + \epsilon_t f_1(k, \omega, \nu_t) = 0, \quad \epsilon_t \ll 1 \quad (3.2)$$

Second, we observe that asymptotic analysis consists of identifying the terms with equal powers of ϵ_t and solving the resulting equations corresponding to each power of ϵ_t . Employing an analogous approach used in asymptotic analysis, we invoke the similarity laws separately to ϵ_t^0 and ϵ_t^1 terms as expressed below:

k -equation:

$$\underbrace{-\nu_t g + \mathbf{u} \cdot \nabla k}_{\epsilon_t^0 \text{-term}} - \underbrace{\{\nu \nabla^2 + \sigma_k (\nu_t \nabla^2 + \nabla \nu_t \cdot \nabla) - \beta \omega\} k}_{\epsilon_t^1 \text{-term}} = 0$$

Magnitude:

$$\underbrace{-\nu_t |g| + u_0 \frac{|k|}{\ell_t}}_{\epsilon_t^0 \text{-term}} - \underbrace{\epsilon_t \left[\frac{\nu}{\nu_t} + 1 \right] \frac{\nu_t}{\nu} u_t \frac{|k|}{\ell_t}}_{\epsilon_t^1 \text{-term}}$$

$$u_0 = |\mathbf{u}|, \quad u_t = k^{1/2}, \quad \epsilon_t = Re_T^{-1} = \left(\frac{u_t \ell_t}{\nu} \right)^{-1} \ll 1,$$

$$\frac{u_t}{u_0} \ll 1, \quad \frac{\nu_t}{\nu} \frac{u_t}{u_0} \approx O(1) \quad (3.3)$$

where u_0 is the magnitude of the mean flow velocity, u_t is a characteristic eddy velocity, ℓ_t is a characteristic length of the eddies, $\epsilon_t = Re_T^{-1} \ll 1$ according to Wilcox (2004, p. 11), $(\nu/\nu_t < 1)$ according to Hinze (1975, p. 626), and we have utilized the fact that ω k -term has a same order of magnitude as the other ϵ_t^1 -terms (Wilcox, 2004, p. 130).

For the ω -equation, it can also be characterized in the form of (3.2). Hence we will not repeat its characterization process offered for the k -equation shown in (3.3). In the subsequent analysis, we stipulate that $\nu/\nu_t < 1$ holds for regions where energy dissipation due to eddy-viscosity occur. This allows us to drop the $(\nu \nabla^2 k)$ -term in the k -equation and the $(\nu \nabla^2 \omega)$ -term in the ω -equation.

We now introduce the following scaling parameters:

$$\begin{aligned} \rho^s &= \bar{\rho} \rho^f, & u^s &= \bar{U} u^f, & \nu^s &= \bar{\nu} \nu^f, & \nu_t^s &= \bar{\nu}_t \nu_t^f, \\ x^s &= \bar{\ell}_u x^f, & x_t^s &= \bar{\ell}_t x_t^f, & k^s &= \bar{u}_t^2 k^f, & \omega^s &= \bar{\omega} \omega^f, & Re &= \bar{U} \bar{\ell}_u / \bar{\nu} \end{aligned} \quad (3.4)$$

where the superscript (f, s) denote the full and scaled models, respectively, and the subscripts (u, t) refer to the mean flow and turbulence quantity, respectively, and Re is a specific mean-flow Reynolds number. It is noted that the spatial operator, $(\partial/\partial x, \nabla)$, takes on two distinct scaling processes:

$$\text{For mean flow: } \left(\frac{\partial u_i}{\partial x_j} \right)^s = \frac{\bar{U}}{\bar{\ell}_u} \left(\frac{\partial u_i}{\partial x_j} \right)^f$$

$$\text{For turbulence parameters: } (\nabla k)^s = \left(\frac{\bar{u}_t^2}{\bar{\ell}_t} \right) (\nabla k)^f, \quad (\nabla \omega)^s = \left(\frac{\bar{\omega}}{\bar{\ell}_t} \right) (\nabla \omega)^f \quad (3.5)$$

Remark 2. We have used one characteristic length scale $\bar{\ell}_t$ for both the turbulence kinetic energy (k) and the dissipation variable (ω), by assuming that most of the turbulence kinetic energy is eventually dissipated. In addition, this characterization was necessitated by the fact that only the turbulence kinetic energy (k) is contained in the energy dissipation expression (see Eq. (2.8)).

