VI-11 - STUDY OF THE STRESSES IN THE MAST

We approach here the true nut of the problem which will lead us to apply the theory which has just been developed, namely that we will use the aerodynamic load developed by each sail to analyse its action on the rigging. I had formerly used this method for calculation of the mast and rigging of origin of *Pen Duick III* whose very particular geometry of the sails, with its foresail with wishbone, did not make it possible to use the usual methods of calculation. Exposed here is the development that shows its limits. It implies of course the estimate of certain data, or of simplifications, that can be easily adjusted by simple observation.

Let us recall that the case studied is that of the close-hauled condition where the loads are greatest. A priori it appears simple to calculate the value of these forces by the usual aerodynamic formulas, unfortunately the coefficients of bearing pressure and real drag are too poorly known to offer sufficient precision. However studies based on the theories of aerodynamics are currently undertaken in this direction and are already applied to the layout of the sails.

One will be able however to use this method when one wants to calculate the loads due to the lighter sails. In this case one will be able to take for the side force:
$\mathbf{F_y = 0{,}0625 \times V^2 \times S \times C}$ where:
V = speed of the wind in m/s;
S = surface area of the sail in m^2;
C = variable coefficient of bearing pressure according to the shape and hollow's of the sail but which we will consider equal to 1,2.
On the other hand we can determine rather well the share of the total force developed by the yacht of the jib, the mainsail, and also the resistance of the rigging and the hull.
In addition we also know rather well the value of the lateral component of the total force since its moment compared to the centre of the immersed canoe body is at any moment equal to the righting moment.

Thus, from the righting moment practically all calculations of rigging are carried out.
The process will thus consist, from the most significant [maximum] righting moment, which produces maximum loads in the rigging, to determine the lever arm of the moment of capsizing and to calculate the corresponding total lateral force.
The second stage will lead us to define the share from each element of the rigging and to deduce from them the forces that they exert to their points of connection with the mast and the hull.

VI-II - 1. - CHOICE OF THE RIGHTING MOMENT

All the methods of calculation of the rigging start from this data, but all do not agree on the value to be taken into account. Some, such as Sparkman & Stephens, take a value equal to 1,5 times the righting moment at 30°, others the moment with 45°, others finally 45 times the moment at 1°. In fact, if one has the stability curve it will always be preferable to record on this the maximum value; table VI-II indeed shows the disparities that can occur with the approximate formulas.
Yacht I is a traditional beam cruiser with moderate freeboards and ratio of ballast of 45 %;
yacht II is a modern half-tonner with beam and high freeboards and low ratio of ballast.

TABLEAU VI-II
COMPARISON OF RIGHTING MOMENT IN m-daN, ESTIMATED ACCORDING TO DIFFERENT METHODS

	1,5 x Mr_{30}	Mr_{45}	45 x Mr_1	Mr maximum
Yacht I	2908	2495	3403	3214
Yacht II	2143	1793	2511	1852

From knowing the maximum righting moment exactly it seems that the righting moment at 30° gives the closest result to reality. For a first approximation for yachts up to approximately 12 m of floating length and 15 tons of displacement when fully loaded one will be able to estimate it at:
$Mr = (b^4 \times P_l)/(0{,}01 \times \Delta)$ where:
b = average of the maximum beams at the floating line;
P_l = weight of the ballast; with
Δ = displacement.
In all the cases the righting moments will be established for a mid-displacement.

VI-II - 2. - EVALUATION OF THE LATERAL FORCE

One starts with the height, H, between the centre of area of the sails and the immersed canoe body. Sparkman & Stephens estimate the centre of the hull at 40 % of the draft under the floating line. It seems that this value, higher than that corresponding to the centre of area from the plan of the hull, at least for the modern aileron yachts, is close to reality if one takes account of the difference in output of the various elements.

In addition, the influence of the hull on the output of the sails, the less favourable orientation of their lower part and the gradient of the wind lift the centre of pressure on the sail generally higher than the geometrical centre. On the other hand the presence of the hull reduces it and one can estimate that one compensates for the other.

One will thus take for the lever arm of the moment of capsizing to be the height of the centre of area of the sails above the floating line plus 40 % of the draft.

There will be thus $F_{y\ tot} = Mr / h$

One will be able to deduce the driving power from F_x knowing that the ratio: $F_{y\ tot} / F_x$ is approximately 4.

VI-II - 3. - DISTRIBUTION OF THE EFFORTS BETWEEN THE SAILS

It will initially be necessary to determine for which configuration of sails we want to carry out the study. For a sloop it will generally be with the maximum heavy genoa and the entire mainsail. They are conditions corresponding in general to navigation in a force 3 wind [7 to 10 knots] and relatively flat sea but where one is likely more to be laid down by a gust while carrying any top. In stronger weather, the state of the sea will generally result in the yacht being held down on its side more easily.

Of course, all other configurations can be adopted to study particular cases. A typical example is that of the sloop lying under spinnaker after a yaw. The F_y force then will be almost completely developed by the spinnaker.

As the sail's centre of pressure is about at middle height of the front triangle, the load exerted at the head of the mast by the halyard will reach $F_y / 2$.

Another interesting configuration can be that of the ketch sailing under jib and mizzen, without mainsail.

For these defined conditions, it is necessary to determine now the proportion, of the calculated force, between the hull unit + rigging and each sail.

According to the resistance of the hull and rigging share of F_y assumed to be 7,5 to 10% and 90 to 92,5 % for the sails.

The share returning to the rigging only (without the hull) is approximately 4 %.

In the longitudinal direction, resistance acts negatively, it accounts for 3,5 to 4 % of F_y including 3 % for rigging. The force F_x developed by the sails alone is thus equal to approximately 30 % of F_y.

By convention we will give to F_x and F_y negative values when the forces are directed backward or leeward.

The distribution of the effort between the sails depends on the output of these. In the lateral direction, the mainsail made little more than the jib; one will thus apply a coefficient of 1,05 to the mainsail against 0,95 for the jib. A mizzen will have a coefficient of 1,1, when it is used with the mainsail, 1,05 without it.

In the longitudinal direction it is the opposite; a jib will have a coefficient of 1,2 against 0,8 for the mainsail and 0,5 for a mizzen.

All these values are not arbitrary but rest on wind tunnel tests, in particular those carried out at the university of Southampton [1].

At this stage we will recapitulate to an example.

That is to say, a sailing ship whose characteristics are shown in (fig. VI-13):
Righting Moment at 30° = 3360 m-daN
Main Sail Surface = 20 m^2
Genoa Surface = 42 m^2
Height CV (*centre of sail area*) on floating = 5,85 m
Draft from waterline = 1,90 m

One will deduce:
Maximum Righting Moment: 1,5 x 3360 = 5040 m-daN
Arm of capsizing lever: 5,85 + (0,4 x 1,9) = 6,61 m

Laterally:
Total F_y: 5040 / 6,61 = -762, 5 daN of which
F_y sails: -762,5 x 0,9 = - 686,25 daN
Surface area of sails for calculation:
Mainsail: 20 x 1,05 = 21 m^2
Genoa: 42 x 0,95 = 39,9 m^2
Total sail area: 60,9 m2
F_y Main sail: 686,25 x 21 / 60,9 = - 236,64 daN
F_y genoa: 686,25 x 39,9 / 60,9 = - 449,61 daN
F_y rigging: 762,5 x 0,04 = - 30,5 daN

Longitudinally *:
F_x aerofoil: 762,5 x 0,3 = 228,75 daN
Sail surface areas for calculation
Mainsail: 20 x 0,8 = 16 m^2
Genoa: 42 x 1,2 = 50,4 m^2
Total: 66,4 m^2
F_x Main sail; 228,75 x 16/66,4 = 55,12 daN
F_x genoa: 228,75 x 50,4/66,4 = 173,63 daN
F_x rigging: 762,5 x 0,03 = 22,88 daN

* These forces are not used in this part of the calculation but are interesting to be known for other uses.