The steady state k - ω equation for the scale model can be related to that for the full model by employing the scaling parameterization relations given in Eqs. (3.4) and (3.5) as

$$k\text{-equation: } \underbrace{\left(\frac{\bar{u}_t^2}{\bar{\ell}_t \bar{\ell}_u} \right) (\bar{\nu} \bar{Re}) \left\{ \mathbf{u}^f \cdot (\nabla k)^f - S_1 (\nu_t g)^f \right\}}_{O(1)\text{-terms}} \quad (3.6a)$$

$$- \underbrace{\sigma_k \left(\frac{\bar{\nu}_t \bar{u}_t^2}{\bar{\ell}_t^2} \right) \left\{ [(\nu_t \nabla^2 k)^f + (\nabla k \cdot \nabla \nu_t)^f] + S_3 \beta (\omega k)^f \right\}}_{\epsilon_t\text{-terms}} = 0 \quad (3.6b)$$

$$S_1 = \left(\frac{\bar{\nu}_t U^2}{\bar{\ell}_u^2} \right) / \left[\left(\frac{\bar{u}_t^2}{\bar{\ell}_t \bar{\ell}_u} \right) (\bar{\nu} \bar{Re}) \right], \quad S_3 = (\Omega \bar{u}_t^2) / \left(\frac{\bar{\nu}_t \bar{u}_t^2}{\bar{\ell}_t^2} \right) \quad (3.6c)$$

$$\omega\text{-equation: } \underbrace{\left(\frac{\Omega}{\bar{\ell}_t \bar{\ell}_u} \right) (\bar{\nu} \bar{Re}) \left\{ \mathbf{u}^f \cdot (\nabla \omega)^f - S_2 \alpha \bar{\ell}_u^2 (g)^f \right\}}_{O(1)\text{-terms}} \quad (3.6d)$$

$$- \underbrace{\sigma_\omega \left(\frac{\bar{\nu}_t \Omega}{\bar{\ell}_t^2} \right) \left\{ [(\nu_t \nabla^2 \omega)^f + (\nabla \omega \cdot \nabla \nu_t)^f] + S_4 \beta^* (\omega^2)^f \right\}}_{\epsilon_t\text{-terms}} = 0 \quad (3.6e)$$

$$S_2 = \left(\frac{U^2}{\bar{\ell}_u^2} \right) / \left[\left(\frac{\Omega}{\bar{\ell}_t \bar{\ell}_u} \right) (\bar{\nu} \bar{Re}) \right], \quad S_4 = (\Omega^2) / \left(\frac{\bar{\nu}_t \Omega}{\bar{\ell}_t^2} \right) \quad (3.6f)$$

Comparing the above expressions (3.6a)–(3.6f) for the scaled model with those for the full model (3.1a)–(3.1c), in order for the similarity to hold between the scaled and full models, we must have

$$S_1 \approx 1, \quad S_2 \approx 1, \quad S_3 \approx 1, \quad S_4 \approx 1 \quad (3.7)$$

from which we obtain the following relations:

$$\text{From } S_1 \approx 1: \bar{u}_t^2 \propto (\bar{\nu} \bar{Re}) \bar{\nu}_t \frac{\bar{\ell}_t}{\bar{\ell}_u^3} \quad (3.8a)$$

$$\text{From } S_2 \approx 1: \Omega \propto (\bar{\nu} \bar{Re}) \frac{\bar{\ell}_t}{\bar{\ell}_u^3} \quad (3.8b)$$

$$\text{From } S_3 \approx 1 \quad \text{and} \quad S_4 \approx 1: \bar{\nu}_t \propto \Omega \bar{\ell}_t^2 \quad (3.8c)$$

$$\Downarrow \quad \bar{\nu}_t \propto \bar{\nu} \bar{Re} \left(\frac{\bar{\ell}_t}{\bar{\ell}_u} \right)^3 \quad \text{and} \quad \frac{\bar{u}_t^2}{U^2} \propto \left(\frac{\bar{\ell}_t}{\bar{\ell}_u} \right)^4 \quad (3.8d)$$

For a cross check, we have performed a similarity analysis using the k - ϵ turbulence model (Harlow and Nakayama, 1968; Yakhot et al., 1992) and confirmed that the same scaling relations are obtained as derived in the preceding relation.