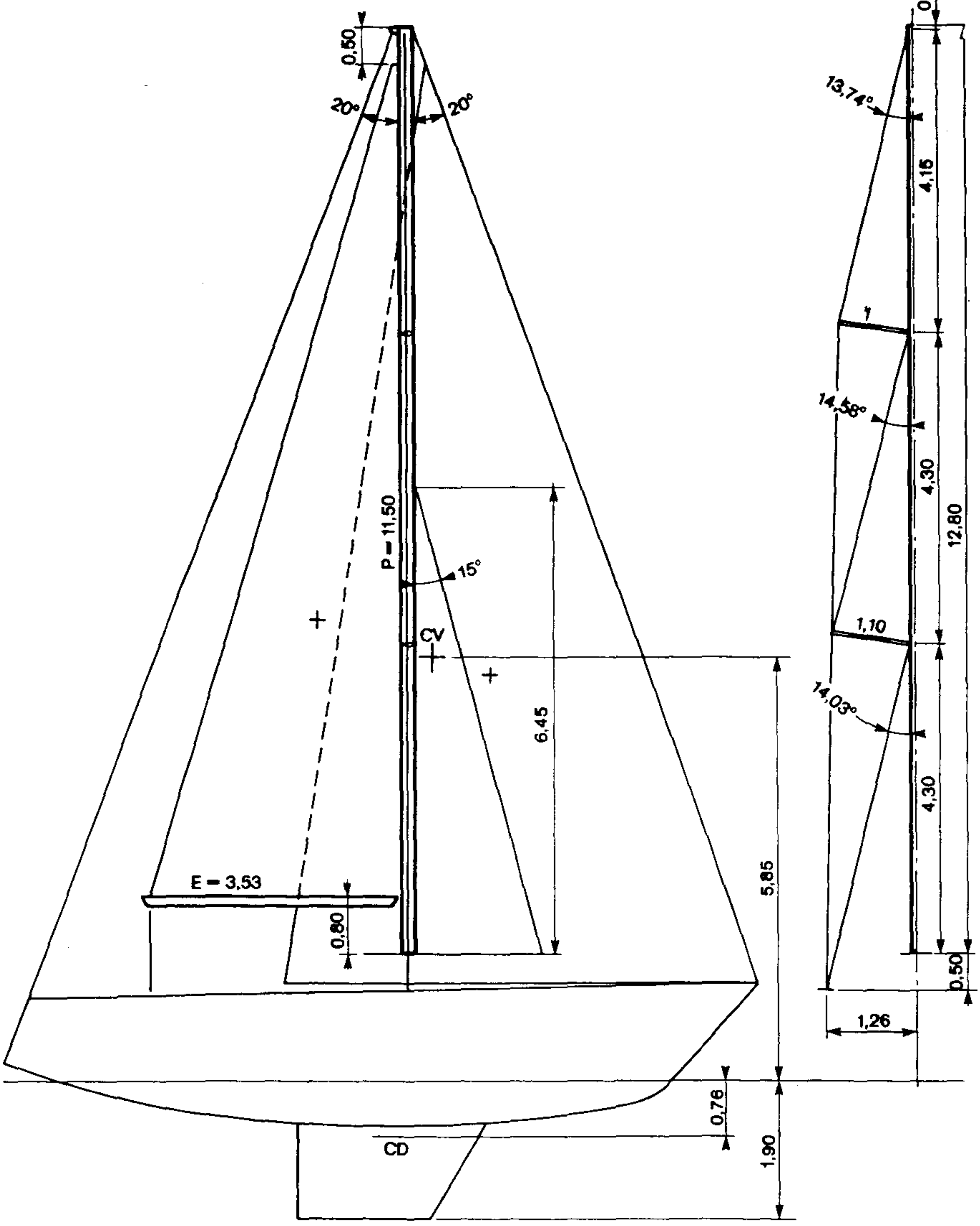

Fig. VI-13. Plan du gréement de l'exemple développé.

VI-II - 4. - EFFORTS OF THE SAILS ON THE MAST

Now that we know the efforts developed by each sail we can determine how these will be distributed on the rigging in the transverse and longitudinal directions.

VI-II - 4.1. Mainsail

Being unable to know exactly the distribution of the efforts in the sail, such as it was explained at the beginning of this chapter, we will adopt the following simplification.
We will consider the mainsail (fig. VI-14) as a juxtaposition of bands parallel with the leach and we will suppose that the whole surface is subjected to a uniform pressure.
The curve which the sail under the effect of this pressure will take will be constant and the whole of the sail will constitute a conical surface whose top will be the point of tack.
We know little about the curvature of this surface and we will consider initially that it is an arc of a circle. On the other hand it is relatively easy to observe the angle β_g, which forms the luff and the leach, it depends on the cut and the adjustment of the sail and generally lies between 30° and 40° in the plane of the boom and the mast. It is this plane which we will take for reference for the moment. Because of its flexibility, the fabric is aligned obligatorily with the force of the load that it transmits. The direction of this is thus given by the tangent to the sail to the point where it meets the spar.

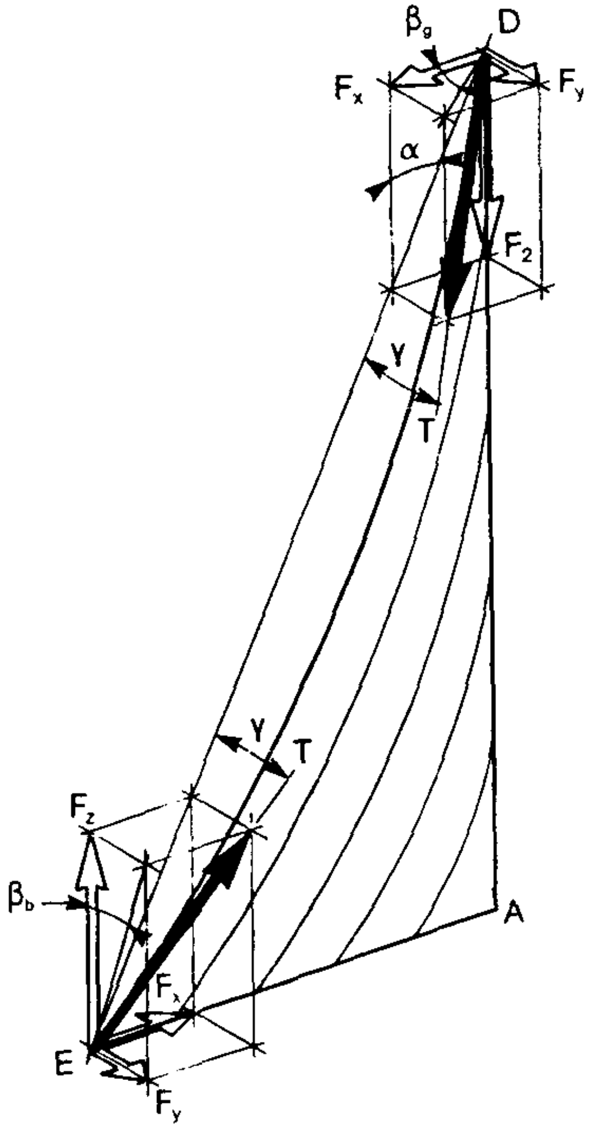

Fig. VI-14. *The mainsail is comparable to a series of bands parallel with the leach. Knowing the angle, β_g, which the sail forms from the luff to the end of the boom it is possible to determine the tensions T in these bands.*

The angle γ which results is given there by the formula $\tan \gamma = \tan \alpha \tan \beta_b \cos \alpha$ where $\tan \alpha = E/P$*

* P being the length of the luff and E that of the foot (symbols of IOR).

Taking again the formula of the sag of a cable, one will have as a total tension between mast and boom:

$T = F_{yg\text{-}v} / (2 \sin \gamma)$ which will give us:

- Transverse force on the mast and the boom

$P_y = T \sin \gamma = F_{yg\text{-}v} /2$

- Longitudinal force on the mast and compression in the boom:

$P_{xm} = P_{xb} = T \cos \gamma \sin \alpha = F_{yg\text{-}v} \cos \gamma \sin \alpha / (2 \sin \gamma)$

Compressive force on the mast and vertical on the boom:

$P_{zm} = P_{zb} = T \cos \gamma \cos \alpha = F_{yg\text{-}v} \cos \gamma \cos \alpha / (2 \sin \gamma)$

To see what that will give for our example by taking $\beta_g = 40°$, P = 11,50 m, E = 3,53 m, from where $\alpha = 17{,}07°$:

$\tan \gamma = \tan 17{,}07° \times \tan 40° \times \cos 17{,}07° = 0{,}246$

$\gamma = 13{,}84°$

$P_y = - 236{,}64/2 = -118{,}32$ daN

$P_{xb} = - P_{xm} = (236{,}64 \cos 13{,}84° \sin 17{,}07°) / (2 \sin 13{,}84°) = 141$ daN

$P_{zb} = - P_{zm} = (236{,}64 \cos 13{,}84° \cos 17{,}07°) / (2 \sin 13{,}84°) = 458{,}74$ daN

Knowing the distribution of these forces on the mast and the boom we will adopt a triangular distribution with a maximum at the points of the halyard and the clew and zero at the point of the tack. Sometimes this will be close to reality, at least for the mast because, for the boom, the cut of the low part of the sail can result in approaching the sail with a free edge where all the effort is concentrated on the clew.

VI-II - 4.2. Boom

We will suppose that the mainsheet is attached at the clew end of the boom and that the vang is not tensioned. When the mainsheet is not attached at the point of the clew, the efforts on its level will be E/x times those calculated at the clew, ‘x’ being its distance from the tack.

The resultant of the vertical and transverse efforts being at the third of the boom starting from the point of clew, one will have two thirds of the efforts on the clew and a third on the tack (fig. VI-15) is:

- at the clew: $p_{ye} = - 118{,}32 \times 2/3 = - 78{,}9$ daN *

  $p_{ze} = 458{,}74 \times 2/3 = 305{,}8$ daN

The load in the clew will be equal to: $T_e = \sqrt{(78{,}9^2 + 305{,}8^2)} = 315{,}8$ daN

The corresponding angle at the clew is given by $\tan \varepsilon = 78{,}9 / 305{,}8$ from where $\varepsilon = 14{,}46°$.

This angle is greater than that which one can observe in reality and which is around 10°.

It results owing from the fact that we chose for the curve of the sail an arc of a circle whereas actually the curve is flatter on the back, with perhaps too large a $\beta_{g\text{-}v}$ angle. The side force remaining constant, it is the vertical force and the load in the clew, which will increase.

If one takes $\varepsilon = 10°$ one will have: - at the clew:

$T_e = p_{ye} / \sin 10° = 78{,}9/\sin 10° = 454{,}2$ daN $\qquad p_{ze} = p_{ye} / \tan 10° = 447{,}3$ daN

- at the tack:

$p_{ya} = - p_{ye} / 2 = - 78{,}9 / 2 = - 39{,}4$ daN $\qquad p_{za} = p_{ze} / 2 = 223{,}7$ daN

---

* One can also regard the mainsail as a triangular surface articulated on the luff. Its centre of area being located at one third from this edge to the clew: $p_{ye} = Fy/3 = - 236{,}64/3 = - 78{,}9$ daN, identical result.

As for the load in the halyard it will be equal to $P_{zb} = p_{ze} + p_{za} = 447{,}3 + 223{,}7 = 671$ daN
These modifications of load on the boom will be reflected of course on the mast since
$P_{zm} = P_{zb} = - 671$ daN, one thus has
$P_{xm} = - 671 \tan \alpha = - 206$ daN and
$P_{xb} = 206$ daN
One notes whereas the load on the boom
$P_{zb} = - 671$ daN is appreciably equal to $3\ F_{yg\text{-}v} \cos \alpha = 678{,}5$ daN and
$P_{xb} \approx 3\ F_{yg\text{-}v} \sin \alpha = 194{,}5$ daN.

We will start from cm values for the calculation of the boom.
Attention, the pressure P_{xb} exerted by the boom on the mast can be much higher when the vang is tensioned, it then relieves vertical effort on the mainsheet but, taking into account the advanced position of its point of attachment on the boom and of its loading angle, the compression which it induces is important and modifies the equilibrium between the panels of the mast, influencing adjustment of its curve.

Up to now we had taken for reference the plane included by the mast and the boom, it is necessary for us now to bring back this plane to the axis of the sailing ship. We will take for the angle of the boom a value of 5°. The problem is quickly solved while passing by a polar/rectangular conversion in a computer. The corrected values will be established as follows:

$P_{xm} = - 206 \Rightarrow - 194{,}9$
$P_{ym} = - 118{,}32 \Rightarrow - 135{,}8$
The compression on the boom p_{xb} will break up at the point of tack into
$p'_{xa} = 206 \cos 5° = 205{,}2$ daN
$p'_{ya} = 206 \sin 5° = 18$ daN
The lateral force at the point of tack p_{ya} will break up into
$p''_{xa} = 39{,}4 \sin 5° = 3{,}4$ daN
$p''_{ya} = - 39{,}4 \cos 5° = - 39{,}3$ daN
And in total:
$p_{xs} = 205{,}2 + 3{,}4 = 208{,}6$ daN
$p_{ys} = 18 - 39{,}3 = - 21{,}3$ daN

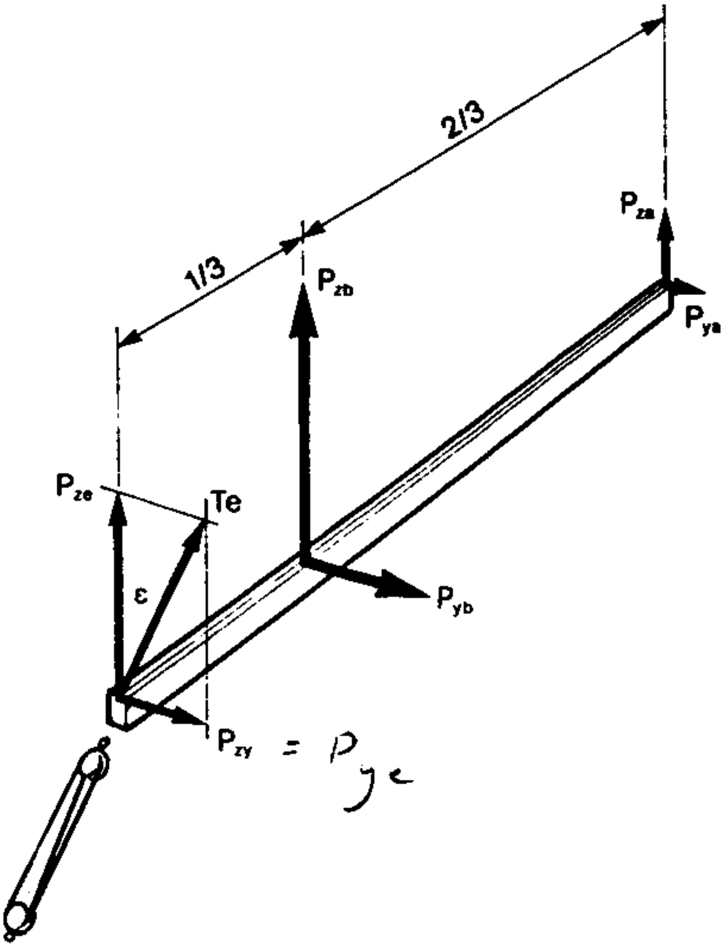

Fig. VI-15. Les efforts sur la bôme.

VI-II - 4.3. Genoa

On the boat used in the example, the clew jib car is at 1,50 m off the centreline of the yacht.
By observation we know that the clew lies in a plane appreciably parallel with the axial plane of the yacht and that the angle of attack of the genoa β is approximately 45°, finally we will allow for a deflection for the luff, envisaged by the yacht, of 0,43 m, in our example, 3,18 %, and a sail hollow of 10 %. We can thus trace a section perpendicular to the stay and passing by the clew (fig. VI-16).