It is observed that the similarity parameters, $\bar{\nu}_t$ and \bar{u}_t^2 (see Eq. (3.8d)), depend on the characteristic length ratio ($\bar{\ell}_t/\bar{\ell}_u$). This ratio can be estimated from the Kolmogorov characteristic length (Landau and Lifshitz, 1959; Hinze, 1975):

$$\text{Kolmogorov characteristic length: } \eta = \left(\frac{\nu^3}{\epsilon} \right)^{1/4} \quad (3.9a)$$

$$\text{Dissipation rate: } \epsilon = \beta^* \omega k \quad (3.9b)$$

The eddy dissipation length scale ratio ($\bar{\ell}_t$) can be obtained by

$$\begin{aligned} \bar{\ell}_t &= \left(\frac{\eta^s}{\eta^f} \right) \propto (\bar{\nu})^{3/4} \left[\frac{(\omega k)^s}{(\omega k)^f} \right]^{-1/4} \\ &\Downarrow \\ &\text{via (3.8a) and (3.8d)} \\ \left(\frac{\bar{\ell}_t}{\bar{\ell}_u} \right) &\propto \bar{Re}^{-1/3}, \quad \bar{Re} = \frac{Re^s}{Re^f} \end{aligned} \quad (3.10)$$

which constitutes an important theoretical result of the present paper.

Remark 3. The above length scale ratio, viz., the ratio of the Kolmogorov scale to the mean flow characteristic length ($\bar{\ell}_t/\bar{\ell}_u$) should not be confused with the ratio of the Kolmogorov scale to the integral length scale (ℓ_{int}) defined by (Wilcox, 1988, 2004)

$$\frac{\ell_t}{\ell_{int}} = R_t^{-3/4}, \quad R_t = u_t \ell_{int} / \nu, \quad u_t^2 = \frac{2k}{3} \quad (3.11)$$

for isotropic turbulence. An interesting relation that relates the mean flow length scale ratio ($\bar{\ell}_u$) (3.10) to the integral length scale (3.11) can be obtained as

$$\bar{\ell}_{int} \propto (\bar{R}_t^{3/4} \bar{Re}^{-1/3}) \bar{\ell}_u \quad (3.12)$$

We defer the validity of the above result for future study.

4. A scaling law for prediction of form drag coefficients

In this section we will utilize Eqs. (3.4), (3.8d) and (3.10) for a theoretical derivation of an eddy resistance law required in the performance assessment of full model vessels from a limited data set of scale model tests and/or high-fidelity CFD calculations.

The energy dissipation due to eddy-viscosity for the scaled model (E_t^s), E_t^s , can be expressed in terms of the full model variables as

$$\begin{aligned} E_t^s &= \left(\frac{\bar{\rho}}{\bar{\ell}_u} \right) \left(\frac{\bar{U}^2}{\bar{\ell}_t} \right) \left[2 \frac{\partial \nu_t}{\partial x_j} \rho u_i S_{ij} \right]^f + \left(\frac{\bar{\rho}}{\bar{\ell}_u} \right) \left(\frac{\bar{\nu}_t \bar{U}^2}{\bar{\ell}_u} \right) \left[2 \rho \nu_t \frac{\partial}{\partial x_j} u_i S_{ij} \right]^f \\ &\quad + \left\{ \bar{\rho} \bar{\nu}_t \left(\frac{\bar{\ell}_t}{\bar{\ell}_u} \right)^{-2} \left(\frac{\bar{U}}{\bar{\ell}_u} \right)^2 \left(\frac{\bar{u}_t}{\bar{U}} \right)^2 \right\} \left[\sigma^* \rho \frac{\partial \nu_t}{\partial x_j} \frac{\partial k}{\partial x_j} + \sigma^* \rho \nu_t \frac{\partial^2 k}{\partial x_j^2} \right]^f \end{aligned} \quad (4.1)$$

In order to relate the energy loss due to eddy viscosity of the scale model (E_t^s) to that of the full model (E_t^f), we introduce the following relations:

$$\begin{aligned} c_1 &= \left[2 \frac{\partial \nu_t}{\partial x_j} \rho u_i S_{ij} \right]^f / E_t^f, \quad c_2 = \left[2 \rho \nu_t \frac{\partial}{\partial x_j} u_i S_{ij} \right]^f / E_t^f, \\ c_3 &= \sigma^* \left[\rho \frac{\partial \nu_t}{\partial x_j} \frac{\partial k}{\partial x_j} + \rho \nu_t \frac{\partial^2 k}{\partial x_j^2} \right]^f / E_t^f, \quad c_1 + c_2 + c_3 = 1 \end{aligned} \quad (4.2)$$

Remark 4. The three turbulent energy dissipation terms associated with the coefficients (c_1, c_2, c_3) represent the energy dissipation due to spatial changes in kinematic eddy viscosity (ν_t), shear strains of mean flows (S_{ij}) à la the Boussinesq hypothesis, and the turbulent kinetic energy (k), respectively.

Substituting (4.2) into (4.1) and making use of (3.10), we arrive at the following scaling relation:

$$E_t^s = \left\{ c_1 \left(\frac{\bar{\ell}_t}{\bar{\ell}_u} \right)^{-1} + c_2 + c_3 \left(\frac{\bar{\ell}_t}{\bar{\ell}_u} \right)^2 \right\} (\bar{\rho} \bar{\nu}_t) \left(\frac{\bar{U}}{\bar{\ell}_u} \right)^2 E_t^f$$

$$E_t^s = \underbrace{\{c_1 \overline{Re}^{1/3} + c_2 + c_3 \overline{Re}^{-2/3}\} (\overline{\rho U})^2}_{\text{via equation (3.10)}} \frac{E_t^f}{\overline{U}} \quad (4.3)$$

Substituting (4.3) into the form drag coefficient ratio given by (2.10), together with (3.8d) for \overline{u}_t , we obtain

$$\overline{C}_p = \frac{C_p^s}{C_p^f} = \left[\frac{E_t^s}{E_t^f} \right] \left[\frac{\overline{U}}{\overline{\rho U}^2 \overline{u}_t} \right] = c_1 + c_2 \overline{Re}^{-1/3} + c_3 \overline{Re}^{-1}, \quad \overline{Re} = Re^s / Re^f \quad (4.4)$$

Conversely, the reciprocal of the preceding relation can be expressed as

$$\hat{C}_p = \frac{1}{\overline{C}_p} = \frac{C_p^f}{C_p^s} = \overline{c}_1 + \overline{c}_2 \overline{Re}^{1/3} + \overline{c}_3 \overline{Re}, \quad \overline{Re} = Re^s / Re^f \quad (4.5)$$

The preceding derivation indicates that the ratio of the form drag coefficients of two geometrically similar vessels depends only on the mean-flow Reynolds number ratio ($\overline{Re} = Re^s / Re^f$).

When one adopts the ITTC practice, the form drag coefficient (C_p) is expressed in terms of the friction resistance coefficient (C_f) and the form factor (K_f) as

$$C_p^f = K_f C_f^f, \quad C_p^s = K_s C_f^s, \quad C_f = 0.075 / (\log_{10} Re - 2)^2 \quad \text{la ITTC practice.} \quad (4.6)$$

where super and subscripts (s,f) refer to the scale model and full model vessels, respectively.

Substitution of (4.6) into (4.4), after some algebraic arrangements, yields

$$\frac{K_s}{K_f} = [c_1 + c_2 \overline{Re}^{-1/3} + c_3 \overline{Re}^{-1}] (C_f^f / C_f^s), \quad \sum_{i=1}^3 c_i = 1 \quad (4.7)$$

Observe that once the coefficients ($c_i, i = 1, 2, 3$) are determined from three data points either from experiments or high-fidelity CFD data, the full-scale form factor (K_f) can then be predicted.