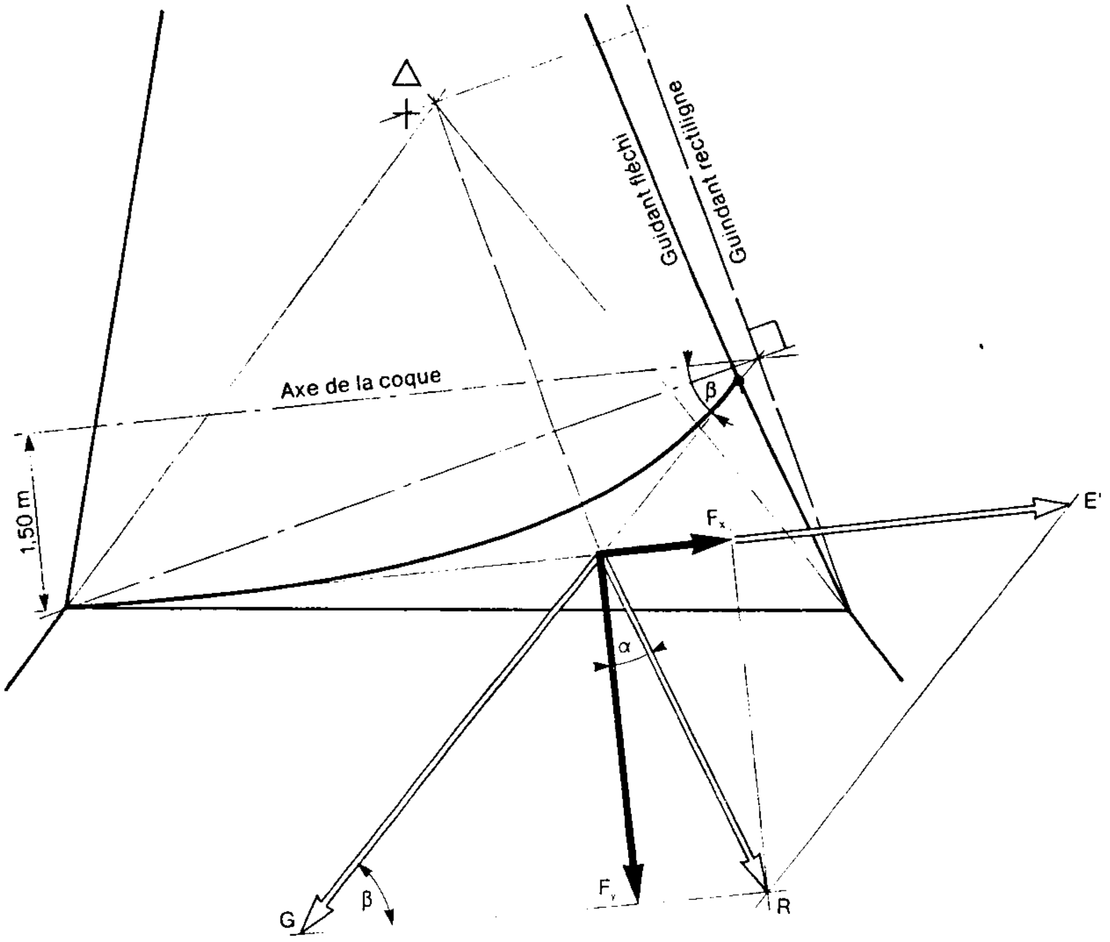

Fig. VI-16. Décomposition de la résultante du foc selon les directions perpendiculaires au guindant dans le plan de celui-ci et dans le plan de l'écoute.

The resultant of F_y and F_x pass by the intersection of the two tangent plans at the luff and the clew. This intersection is thus a place of the point of convergence of the three forces balancing the resultant. This cuts the line joining the clew to the stay at 3/10 of this, that is to say a little in front of the third, which is normal. It will be also noted that it is located a little in front of the perpendicular to this line, which is also normal if one takes account of a certain cant (inclination) of the sail on the whole his height.
By breaking up the resultant according to two tangent planes we will have load E' in the plane of the clew and the load G distributed on the luff.
$E' = F_y (\sin \alpha + \tan \beta) = 449{,}61 (\sin 19{,}95° + \tan 45°) = 603$ daN
$G = Fy / \sin \beta = 449{,}61 / \sin 45° = 636$ daN

We can now trace a side projection of the genoa on which we will carry the position of the point of convergence of the forces (fig. VI-17). We will place it at the same height as the geometrical centre of area and on the straight line parallel with the luff previously seen.
From this point we have the direction of the three forces A, D and E passing respectively by the three points of tack, halyard and clew.
From E', projection of E on the perpendicular with the luff, we obtain E = 709,46 daN, as the load in the clew.
By composition of E and F_x and decomposition of their resultant we obtain projections on the axial level of D and A.
A folding back on a transverse plane will give us, starting from F_y, the projection of the three forces from this point of view. We can measure or calculate their full-scale value and their angles in the longitudinal and transverse planes.

For our example one will find as follows:
A = 812 daN
D = 1226 daN
$\alpha = 9{,}94°$
$\gamma = 10{,}06°$

One notes, within sight of these angles, the importance that to provide the points with high and low attachment of the stay and the point of tack of an articulation with sufficient clearance and to correctly draw the exit of the halyard that these do not wear on the edges of the sheave box.
Up to now we reasoned as if the genoa were not affixed to a stay.
The presence of the stay and its natural tension will balance a part of the normal forces in the luff and will reduce much of the component in the load of the halyard. It is very difficult to define this proportion exactly the more so as it is eminently variable since it is dependent on the cut and the adjustment of the sail and of the rigging. The distribution along the luff is also poorly known. We will estimate, a little arbitrarily, I recognize, that the force on the luff G will be distributed for 2/3 on the forestay and 1/3 on the luff.
The tension in the stay which results from it will be then T = G / 12 x c, where c is the curve expressed by the deflection/luff ratio.
In our example:
T = 636 x 100 / 12 x 3,18 = 1667 daN.

The reduction of the tension in the halyard, that shares only part of the corresponding tension in the luff, will be estimated to be 1/4 of the tension of the luff. We will thus have:
D = 1226 x 0,75 = 919,5 daN
The sum of the two tensions will determine the total efforts with the rigging of the stay on the mast and the fitting of the stem.
At the head of the mast we will have then: Halyard + stay = 919,5 + 1667 = 2586,5 daN which will break up into:
Mast Compression $F_z = \sqrt{(2586^2 / (\sin^2\gamma + 1/\cos^2\alpha))} = 2511$ daN
Side Force $F_y = 2511 \times \sin\gamma = 438{,}5$ daN
Longitudinal Force $F_x = 2511 \times \sin\alpha = 433{,}5$daN

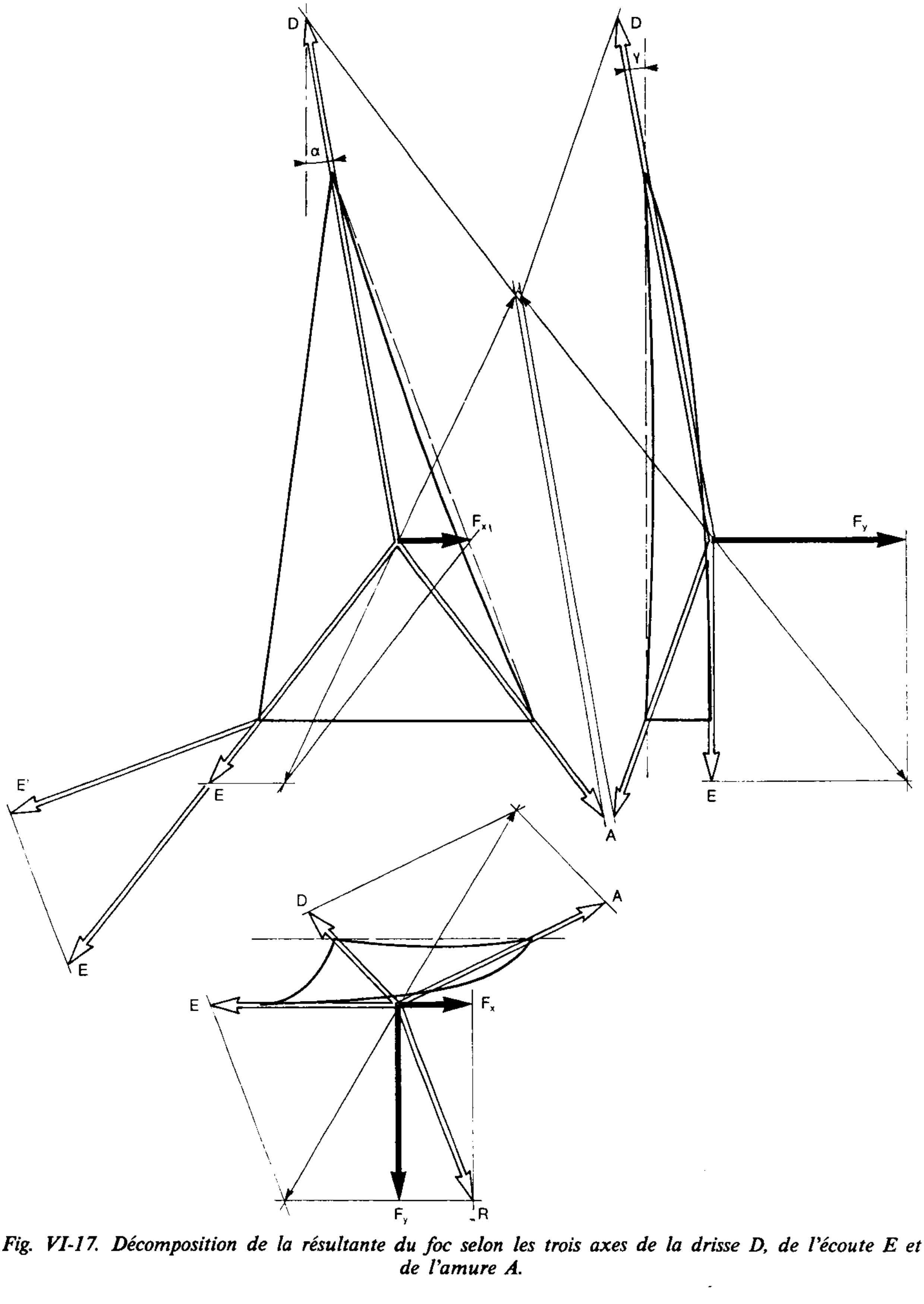

Fig. VI-17. Décomposition de la résultante du foc selon les trois axes de la drisse D, de l'écoute E et de l'amure A.