Alternatively, one may utilize (4.5) to obtain the following form factor prediction formula:

$$\frac{K_f}{K_s} = [\overline{c}_1 + \overline{c}_2 \overline{Re}^{1/3} + \overline{c}_3 \overline{Re}] (C_f^s / C_f^f), \quad \sum_{i=1}^3 \overline{c}_i = 1 \quad (4.8)$$

Remark 5. A least-squares formula is often employed to utilize the experimental data to obtain full model form factors, which can be expressed as (see Tzabiras, 1992)

$$K_f = K_s + c(\lambda - 1), \quad \lambda = \frac{L^f}{L^s} \quad (4.9)$$

where λ is the ratio of the full model to the scale model lengths and c is a proportionality constant.

As geometrically similar scale models adopt the same Froude number (Fr), we have

$$Fr^f = Fr^s \Rightarrow \frac{U^f}{\sqrt{gL^f}} = \frac{U^s}{\sqrt{gL^s}} \Rightarrow \frac{U^s}{U^f} = \sqrt{\frac{L^s}{L^f}} = \lambda^{-1/2} \quad (4.10)$$

Therefore, the scale ratio (λ) can be expressed in terms of the Reynolds number ratio $\overline{Re} = Re^s / Re^f$ using (4.10) as

$$\frac{Re^s}{Re^f} = \overline{v}^{-1} \frac{U^s L^s}{U^f L^f} = \overline{v}^{-1} \left(\frac{L^s}{L^f} \right)^{3/2} = \overline{v}^{-1} \lambda^{-3/2}$$

$$\Downarrow \quad \lambda = \overline{Re}^{-2/3} \quad \text{provided} \quad \overline{v} = 1 \quad (4.11)$$

Hence, the least-square fit based on a linear scale ratio (4.9) can be expressed as

$$\frac{K_s}{K_f} = (1 + \overline{c}) - \overline{c} \overline{Re}^{-2/3}, \quad \overline{c} = c / K_f \quad (4.12)$$

Comparing the present theory (4.7) and the least-square fit based on the linear scale law (4.9), we observe that the least-square data-fit formula (4.9) can be thought of as an exponentially averaged formula

$$\overline{\alpha Re}^{-2/3} \quad \xleftarrow{\text{Exponential Averaging: } -2/3 \leftarrow 1/2(-1/3-1)} \quad \alpha_1 \overline{Re}^{-1/3} + \alpha_2 \overline{Re}^{-1} \quad (4.13)$$

However, we submit that our observation has no rational basis at present. We will evaluate these two formulas in the next section.

5. Correlation studies

A standard procedure for validation of a new theory would involve correlations with experiments and/or comparison with existing accepted theories. As alluded to in Introduction, there exists no established theory for predicting form factors. In addition, to the best of the present authors' knowledge, there appears to be no full-scale experiment data available in the open literature. Thus, we are motivated to seek for an alternative route, which is to rely on CFD-generated full-model form factors for validating the present theory (4.7). To this end, we first obtain the undetermined closure coefficients (c_1, c_2, c_3) in the present theory. Then, the full model form factor (K_f) can be predicted in terms of ($\overline{Re}, K_s, C_f^s / C_f^f$).

5.1. Determination of closure coefficients (c_1, c_2, c_3) of present theory

After searching through open literature, we have chosen Tzabiras (1992) and Kouh et al. (2009) for the determination of the three closure coefficients of the present theory. Among the six CFD-based results reported in Kouh et al. (2009), we have decided to use the DTMB 4515 while its shape is quite different from a fishing vessel-type used in Tzabiras (1992), their form factor variations statistically resemble each other.

In determining the closure coefficients, we have used the three data sets listed in Table 1.

Utilizing the Tzabiras CFD data set (Tzabiras, 1992) shown in Table 1, we obtain

Using P1 and P2 set :

$$\frac{0.101}{0.133} = \left[c_1 + c_2 \left(\frac{1.26 \times 10^6}{10^7} \right)^{-1/3} + c_3 \left(\frac{1.26 \times 10^6}{10^7} \right)^{-1} \right] / \overline{C}_f^{12}$$

$$\overline{C}_f^{12} = \frac{(\log(10^7) - 2)^2}{(\log(1.26 \times 10^6) - 2)^2} \quad \text{via ITTC formula}$$

Using P2 and P3 set :

$$\frac{0.133}{0.158} = \left[c_1 + c_2 \left(\frac{10^7}{10^8} \right)^{-1/3} + c_3 \left(\frac{10^7}{10^8} \right)^{-1} \right] / \overline{C}_f^{23}$$

$$\overline{C}_f^{23} = \frac{(\log(10^8) - 2)^2}{(\log(10^7) - 2)^2}$$

Consistency condition :

$$c_1 + c_2 + c_3 = 1$$

\Downarrow

$$c_1 = 0.7919, \quad c_2 = 0.2116, \quad c_3 = -0.0036, \quad (5.1)$$

Table 1
Experimental and CFD-generated data for present study.

Vessel		CFD scale model result			Full model form factor		
		P1	P2	P3	CFD full model result	Least-squares prediction (5.4)	Present theory (4.7) and (5.3)
Tzabiras (1992)	Re	1.26e+6	1.0e+7	1.0e+8	1.0e+9		
	K	0.101	0.133	0.158	0.1840	0.1958	0.1837
Kouh DTMB (Kouh et al., 2009)	Re	3.0e+7	1.0e+8	3.0e+8	1.0e+9		
	K	0.2017	0.2150	0.2305	0.2450	0.2433	0.2479
ℓ_2 -norm error					0.0%	6.45%	1.19%

Likewise, utilizing the DTMB CFD data set (Kouh et al., 2009) shown in Table 1, we obtain

Using P1 and P2 set :

$$\frac{0.2017}{0.2150} = \left[c_1 + c_2 \left(\frac{30^7}{10^8} \right)^{-1/3} + c_3 \left(\frac{30^7}{1.0^8} \right)^{-1} \right] / \bar{C}_f^{12}$$

$$\bar{C}_f^{12} = \left(\frac{(\log(10^8) - 2)^2}{(\log(3 \times 10^7) - 2)^2} \right) \text{ via ITTC formula}$$

Using P2 and P3 set :

$$\frac{0.2150}{0.2305} = \left[c_1 + c_2 \left(\frac{10^8}{3 \times 10^8} \right)^{-1/3} + c_3 \left(\frac{10^8}{3 \times 10^8} \right)^{-1} \right] / \bar{C}_f^{23}$$

$$\bar{C}_f^{23} = \frac{(\log(3 \times 10^8) - 2)^2}{(\log(10^8) - 2)^2}$$

Consistency Condition :

$$c_1 + c_2 + c_3 = 1$$

$$\Downarrow$$

$$c_1 = 0.7728, \quad c_2 = 0.2358, \quad c_3 = -0.0084, \quad (5.2)$$

Averaging the preceding two closure sets, we obtain

$$c_1 = 0.782388, \quad c_2 = 0.223720, \quad c_3 = -0.006108 \quad (5.3)$$

which will be used in subsequent validation assessment.

For a consistent comparison purpose, we have modified the least-squares data-fit formula proposed by García-Gómez (2000) to fit the CFD results given by Tzabiras (1992) and the DTMB 5415 of the Kouh et al. (2009) to arrive at

$$K_f = K_s + 1.038178 \times 10^{-3}(\lambda - 1), \quad \lambda = \overline{Re}^{-2/3} \quad (5.4)$$

5.2. Initial validation of present theory

The closure coefficients obtained for the present theory (5.3) based on CFD-generated data are employed to predict the full-model form factors, which are then compared with the CFD-generated full model form factors. In so doing, we have found that a too large jump of the full model Reynolds number (Re^f) from that of the scale model (Re^s) can lead to erroneous predictions of K_f . Hence, we have adopted the following incremental extrapolation procedure akin to Richardson's extrapolation:

$$\text{Compute : } dRe = (Re^f - Re^s)/N \text{ such that } dRe/Re_s < 1 \quad (5.5a)$$

Loop : for $n = 1 : N$ do

$$\text{Increment : } Re^n = Re^s + dRe \rightarrow \bar{Re} = Re^s / Re^n \quad (5.5b)$$

$$\text{Compute : } K_f^n \text{ via Eq. (4.7) using (5.3)} \quad (5.5c)$$

$$\text{Update : } K_s^n = K_f^n, \quad Re^s = Re^n \quad (5.5d)$$

Go to Loop

Convergence of the preceding procedure is verified by decreasing the size of Reynolds number increments (dRe), and it is found that $N = 10,000$ gave adequate convergence for all the cases computed. It should be emphasized that we have used the same coefficients (5.3) for predicting the form factors of the six vessels in the column of Present theory in Table 2.