VI-II - 4.4. Distribution of the effort of the sails on the mast

We can now represent the distribution (which we have triangulated) of the efforts of the mainsail on the mast in the longitudinal and transverse planes (fig. Vl-18 a and b) by considering, as for all the calculations, that the mast remains straight. For a triangular distribution of the load of the mainsail, the value of the load exerted per unit of length, on the level of the point of the halyard is equal to **2F/P** is:

Longitudinal: $p_x = 2 \times 194{,}9 / 11{,}5 = 33{,}90$ daN/m
Transverse: $p_y = 2 \times 135{,}8 / 11{,}5 = 23{,}62$ daN/m
From the site of the points of attachment we can draw up a first table of the loads that are exerted there:

	Longitudinal	Transversal
Head of mast	433,5 - 75,86 = 357,64	
Babystay	-79,43	
Cap shroud		- 470,93
2^{nd} spreader		- 69,00
1^{st} spreader		- 36,05
Foot of mast	168,98	- 25,72
	447,19	- 601,70

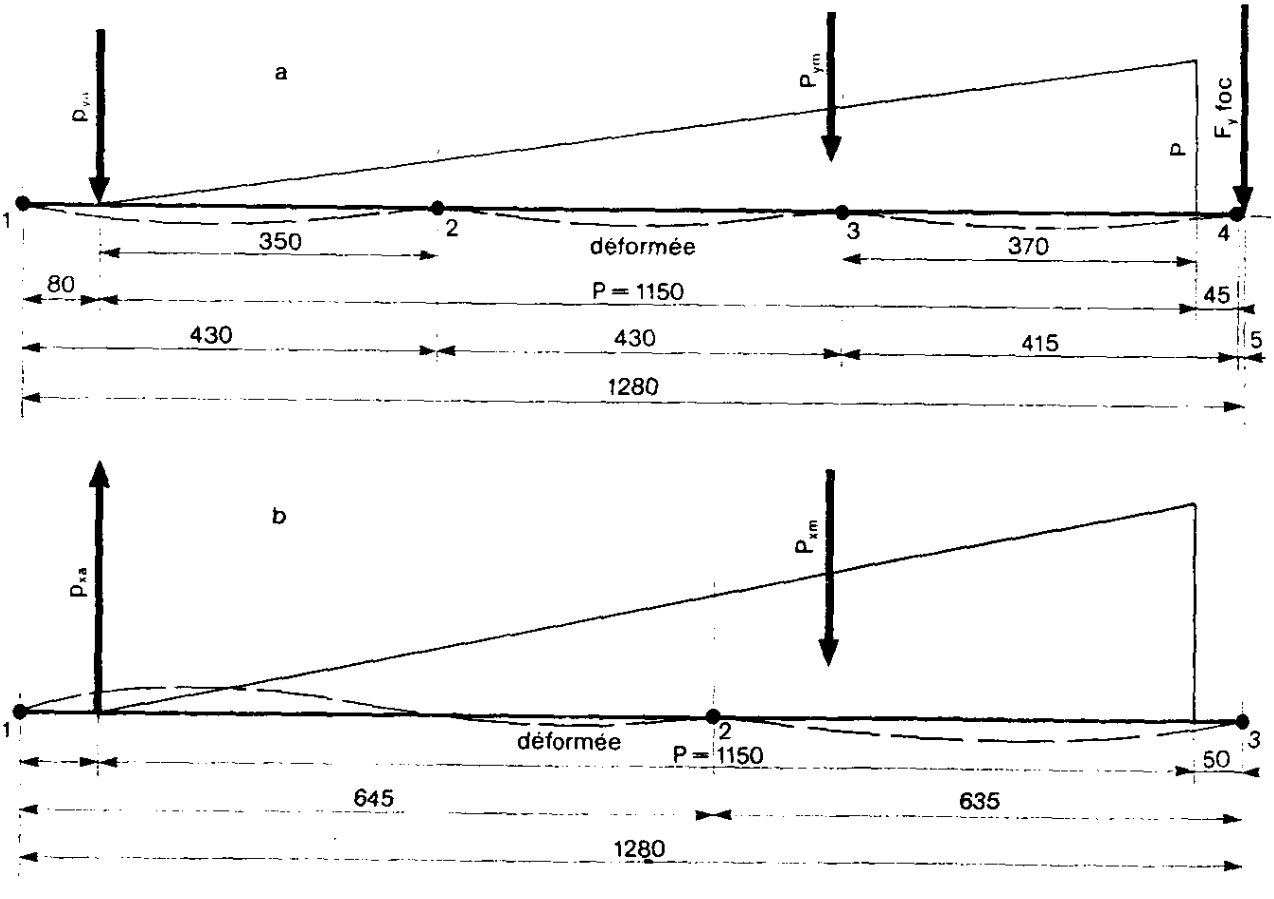

Fig. VI-18. Géométrie du mât dans le sens transversal (a) et longitudinal (b).

If the mast were made up segments articulated at each point of fixing these are the loads which act on the shrouds. But actually the mast is a continuous profile. At each node will be created equal bending moments of opposite direction which will modify the load that is exerted there. The calculation, which can be solved by "the three moments theorem" is extremely complex and I will not develop it here.
For our example one leads to the corrected table:

	Longitudinal	Transversal
Head of mast	362,54	
Babystay	-89,16	
Cap shroud		-464,14
2nd spreader		- 77,98
1st spreader		- 43,32
Foot of mast	173,81	- 16,26
	447,19	- 601,70 1

In the case of a keel-stepped mast the results would be a little different because of the presence of an additional section between the partners and the foot of the mast.

VI-II - 5. LOADS IN THE STAYS

From knowing the load in each point we will be able to calculate those of the stays and the compression that they will induce in the mast.

VI-II - 5.1 longitudinal Staying (fig. VI-19)

The load at the head will be balanced by the load of the backstay:
$T = F_x/\sin \alpha$ inducing a compression in the mast:
$F_z = F_x/\tan \alpha$
The same formulas will apply to runners and the inner forestays.

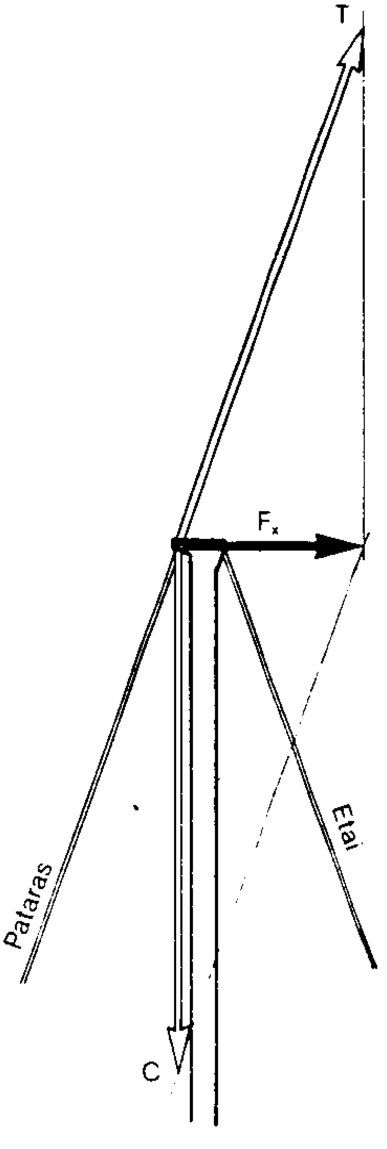

Fig. VI-19. Load at the head of mast F_x generates a load in the backstay T and a compression in the mast C. According to their distance from the axis of the mast, compressions due to the forestay and the backstay can generate a bending moment in the top part of the mast.