In addition, we have computed the full model form factors using three data points. For example, for the case of the Tzabiras model ship listed in Table 1, three test data sets, viz., $\{P1 = (Re^s = 1.26 \times 10^6, K_s = 0.101), P2 = (Re^s = 10^7, K_s = 0.133), P3 = (Re^s = 10^8, K_s = 0.158)\}$, have been used to predict the three full model form factors K_f corresponding to full model Reynolds number $Re = 10^9$. We then obtained the average of the three full model form factors. The one that has the least deviation from the average value is the one we have selected, which happens to be $K_f = 0.1837$ which is listed in Table 1. A similar procedure is used to predict the full model form factors for the least squares formula (5.4).

The full model form factors predicted by the present theory (4.7) and the least-squares data-fitting formula (5.4) are summarized in Table 1. Note that the CFD-predicted form factors for full models are regarded as reference values. Comparing the ℓ_2 -norm errors of the present theory vs. the least-squares predictions of full model form factors, we find that the least-squares fit formula (5.4) leads to 5.5 times (6.45%) that of the present theory (1.19%). Hence, at least in terms of CFD-generated form factors, the present theory outperforms the least-squares formula that has been utilized in ship resistance prediction.

5.3. Performance of present theory using experimental scale model tests data

As a second validation step, we will examine the four scale model test data reported in García-Gómez (2000). It should be emphasized, as noted already, that no full-scale model form factors have been reported in open literature. This is summarized in Table 2.

Once again, it is reminded that the four full model form factors listed in Individual estimation shown in Table 2 are predicted values employing case-by-case least-squares formula based on scale model test data, with the exception of the CFD-generated cases for which full model form factors are obtained via CFD simulations. Nevertheless, in the absence of full model form factors, we have decided to use it as reference values. When applied to the form factor predictions of the four test vessels, the present theory (4.7) with the closure coefficients obtained from two CFD-simulation data (5.3) yields far lower ℓ_2 norm errors (11.5%) as compared to the least-squares formula (5.4) (30.5%).

Out of curiosity, we used the least-squares formula developed in García-Gómez (2000) which reads

$$K_f = K_s + 1.91 \times 10^{-3}(\bar{Re}^{-2/3} - 1) \quad (5.6)$$

Table 2

Experimental and CFD-generated data for present study.

Vessel		Scale model test				Full model form factor		
						Individual estimates	Least squares ^d	Present theory ^e
Ore	Re	4.19e+6	9.65e+6	1.56e+7	Not available	1.95e+9		
Carrier	K	0.198	0.227	0.272	available	0.307 ^a	0.2969	0.3231
Tina	Re	2.0e+6	3.07e+6	5.65e+6	7.9e+6	1.43e+9		
Onassis	K	0.044	0.071	0.128	0.132	0.197 ^a	0.1381	0.1973
Victory	Re	1.86e+6	3.41e+6	7.83e+6	1.13e+7	8.64e+8		
	K	0.077	0.105	0.130	0.152	0.173 ^a	0.1697	0.1682
Lucy	Re	5.96e+6	9.18e+6	1.68e+7	2.36e+7	3.77e+8		
Ashton	K	0.016	0.029	0.039	0.04	0.055 ^a	0.0455	0.0496
Tzabiras ^b	Re	1.26e+6	1.0e+7	1.0e+8	Not available	1.0e+9		
	K	0.101	0.133	0.158	available	0.184 ^c	0.1958	0.1837
Kouh ^b	Re	3.0e+7	1.0e+8	3.0e+8	Not available	1.0e+9		
DTMB	K	0.2017	0.2150	0.2305	available	0.2450 ^c	0.2433	0.2479
ℓ_2 -norm error						0.0% ^f	30.54%	11.5%