VI-II - 5.2. Transverse staying (fig. VI-20)

The spreaders are fixed according to the bisection of the angle which the stay makes at its end.

When the stay goes down again parallel to the mast, its angle is:

$\alpha = 2 \sin^{-1} (f/(2 \times l))$ has, with f: the length of the spreader.

In this case the angle of the spreader $\beta = \alpha/2$

The compression induced in the mast is:

$F_z = F_y / \tan \alpha$ and force in the stay:

$T = F_y / \sin \alpha$

On the level of the spreader one has

- Compression in the spreader: $C = (2 F_y \sin \alpha/2) / (\sin \alpha)$
- Lateral Component on the mast $f_y = F_y$
- Vertical Component on the mast: $f_z = (2 F_y \sin^2\alpha/2) / (\sin \alpha)$

When the stay does not go down again parallel to the mast (case of spreaders length gradually reduced) but formed with the vertical an angle β, angle α practically does not change. That of the spreader is increased by γ/2.

One has then on the level of the spreader

$c = (2 F_y/\sin \alpha) \times \sin((\alpha-\gamma)/2)$

$f_y = F_y (\sin \alpha - \sin \gamma)/\sin \alpha$

$f_z = F_y (\cos \alpha - \cos \gamma)/\sin \alpha$

The influence of the angle γ is very large and can involve important reductions of c and F_y with a corresponding transfer of the load in the higher shroud when one gradually reduces the length of the spreaders from the lower stages towards the higher stages.

For the shrouds in the plane of the mast, the formulas are the same ones as for the backstay. In the case of double lower shrouds, forward lower will support the transverse load, but also the longitudinal loading reserved previously for the baby stay. It will thus have a double vertical component and its load will increase the resultant longitudinal loading.

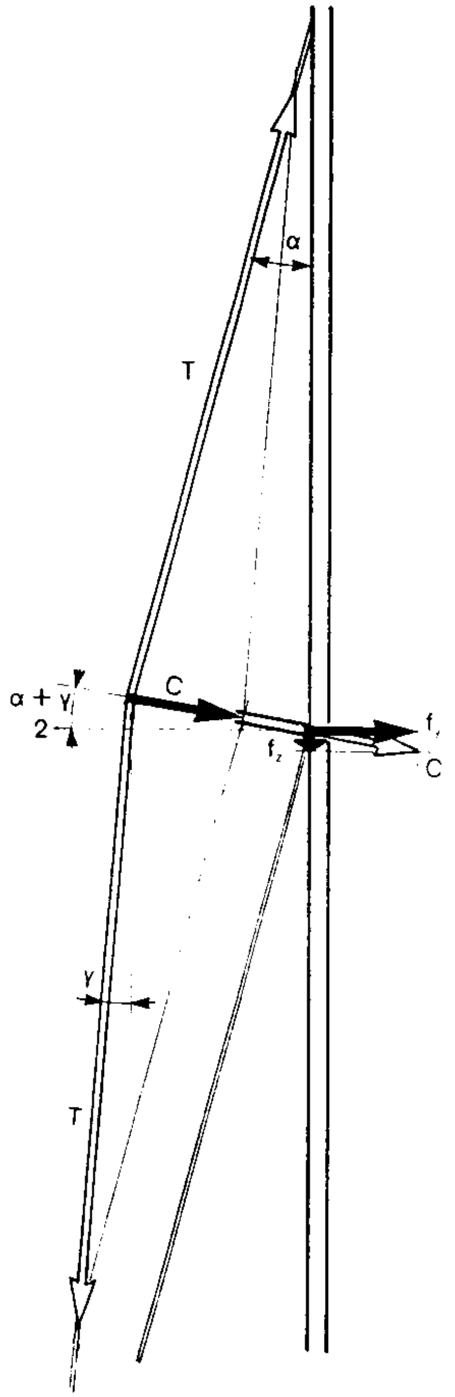

Fig VI-20. *The tension of the cap-shroud transmits to the mast, via the spreader, a force 'c' which breaks up into a compression f_z and a transverse force f_y the value of 'c' varies very quickly when the angle γ is modified.*

The resultant of the transverse load will be also distributed between the two lower shrouds and will be increased by taking account of their angles γ fore and aft.
One will have thus for the aft lower Shroud:
$T_{AL} = F_y / (2 \sin \alpha \cos \gamma)$
and for the forward lower shroud:
$T_{FL} = F_y / (2 \sin \alpha \cos \gamma) + F_x / \sin \gamma$
We will now be able to draw up a table (below) making it possible to define all the components of compression in the mast and loads in the shrouds.

POINT OF RIGGING	LOAD ON THE ELEMENT OF INTEREST*		LOADS ON THE MAST*			
			Longitudinal Rigging		Transverse Rigging	
			F_x	F_z	F_y	F_z
Head of mast	Forestay	1667	362,5	-2511		
	Jib Halyard	919,5				
	Jib Halyard Return			-919,5		
	Main halyard	671		-671		
	Main Hal. Return			-671		
	Backstay	*1059,9*		-996		
	Cap Shroud	*1954*			- 464,1	*- 1898*
2nd spreader	spreader	*406,5*			- 402,7	*- 55*
	Inter. Shroud	*1909,7*			- 56,37 - 402,7	*- 1848,2*
Babystay	Babystay	*344,6*	- 89,2	*- 332,9*		
1st spreader	spreader	*435*			- 430,7	*- 60,8*
	Lower Shroud	*1954*			- 28,98 - 430,7	*- 1895,6*
Gooseneck			208,6		- 27,4	
				- 6101,4		***- 5757,6***

*The figures in normal font result from preceding calculations, those in italic are due to the load of the staying which results from it. Not to complicate the demonstration too much, the resistance of the mast and rigging is not taken account of, in any event being small.

Compression in the three panels of the mast will thus be distributed as follows:
Higher panel: - 2511 - 919,5 - 2 x 671 - 996 - 1898 = - 7666,5 daN?
Intermediate panel: - 7666,5 - 55 - 1848,2 = - 9569,7 daN above the baby stay?
Intermediate panel: - 9569,7 - 332,9 = - 9902,6 daN below the baby stay?
Lower panel: - 9902,6 - 60,8 - 1895,6 = - 11859 daN.

VI-II - 6. - SAFETY COEFFICIENTS

Up to now we considered only the static loads. It is quite obvious that usually the coefficients intended to take account of the tolerances of the mechanical characteristics; the loads on the stays and the mast will undergo significant variations due to the dynamic stresses from the movement of the yacht.
At the time of studies carried out in Sweden at the higher Technical training school Chalmers de Malmoe one measured the following variations by ratio with the average loads. The sailing ship used, L-32, measured 9,81 m H.T., displacement 4,6 t; the sloop sail area of 47,85 m^2 was supported by rigging at the head with one set of spreaders and a pair of fore and aft lowers.

Tension in the Shrouds	Forestay	+ 32 %
	Cap-shroud	+ 45 %
	Aft lowers	+ 71 %
	Forward lowers	+ 42 %
Stresses in the mast	Higher Panel	+ 133 %
	Lower Panel	+ 34 %

One will note the important variations of the tension of the aft lower and the load in the higher panel of the mast. They are explained very well by the fact that decelerations are much higher than accelerations; the mast then tends to bow forwards, from where an important overload of the aft lower and a release of the forward lower. As for the higher panel of the mast, being further from the centre of gravity it is subjected to higher decelerations, while at the same time, the high part of the sail flogs, it is now not supported.

Nevertheless the variations measured in the stay appear small compared to those of the higher part of the mast and I think that reality must be closer to their average.
It should be noted in addition that accelerations to which the rigging is subjected vary by the square speed. One imagines what can be the consequences for a multi-hull sailing in 30 knots.
All in all, by including the mechanical, dynamic factors and of fatigue one will be able to adopt the safety coefficients of the table below, for a monohull. For the multi-hulls, the coefficients of longitudinal rigging could be doubled.

	1 spreader	2 spreader
Forestay, backstay, runner, 7/8 rigging	3 -3,5	
Baby stay	2,2 - 2,4	
Cap Shroud	2 - 2,2	1,9 - 2,1
In line lowers	2,1 - 2,3	2,2 – 2,4
Forward lower	1,8 - 2	1,9 – 2,1
Aft lower, check stays	2,2 - 2,4	2,3 - 2,5
halyards	2,5	
Sheets	5	

The margin indicated takes account of the conditions of use. The lowest value corresponds to small yachts for coastal cruising, the highest values for deep-sea yachts. It will be noted that the coefficients are lower for a single in line lower, little bothered by decelerations, than for an aft lower which, supports all the effects of decelerations to the mast.
If we apply these coefficients to the loads in the stays calculated above we will find respectively:
Forestay: 1667 x 3,5 = 5834 daN
Baby stay: 344,6 x 2,4 = 827 daN
Cap-shroud: 1954 x 2,1 = 4103,4 daN
Intermediate: 1909,7 x 2,1 = 4010,4 daN
Lower: 1954 x 2,4 = 4689,6 daN

VI-II - 7. - CALCULATION OF THE MOMENTS OF INERTIA OF THE MAST

The formula most usually allowed for the calculation of the masts is that of Euler * which gives:
$I_G = (P\ L^2) / (k\ \pi^2\ E)$ with
P: axial loading;
L: free length of the section;
k: coefficient of fixture;
E: modulus of elasticity.

* See page 122.

This formula can however apply only to little curved masts, whose deflection - within each panel - does not exceed the corresponding width of the profile. When the bending of the mast becomes too great it should be considered that the bending moment causing bending is balanced by the load of the sail. The mast does not work any more in compression, but its stability depends only on the sail. When the sail is not set (taking in a reef, bad adaptation of the sail to the mast by its cut or its adjustment, flogging, etc.) the mast comes to work in combined compression and deflection. It is a very different mode of calculation, in which all rigging must be considered to be hyper-static (lengthening of the shrouds and shortening of the profile), which would be far too involved for us here.
We will thus leave it for the moment with simple calculation of buckling, by taking account nevertheless of the transverse loads.

VI-II - 7.1. Coefficient of fixture

In the case of a mast, the coefficient of fixture is difficult to determine because, except at the head or at the foot where one can regard the support as articulated, at the other nodes, the presence of the adjacent panels will introduce a moment of resistance which will constitute a partial fixture.

A variable coefficient will result according to the relationship between the loads and the lengths in the panels.
The form of the deformation of the mast resulting from the provision of the points of anchoring of the staying will also have importance in the fact that it will allow or not, in one direction or the other, the participation of the adjacent panels. Theoretically, if the load were constant top to bottom of the mast and that it is perfectly straight one should have K = 2 for the end panels and 4 for the intermediate panels.
Taking into account in respect of the above conditions, the coefficient of fixture will lie between 1 and 2.
For example, for a mast with only one set of spreaders and stepped on the deck, the coefficient would be 1 if the load were equal in the two panels. However the higher panel, loaded differently, will bring to this an additional rigidity that will increase its critical load. The graphics of figure VI-21 give the value of K, according to the ratio of the loads in the upper (superior) and lower (inferior) panels and, P_i/P_s.

In the same way, the coefficient of fixture of a mast passing through the deck and stepped on the keel will depend on the l_2/l_1 ratio of lengths of the panels ranging between the deck and the keel and the lower spreaders and the deck. The graphics of figure VI-22 give the values of k_2 under these conditions.

The two coefficients apply to the lower panel.

If for example the P_i/P_s ratio = 1,25, one will have k_1 = 1,1 (fig. VI-21).
If, however, it is stepped at a height equal to 1/3 of the lower panel, there will be k_2 = 1,29 and the coefficient of fixture K = k_1 x k_2 = 1,1 x 1,29 = 1,42.

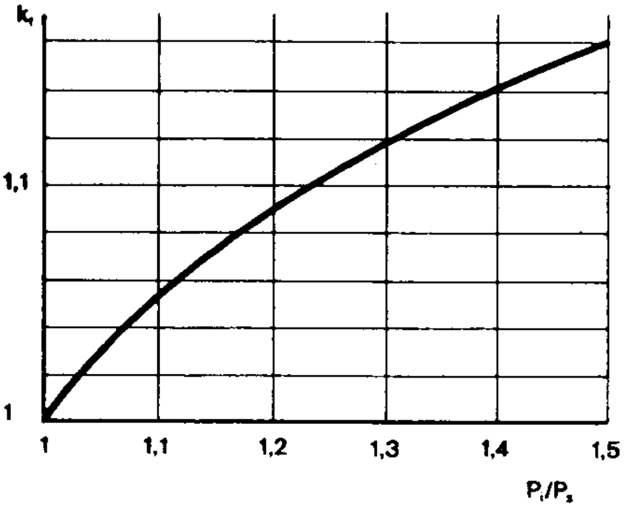

Fig. VI-21. Courbe du coefficient k_1 correspondant au rapport entre les forces de compression dans les panneaux adjacents.

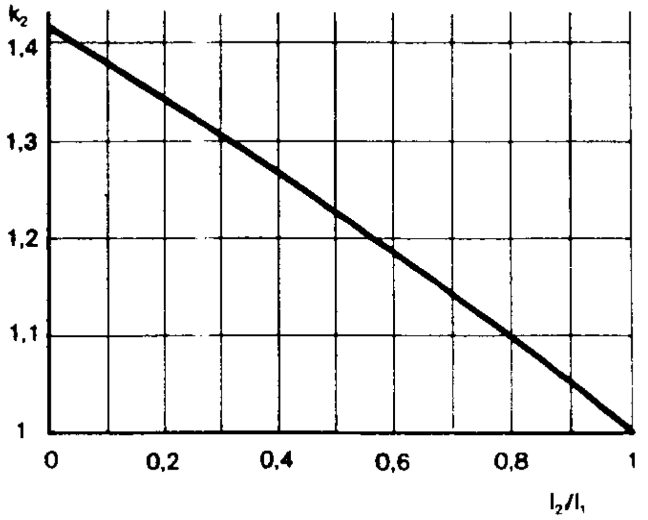

Fig. VI-22. Courbe du coefficient k_2 correspondant au rapport des longueurs des panneaux adjacents.

By combining these two coefficients one can about cover all the cases.
In our example we will have thus

Longitudinal:
The loads are not constant in all the height of the panels so we will take the averages of those located at the head and above the spreaders, and below and at the foot. One will thus have:
Pi/Ps = [(9902,60 + 11859)/2] / [(7666,5 + 9569,7)/2] = 10880,8 / 8618, 1 = 1,26 from where k = 1,05 for the lower panel and 1/1,05 = 0,95 for the higher panel.
The two panels being of equal length there is not another coefficient to apply. If it were not thus one could use the coefficient k_2 corresponding for the longest panel and $1/k_2$ for shortest.

Transverse:
In the first stage we will regard the mast as fixed between the spreaders and the deck. We will have:
Pi/Ps = 9902,6 / 7666,5 = 1,29 from where,
for the intermediate panel, a first coefficient k_1 = 1,11.
Likewise l_2 / l_1 = 4,3 / 4,3 = 1, k_2 = 1
For the higher panel k = $1/k_1$ = 0,90.
In the second stage one will consider that the whole is reversed and that it is the higher panel which is fixed. We will have then;
Pi/Ps = 9569,7 / 10880,8 = 0,88 from where,
for the intermediate panel, a second coefficient k'_1 = 0,94. Likewise l_1 / l_2 = 4,15 / 4,3 = 0,97, there will be k'_2 = 1,02.
The coefficient of total fixture of the intermediate panel will be:
$k_1.k_2.k'_1.k'_2$ = 1,11 x 1 x 0,94 x 1,02 = 1,06
For the lower panel one will have k = $1/k'_1$ = 1,07

These coefficients of fixture are in fact very pessimistic and would correspond to an alternate deformation of the panels. This case can nevertheless occur when there is no mainsail. Under normal conditions it would not be thus, at least in the transverse direction, and the coefficients of fixture would be practically doubled. We will preserve them despite everything, considering that these low values correspond to a safety coefficient.
Knowing the coefficients of fixture, we will be able to calculate the minimum moments of inertia necessary in each panel.
To reduce the numbers to be treated, one takes the centimetre as the unit of length. This will give for the alloy A-SGMT6, $E = 7 \times 10^5$ and for the yield stress $Re = 2400$ daN/cm^2