^a Estimated form factor for full models reported in García-Gómez (2000) based on least-square fit formula using the experimental data of each vessel.^b CFD simulated results (Tzabiras, 1992).^c Full model form factor computed via CFD.^d Based on the least-squares formula (5.4).^e Based on the present theory (4.7) whose coefficients (5.3) are determined with the Tzabiras CFD-generated data and the DTMB 5415 data set reported in Kouh et al. (2009).^f In the absence of the measured full model form factors for experimental scale model tests, the individually estimated full model form factors are used as reference values.

which gives the corresponding ℓ_2 error norm of 18.86%. It must be emphasized that the least-squares formula offered by García-Gómez (2000) was tailored to take full advantage of the four scale model experiment data. Nevertheless, García-Gómez's formula (5.6) yields its ℓ_2 -norm error higher than the six data prediction error of the present theory (11.5%).

6. Discussions

A theory for the prediction of the form factors for surface vessels subjected to turbulent incompressible flows is developed, which can be used to estimate the form drags for new geometrically similar vessels, provided there exists form drag database of existing scale model vessels. Derivation of the present form drag scaling law is founded on a hypothesis that stipulates the energy loss due to eddy viscosity dissipation away from the wetted surface which constitutes the dominant mechanisms of energy loss due to viscous pressure-caused form drag. In other words, whereas the derivation of the turbulent skin friction is founded on a direct method utilizing the local characteristics of turbulent flow in the boundary layer, the present derivation of the form drag scaling law is obtained by an indirect method. An appealing feature of the present form drag scaling law is that it depends only on the Reynolds number ratio of new and existing vessels, just as the skin friction laws do. The two independent closure coefficients can be determined from three distinct scale model data either from experiments or CFD-generated data set. To the best of our knowledge, the proposed form drag prediction theory (4.7) is its first kind. We now offer further comments regarding the limitations of the present work:

- The validity of the present viscous resistance coefficient formula (4.7) for a wide class of vessels is not fully established, and is restricted to the four test data plus two CFD-results. A more comprehensive correlation study needs to be undertaken before it can be accepted to the hydrodynamics community in general.
- The range of Reynolds numbers for which the present form factor formula remains valid has not been identified. One

fruitful area may be to explore the use of the so-called effective Reynolds number (see, e.g., Aspden et al., 2008) for establishing the Reynolds number ranges for which the present form factor formula remain valid. This merits a further study.

- The present analysis assumes that the present similarity relations derived for the characterization of form drag uniformly hold beyond the viscous sublayer (see, e.g., Barenblatt et al., 2000; Barenblatt, 2003 for the modeling of an intermediate region for which the similarity exponents become Reynolds number dependent). An adaptation of such a theory for an accurate prediction of ship resistance requires carefully carried-out experimental data, which is wanting to the best of our knowledge.
- The present analysis is restricted to steady state flows. Since energy dissipation occurs at the smallest eddy scales (Hinze, 1975), we have invoked Kolmogorov's original hypothesis: "...the fine pulsations of the higher orders are subjected to approximately space-isotropic statistical régime. Within small time-intervals it is natural to consider this régime approximately steady even in the case, when the flow in the whole is not steady (Kolmogorov, 1991, p. 10)." However, the characteristic wavelength changes from the mean flow all the way to the smallest eddy scales may be influenced by transient phenomena. The absence of time scales in the present analysis may turn out to be another attributes that need to be addressed.
- In applying the k - ω turbulence model, we have found that it does not enjoy a complete similarity, and thus we have employed a perturbation approach such that similarity relations hold separately for $O(1)$ and $\epsilon_t \ll 1$ -terms (see Eqs. (3.6)–(3.8)). This is introduced as a mathematical convenience, rather than on a physically rational ground. An analysis of fully coupled nonlinear behavior may shed further insight into the eddy viscosity and length scale relations. Nevertheless, the step-by-step procedure for deriving the form drag scaling law developed in the present paper can be applied to other turbulence models.
- The present work indicates that it may be argued that the k - ω turbulence model may implicitly handle a breakdown of a complete similarity. To the best of our knowledge, no credible theory exists to confirm or deny such a conjecture.

- Future work would focus on a classification of the vessel types and their *geosym* experimental and CFD-generated data analysis. This would increase the fidelity of the present theory for predicting form factors more realistically. This and related thrusts are being investigated.

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