Longitudinal:
Lower panel: $P = 11859$ daN, $l = 645$ cm, $k = 1{,}05$
$I_L = (11859 \times 645^2)/(1{,}05 \times \pi^2 \times 7 \times 10^5) = 680 \text{ cm}^4$
Higher Panel: $P = 9569{,}7$ daN, $l = 635$ cm, $k = 0{,}95$
$I_T = (9569{,}7 \times 635^2)/(0{,}95 \times \pi^2 \times 7 \times 10^5) = 588 \text{ cm}^4$
Transverse:
Lower panel: $P = 11859$ daN, $l = 430$ cm, $k = 1{,}07$
$I_T = (11859 \times 430^2)/(1{,}07 \times \pi^2 \times 7 \times 10^5) = 297 \text{ cm}^4$
Intermediate Panel: $P = 9\,902{,}6$ daN, $l = 430$ cm, $k = 1{,}06$
$I_T = (9902{,}6 \times 430^2)/(1{,}06 \times \pi^2 \times 7 \times 10^5) = 250 \text{ cm}^4$
Higher Panel: $P = 7\,666{,}5$ daN, $l = 415$ cm, $k = 0{,}90$
$I_T = (7\,666{,}5 \times 415^2)/(0{,}9 \times \pi^2 \times 7 \times 10^5) = 212 \text{ cm}^4$

The most similar profiles that we will find will be Francespar F 380 A or Z Spar Z 500, whose characteristics are as follows:

	Francespar F380 A	Z Spar Z 500
External Dimensions mm	186 X 130	185 x 125
e mm	3,6	3,3
S (area) cm^2	19	18,5
I_T cm^4	380	350
I_T / v	58,5	56
I_L cm^4	730	730
I_L / v	78,5	77

We will calculate the buckling critical load,
$P_{crit} = k \times \pi^2 \times E \times I / l^2$ and working stress
$\sigma_a = k \times \pi^2 \times E / \lambda^2$ in the various panels for the Francespar profile.
For that, one will calculate initially the radius of gyration
$\rho = \sqrt{}\,(I / S)$ and the slenderness $\lambda = L / \rho$.

Longitudinal: $\rho = \sqrt{}\,(730/19) = 6{,}20$ cm
Lower Panel: $\lambda = 645/6{,}20 = 104{,}1$
$P_C = 1{,}05 \times \pi^2 \times 7 \times 10^5 \times 730 / 645^2 = 12\,729$ daN
$\sigma_a = 1{,}05 \times \pi^2 \times 7 \times 10^5 / 104{,}1^2 = 669{,}4$ daN/cm^2
Higher Panel: $\lambda = 635/6{,}20 = 102{,}4$
$P_C = 0{,}95 \times \pi^2 \times 7 \times 10^5 \times 730/6352 = 11\,882$ daN
$\sigma_a = 0{,}95 \times \pi^2 \times 7 \times 10^5 / 102{,}4^2 = 625{,}9$ daN/ cm^2

Transverse: $\rho = \sqrt{(380/19)} = 4{,}47$ cm

Lower Panel: $\lambda = 430/4{,}47 = 96{,}15$

$P_C = 1{,}07 \times \pi^2 \times 7 \times 10^5 \times 380/430^2 = 15\,192$ daN

$\sigma_a = 1{,}07 \times \pi^2 \times 7 \times 10^5 / 96{,}15^2 = 799{,}6$ daN/ cm^2

Intermediate panel: $\lambda = 430 / 4{,}47 = 96{,}15$

$P_C = 1{,}06 \times \pi^2 \times 7 \times 10^5 \times 380/430^2 = 15\,050$ daN

$\sigma_a = 1{,}06 \times \pi^2 \times 7 \times 10^5 / 96{,}15^2 = 792{,}1$ daN/ cm^2

Higher Panel: $\lambda = 415 / 4{,}47 = 92{,}8$

$P_C = 0{,}9 \times \pi^2 \times 7 \times 10^5 \times 380/415^2 = 13\,719$ daN

$\sigma_a = 0{,}9 \times \pi^2 \times 7 \times 10^5 / 92{,}8^2 = 722$ daN/ cm^2

VI-II - 7.2. Side load

The critical load which a column can accept working with buckling can be considerably reduced if a side load is applied to it. It is unfortunately the case of a mast, which is subjected to the load of the mainsail and the boom push.

It is hardly easy to introduce this additional element into a formula to make it possible to obtain the moment of inertia directly. It will be easier to check afterwards, once the profile of the mast is chosen from the calculation of the moments of inertia due to buckling alone.

One will first of all calculate the bending moment in the panel from the plan considered. To simplify, one will take account only of one concentrated resultant force for the lower panel, or a force uniformly distributed for the intermediate and higher panels and without fixture at the ends. Once again these conditions are definitely more unfavourable than reality since they do not take consideration of the adjacent panels. For the calculation of the moment one will use the following formulas:

Concentrated loading, $M = (p \sin k \times b \sin k \times a) / (k \sin kl)$

Distributed loading, $M = q/k^2 \left((1/\cos (kl/2))-1\right)$, with

p: specific side load;

q: load per unit of length = p/*l*;

a and b: distances to the ends from the concentrated loading;

L: length of the panel;

k: $\sqrt{(P/EI)}$

From this moment one will deduce the maximum stress by the formula:

$\sigma_{max} = P/S + M/[(I/v)(1-P/Pcrit)]$ and the safety coefficient its $Cs = \sigma_a / \sigma_{max}$

For our example one arrives at the following results:

Longitudinal, I = 730 cm, I/v = 78,5 cm^3, S = 19 cm^2

Lower Panel:

P = 11859 daN, P_c 12729 daN, *l* = 645 cm

p = 161,6 daN, a = 29,8, b = 615,2,

k = 0,0048 M = 80,68 daN-cm

$\sigma_{max} = 639{,}2$ daN/cm^2 Cs = 1,05

Higher Panel:

P = 9569,7 daN, P_c = 11882 daN, *l* = 635 cm

p = 147,86 daN, q = 0,23 daN/cm

k = 0,0043 M = 3,53 daN-cm

$\sigma_{max} = 504$ daN/cm^2 Cs = 1,24

Transverse, $I = 380$ cm^4, $I/v = 58,5$ cm^3, $S = 19$ cm^2
Lower Panel:
$P = 11859$ daN, $P_c = 15192$ daN, $\lambda = 430$ cm
$p = 40$ daN, $a = 153,3$, $b = 276,7$
$k = 0,0067$ $M = 68,85$ daN-cm
$\sigma_{max} = 629$ daN/cm^2 $Cs = 1,27$
Intermediate panel:
$P = 99026$daN, $P_c = 15050$ daN, $L = 430$ cm
$p = 49,9$ daN, $q = 0,12$ daN/cm
$k = 0,0061$ $M = 0,82$ daN-cm
$\sigma_{max} = 521$ daN/cm^2 $Cs = 1,52$
Higher Panel:
$P = 7666,5$ daN, $P_c = 13719$ daN, $l = 415$ cm
$p = 73,3$ daN, $q = 0, 18$ daN/cm
$k = 0,0054$ $M = 1,16$ daN-cm
$\sigma_{max} = 403,5$ daN/cm^2 $Cs = 1,79$

All stresses are substantially lower than the critical stresses; therefore the mast works well with buckling.

The safety coefficients can appear low, in particular in the longitudinal direction. In reality it will be very rare to reach these values, because, if the mast is slightly curved forwards by the baby stay, there is a balance between the moment thus created and that, of the direction opposed by the load of the mainsail creating a stabilization of the mast corresponding to a coefficient of fixture higher than those calculated.

When this stabilizing effect of the mainsail disappears, for example when taking in a reef, a moment when, moreover, the balance between the moments of the forestay and the mainsail and of the cap-shroud is not ensured any more, accidents can occur.
With the study of the results one also sees that those could be improved by displacement of certain points of anchoring of the shrouds. As a whole, a descent of those would make it possible to better balance the stresses in the panels, but it should be known that small modifications can have large consequences. In addition it is good to have higher coefficients in the upper panels where the dynamic stresses are greater.

VI-11 - 7.3. Local buckling

Thin tubes, like the profiles of a mast, can fail at a rate of constraint lower than that determined by a general buckling. It is normally in the curved part of the section that local buckling occurs; for an elliptic profile it will be the sidewall of average radius ‘r’.
Working stress for a tube will be: $\sigma_{co} = Re/[1 + 3\ (r/e)\ (Re/E)]$
In our example, with: $Re = 24$ daN/mm^2, $r = 122,5$ mm and $e = 3,6$ mm one has $\sigma_{co} = 17,78$ daN/mm^2, which remains much higher than the working stresses of general buckling.
This checking is nevertheless necessary, in particular for the large profiles or when, wishing to increase the moment of inertia, leads one to too low wall thicknesses.